

# thm\_2Etransc\_2EASN (TMbQiUHJDHqDbE1fUoiq4SS6iv6phTgEqvc)

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**Definition 1** We define `c_2Emin_2E_40` to be  $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \text{ (ap } P \ x)) \text{ of type } \iota \Rightarrow \iota.$

**Definition 2** We define `c_2Emin_2E_3D` to be  $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj\_o } (x = y)$  of type  $\iota \Rightarrow \iota.$

**Definition 3** We define `c_2Ebool_2E_3F` to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } V0P \text{ (ap } (c_2Emin_2E_40 \ A \ P))$

**Definition 4** We define `c_2Ebool_2E_T` to be  $(\text{ap } (\text{ap } (c_2Emin_2E_3D \ (2^2)) \ (\lambda V0x \in 2. V0x)) \ (\lambda V1x \in 2. V1x))$

**Definition 5** We define `c_2Ebool_2E_21` to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (c_2Emin_2E_3D \ (2^{A-27a}) \ P))$

**Definition 6** We define `c_2Ebool_2E_F` to be  $(\text{ap } (c_2Ebool_2E_21 \ 2) \ (\lambda V0t \in 2. V0t)).$

**Definition 7** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2. \lambda Q \in 2. \text{inj\_o } (p \Rightarrow q)$  of type  $\iota.$

**Definition 8** We define `c_2Ebool_2E_7E` to be  $(\lambda V0t \in 2. (\text{ap } (\text{ap } c_2Emin_2E_3D_3D_3E \ V0t) \ c_2Ebool_2E_F))$

**Definition 9** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (c_2Ebool_2E_21 \ 2) \ (\lambda V2t \in 2. V2t))$

**Definition 10** We define `c_2Ebool_2E_3F_21` to be  $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } c_2Ebool_2E_2F_5C \ A \ P))$

Let `c_2Enum_2EZERO_REP` :  $\iota$  be given. Assume the following.

$$c_2Enum_2EZERO\_REP \in \text{omega} \tag{1}$$

Let `ty_2Enum_2Enum` :  $\iota$  be given. Assume the following.

$$\text{nonempty } ty\_2Enum\_2Enum \tag{2}$$

Let `c_2Enum_2EABS_num` :  $\iota$  be given. Assume the following.

$$c_2Enum_2EABS\_num \in (ty\_2Enum\_2Enum^{\text{omega}}) \tag{3}$$

**Definition 11** We define `c_2Enum_2E0` to be  $(\text{ap } c_2Enum_2EABS\_num \ c_2Enum_2EZERO\_REP).$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (4)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (5)$$

**Definition 12** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (6)$$

**Definition 13** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Let  $ty\_2Erealx\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealx\_2Ereal \quad (7)$$

Let  $c\_2Ereal\_2Epow : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealx\_2Ereal^{ty\_2Enum\_2Enum})^{ty\_2Erealx\_2Ereal}) \quad (8)$$

Let  $c\_2Earithmetic\_2EFACT : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EFACT \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}) \quad (9)$$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealx\_2Ereal^{ty\_2Enum\_2Enum}) \quad (10)$$

**Definition 14** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

**Definition 15** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

**Definition 16** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (11)$$

Let  $c\_2Earithmetic\_2EDIV : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (12)$$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (13)$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (14)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal}) \quad (15)$$

**Definition 17** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (c\_2Emin\_2E40\ (t$

Let  $c\_2Erealax\_2Etrealm\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal}) \quad (16)$$

Let  $c\_2Erealax\_2Etrealm\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal}) \quad (17)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})} \quad (18)$$

**Definition 18** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 19** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal$

Let  $c\_2Erealax\_2Etrealm\_inv : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_inv \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal}) \quad (19)$$

**Definition 20** We define  $c\_2Erealax\_2Einv$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal\_ABS$

Let  $c\_2Erealax\_2Etrealm\_mul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal})^{ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal}) \quad (20)$$

**Definition 21** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 22** We define  $c\_2Ereal\_2E2F$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.($

Let  $c\_2Earithmetic\_2EEVEN : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (21)$$

**Definition 23** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A\_27a. (\lambda V2t2 \in A\_27a. ($

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \end{aligned} \quad (22)$$

**Definition 24** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in A\_27a. \lambda V1y \in A\_27b. (ap\ (c\_2E$

Let  $c\_2Ereal\_2Esum : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Esum \in ((ty\_2Erealax\_2Ereal^{(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum)}) \quad (23)$$

**Definition 25** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 26** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 27** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

**Definition 28** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_2Erealax\_2Etrealm\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal \\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (24)$$

**Definition 29** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal. \lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 30** We define  $c\_2Ereal\_2Ereal\_sub$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal. \lambda V1y \in ty\_2Erealax\_2Ereal$

Let  $c\_2Erealax\_2Etrealm\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (25)$$

**Definition 31** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal. \lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 32** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal. \lambda V1y \in ty\_2Erealax\_2Ereal$

**Definition 33** We define  $c\_2Ereal\_2Eabs$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal. (ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND \\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \end{aligned} \quad (26)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST \\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \end{aligned} \quad (27)$$

**Definition 34** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. \lambda V0f \in ((A\_27c)^{A\_27a})$

Let  $ty\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Emetric\_2Emetric\ A0) \quad (28)$$

Let  $c\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Emetric\ A\_27a \in ((ty\_2Emetric\_2Emetric\ A\_27a)^{(ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})}) \quad (29)$$

**Definition 35** We define  $c\_2Emetric\_2Emr1$  to be  $(ap\ (c\_2Emetric\_2Emetric\ ty\_2Erealax\_2Ereal)\ (ap\ (c\_2Emetric\_2Emetric\ ty\_2Erealax\_2Ereal)\ ty\_2Erealax\_2Ereal))$

Let  $c\_2Emetric\_2Edist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Edist\ A\_27a \in ((ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})^{(ty\_2Emetric\_2Edist\ A\_27a)}) \quad (30)$$

Let  $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \quad (31)$$

Let  $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^{A\_27a})})}) \quad (32)$$

**Definition 36** We define  $c\_2Emetric\_2Emtop$  to be  $\lambda A\_27a : \iota. \lambda V0m \in (ty\_2Emetric\_2Emetric\ A\_27a). (ap\ (c\_2Emetric\_2Emetric\ ty\_2Erealax\_2Ereal)\ ty\_2Erealax\_2Ereal)$

Let  $c\_2Enets\_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Enets\_2Etends\ A\_27a\ A\_27b \in (((ty\_2Epair\_2Eprod\ (ty\_2Etopology\_2Etopology\ A\_27a)\ ((2^{A\_27b})^{A\_27b})))^{A\_27a})^{(A\_27a)^{A\_27b}} \quad (33)$$

**Definition 37** We define  $c\_2Eseq\_2E\_2D\_2D\_23E$  to be  $\lambda V0x \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). \lambda V1x \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). \lambda V2x \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).$

**Definition 38** We define  $c\_2Eseq\_2Esums$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). \lambda V1s \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).$

**Definition 39** We define  $c\_2Eseq\_2Esuminf$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). (ap\ (c\_2Eseq\_2Esuminf\ \lambda V1s \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). \lambda V2s \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).))$

**Definition 40** We define  $c\_2Etransc\_2Esin$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal. (ap\ c\_2Eseq\_2Esuminf\ (\lambda V1s \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). \lambda V2s \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).))$

**Definition 41** We define  $c\_2Etransc\_2Ecos$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal. (ap\ c\_2Eseq\_2Esuminf\ (\lambda V1s \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). \lambda V2s \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).))$

**Definition 42** We define  $c\_2Etransc\_2Epi$  to be  $(ap\ (ap\ c\_2Erealax\_2Ereal\_mul\ (ap\ c\_2Ereal\_2Ereal\_of\_mul)))$

**Definition 43** We define  $c\_2Etransc\_2Easn$  to be  $\lambda V0y \in ty\_2Erealax\_2Ereal. (ap\ (c\_2Emin\_2E\_40\ ty\_2Erealax\_2Ereal))$

Assume the following.

$$True \quad (34)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in \\ A.27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} & (\forall V0y \in ty.2Erealx.2Ereal.(((p (ap (ap c.2Ereal.2Ereal\_lte \\ & (ap c.2Erealx.2Ereal\_neg (ap c.2Ereal.2Ereal\_of\_num (ap \\ & c.2Earithmetic.2ENUMERAL (ap c.2Earithmetic.2EBIT1 c.2Earithmetic.2EZERO)))))) \\ & V0y)) \wedge (p (ap (ap c.2Ereal.2Ereal\_lte V0y) (ap c.2Ereal.2Ereal\_of\_num \\ & (ap c.2Earithmetic.2ENUMERAL (ap c.2Earithmetic.2EBIT1 c.2Earithmetic.2EZERO)))))) \Rightarrow \\ & (p (ap (c.2Ebool.2E.3F.21 ty.2Erealx.2Ereal) (\lambda V1x \in ty.2Erealx.2Ereal. \\ & (ap (ap c.2Ebool.2E.2F.5C (ap (ap c.2Ereal.2Ereal\_lte (ap c.2Erealx.2Ereal\_neg \\ & (ap (ap c.2Ereal.2E.2F c.2Etrasc.2Epi) (ap c.2Ereal.2Ereal\_of\_num \\ & (ap c.2Earithmetic.2ENUMERAL (ap c.2Earithmetic.2EBIT2 c.2Earithmetic.2EZERO)))))) \\ & V1x)) (ap (ap c.2Ebool.2E.2F.5C (ap (ap c.2Ereal.2Ereal\_lte V1x) \\ & (ap (ap c.2Ereal.2E.2F c.2Etrasc.2Epi) (ap c.2Ereal.2Ereal\_of\_num \\ & (ap c.2Earithmetic.2ENUMERAL (ap c.2Earithmetic.2EBIT2 c.2Earithmetic.2EZERO)))))) \\ & (ap (ap (c.2Emin.2E.3D ty.2Erealx.2Ereal) (ap c.2Etrasc.2Esin \\ & V1x)) V0y)))))) \end{aligned} \quad (37)$$

### Theorem 1

$$\begin{aligned} & (\forall V0y \in ty.2Erealx.2Ereal.(((p (ap (ap c.2Ereal.2Ereal\_lte \\ & (ap c.2Erealx.2Ereal\_neg (ap c.2Ereal.2Ereal\_of\_num (ap \\ & c.2Earithmetic.2ENUMERAL (ap c.2Earithmetic.2EBIT1 c.2Earithmetic.2EZERO)))))) \\ & V0y)) \wedge (p (ap (ap c.2Ereal.2Ereal\_lte V0y) (ap c.2Ereal.2Ereal\_of\_num \\ & (ap c.2Earithmetic.2ENUMERAL (ap c.2Earithmetic.2EBIT1 c.2Earithmetic.2EZERO)))))) \Rightarrow \\ & ((p (ap (ap c.2Ereal.2Ereal\_lte (ap c.2Erealx.2Ereal\_neg ( \\ & ap (ap c.2Ereal.2E.2F c.2Etrasc.2Epi) (ap c.2Ereal.2Ereal\_of\_num \\ & (ap c.2Earithmetic.2ENUMERAL (ap c.2Earithmetic.2EBIT2 c.2Earithmetic.2EZERO)))))) \\ & (ap c.2Etrasc.2Easn V0y))) \wedge ((p (ap (ap c.2Ereal.2Ereal\_lte \\ & (ap c.2Etrasc.2Easn V0y)) (ap (ap c.2Ereal.2E.2F c.2Etrasc.2Epi) \\ & (ap c.2Ereal.2Ereal\_of\_num (ap c.2Earithmetic.2ENUMERAL ( \\ & ap c.2Earithmetic.2EBIT2 c.2Earithmetic.2EZERO)))))) \wedge ((ap \\ & c.2Etrasc.2Esin (ap c.2Etrasc.2Easn V0y)) = V0y)))) \end{aligned}$$