thm_2Etoto_2Etoto__trans__less (TMVSymK1ykMMcT1McKTY4gqyT4TD3UF7dxD)

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Let $ty\_2EternaryComparisons\_2Eordering : \iota$ be given. Assume the following.

$$nonempty\ ty\_2EternaryComparisons\_2Eordering \tag{1}$$

Let $c\_2EternaryComparisons\_2EGREATER : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2EGREATER \in ty\_2EternaryComparisons\_2Eordering \tag{2}$$

Let $c\_2EternaryComparisons\_2ELESS : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2ELESS \in ty\_2EternaryComparisons\_2Eordering \tag{3}$$

Let $c\_2EternaryComparisons\_2EEQUAL : \iota$ be given. Assume the following.

$$c\_2EternaryComparisons\_2EEQUAL \in ty\_2EternaryComparisons\_2Eordering \tag{4}$$

Let $ty\_2Etoto\_2Etoto : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etoto\_2Etoto\ A0) \tag{5}$$

Let $c\_2Etoto\_2Eapto : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etoto\_2Eapto\ A\_27a \in (((ty\_2EternaryComparisons\_2Eordering^{A\_27a})^{A\_2} \tag{6}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0c \in (ty\_2Etoto\_2Etoto\ A\_27a).(\forall V1x \in A\_27a.(\forall V2y \in A\_27a.(\forall V3z \in A\_27a.((((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V0c)\ V1x)\ V2y) = c\_2EternaryComparisons\_2ELESS) \wedge ((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V0c)\ V2y)\ V3z) = c\_2EternaryComparisons\_2ELESS)) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V0c)\ V1x)\ V3z) = c\_2EternaryComparisons\_2ELESS)))))) \quad (7)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0c \in (ty\_2Etoto\_2Etoto\ A\_27a).(\forall V1x \in A\_27a.(\forall V2y \in A\_27a.(\forall V3z \in A\_27a.((((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V0c)\ V1x)\ V2y) = c\_2EternaryComparisons\_2ELESS) \wedge ((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V0c)\ V3z)\ V2y) = c\_2EternaryComparisons\_2EGREATER)) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V0c)\ V1x)\ V3z) = c\_2EternaryComparisons\_2ELESS)))))) \quad (8)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0c \in (ty\_2Etoto\_2Etoto\ A\_27a).(\forall V1x \in A\_27a.(\forall V2y \in A\_27a.(\forall V3z \in A\_27a.((((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V0c)\ V2y)\ V1x) = c\_2EternaryComparisons\_2EGREATER) \wedge ((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V0c)\ V3z)\ V2y) = c\_2EternaryComparisons\_2EGREATER)) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V0c)\ V1x)\ V3z) = c\_2EternaryComparisons\_2ELESS)))))) \quad (9)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0c \in (ty\_2Etoto\_2Etoto\ A\_27a).(\forall V1x \in A\_27a.(\forall V2y \in A\_27a.(\forall V3z \in A\_27a.((((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V0c)\ V2y)\ V1x) = c\_2EternaryComparisons\_2EGREATER) \wedge ((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V0c)\ V2y)\ V3z) = c\_2EternaryComparisons\_2ELESS)) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V0c)\ V1x)\ V3z) = c\_2EternaryComparisons\_2ELESS)))))) \quad (10)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0c \in (ty\_2Etoto\_2Etoto\ A\_27a).(\forall V1x \in A\_27a.(\forall V2y \in A\_27a.(\forall V3z \in A\_27a.((((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V0c)\ V1x)\ V2y) = c\_2EternaryComparisons\_2ELESS) \wedge ((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V0c)\ V2y)\ V3z) = c\_2EternaryComparisons\_2EEQUAL)) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V0c)\ V1x)\ V3z) = c\_2EternaryComparisons\_2ELESS)))))) \quad (11)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0c \in (ty\_2Etoto\_2Etoto\ A\_27a).(\forall V1x \in A\_27a.(\forall V2y \in A\_27a.(\forall V3z \in A\_27a.((((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V0c)\ V1x)\ V2y) = c\_2EternaryComparisons\_2EEQUAL) \wedge ((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V0c)\ V2y)\ V3z) = c\_2EternaryComparisons\_2ELESS)) \Rightarrow ((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V0c)\ V1x)\ V3z) = c\_2EternaryComparisons\_2ELESS)))))) \quad (12)$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0c \in (ty\_2Etoto\_2Etoto$$
$$A\_27a).(\forall V1x \in A\_27a.(\forall V2y \in A\_27a.(\forall V3z \in$$
$$A\_27a.((((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V0c)\ V1x)\ V2y) = c\_2EternaryComparisons\_2ELESS$$
$$((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V0c)\ V2y)\ V3z) = c\_2EternaryComparisons\_2ELESS)) \Rightarrow$$
$$((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V0c)\ V1x)\ V3z) = c\_2EternaryComparisons\_2ELESS)))))) \wedge$$
$$((\forall V4c \in (ty\_2Etoto\_2Etoto\ A\_27a).(\forall V5x \in A\_27a.$$
$$(\forall V6y \in A\_27a.(\forall V7z \in A\_27a.((((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto$$
$$A\_27a)\ V4c)\ V5x)\ V6y) = c\_2EternaryComparisons\_2ELESS) \wedge ((ap\ ($$
$$ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V4c)\ V7z)\ V6y) = c\_2EternaryComparisons\_2EGREATER)) \Rightarrow$$
$$((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V4c)\ V5x)\ V7z) = c\_2EternaryComparisons\_2ELESS)))))) \wedge$$
$$((\forall V8c \in (ty\_2Etoto\_2Etoto\ A\_27a).(\forall V9x \in A\_27a.$$
$$(\forall V10y \in A\_27a.(\forall V11z \in A\_27a.((((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto$$
$$A\_27a)\ V8c)\ V10y)\ V9x) = c\_2EternaryComparisons\_2EGREATER) \wedge ($$
$$(ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V8c)\ V11z)\ V10y) = c\_2EternaryComparisons\_2EGREATER)$$
$$((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V8c)\ V9x)\ V11z) = c\_2EternaryComparisons\_2ELESS))))))$$
$$((\forall V12c \in (ty\_2Etoto\_2Etoto\ A\_27a).(\forall V13x \in A\_27a.$$
$$(\forall V14y \in A\_27a.(\forall V15z \in A\_27a.((((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto$$
$$A\_27a)\ V12c)\ V14y)\ V13x) = c\_2EternaryComparisons\_2EGREATER) \wedge$$
$$((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V12c)\ V14y)\ V15z) = c\_2EternaryComparisons\_2ELESS)) \Rightarrow$$
$$((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V12c)\ V13x)\ V15z) = c\_2EternaryComparisons\_2ELESS)))))$$
$$((\forall V16c \in (ty\_2Etoto\_2Etoto\ A\_27a).(\forall V17x \in A\_27a.$$
$$(\forall V18y \in A\_27a.(\forall V19z \in A\_27a.((((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto$$
$$A\_27a)\ V16c)\ V17x)\ V18y) = c\_2EternaryComparisons\_2ELESS) \wedge (($$
$$ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V16c)\ V18y)\ V19z) = c\_2EternaryComparisons\_2EEQUAL)) \Rightarrow$$
$$((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V16c)\ V17x)\ V19z) = c\_2EternaryComparisons\_2ELESS)))))$$
$$(\forall V20c \in (ty\_2Etoto\_2Etoto\ A\_27a).(\forall V21x \in A\_27a.$$
$$(\forall V22y \in A\_27a.(\forall V23z \in A\_27a.((((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto$$
$$A\_27a)\ V20c)\ V21x)\ V22y) = c\_2EternaryComparisons\_2EEQUAL) \wedge ($$
$$(ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V20c)\ V22y)\ V23z) = c\_2EternaryComparisons\_2ELESS)) \Rightarrow$$
$$((ap\ (ap\ (ap\ (c\_2Etoto\_2Eapto\ A\_27a)\ V20c)\ V21x)\ V23z) = c\_2EternaryComparisons\_2ELESS))))))$$