

# thm\_2Etransc\_2EATN\_\_BOUNDS (TMUAGT- Mihu3rVwzMh6W5eGJHXgdojTW4Y8C)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a})) (\lambda V0t \in 2.V0t)) (\lambda V1t \in 2.V1t))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_27E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F))$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{1}$$

Let  $ty\_2Eenum\_2Eenum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Eenum\_2Eenum \tag{2}$$

Let  $c\_2Ereal\_2Epow : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealax\_2Ereal\ ty\_2Eenum\_2Eenum)\ ty\_2Erealax\_2Ereal) \tag{3}$$

Let  $c\_2Eenum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Eenum\_2EZERO\_REP \in \omega \tag{4}$$

Let  $c\_2Eenum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Eenum\_2EABS\_num \in (ty\_2Eenum\_2Eenum^{\omega}) \tag{5}$$

**Definition 7** We define  $c\_2Eenum\_2E0$  to be  $(ap c\_2Eenum\_2EABS\_num\ c\_2Eenum\_2EZERO\_REP)$ .

Let  $c\_Ereal\_Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_Ereal\_Ereal\_of\_num \in (ty\_Erealax\_Ereal^{ty\_Eenum\_Eenum}) \quad (6)$$

Let  $c\_Earithmetic\_EFACT : \iota$  be given. Assume the following.

$$c\_Earithmetic\_EFACT \in (ty\_Eenum\_Eenum^{ty\_Eenum\_Eenum}) \quad (7)$$

**Definition 8** We define  $c\_Earithmetic\_EZERO$  to be  $c\_Eenum\_E0$ .

Let  $c\_Eenum\_EREP\_num : \iota$  be given. Assume the following.

$$c\_Eenum\_EREP\_num \in (\omega^{ty\_Eenum\_Eenum}) \quad (8)$$

Let  $c\_Eenum\_ESUC\_REP : \iota$  be given. Assume the following.

$$c\_Eenum\_ESUC\_REP \in (\omega^{\omega}) \quad (9)$$

**Definition 9** We define  $c\_Eenum\_ESUC$  to be  $\lambda V0m \in ty\_Eenum\_Eenum.(ap\ c\_Eenum\_EABS\_num)$

Let  $c\_Earithmetic\_E2B : \iota$  be given. Assume the following.

$$c\_Earithmetic\_E2B \in ((ty\_Eenum\_Eenum^{ty\_Eenum\_Eenum})^{ty\_Eenum\_Eenum}) \quad (10)$$

**Definition 10** We define  $c\_Earithmetic\_EBIT2$  to be  $\lambda V0n \in ty\_Eenum\_Eenum.(ap\ (ap\ c\_Earithmetic\_EBIT1))$

**Definition 11** We define  $c\_Earithmetic\_ENUMERAL$  to be  $\lambda V0x \in ty\_Eenum\_Eenum.V0x$ .

Let  $c\_Earithmetic\_EDIV : \iota$  be given. Assume the following.

$$c\_Earithmetic\_EDIV \in ((ty\_Eenum\_Eenum^{ty\_Eenum\_Eenum})^{ty\_Eenum\_Eenum}) \quad (11)$$

**Definition 12** We define  $c\_Earithmetic\_EBIT1$  to be  $\lambda V0n \in ty\_Eenum\_Eenum.(ap\ (ap\ c\_Earithmetic\_EBIT1))$

Let  $ty\_Ehreal\_Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_Ehreal\_Ehreal \quad (12)$$

Let  $ty\_Epair\_Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_Epair\_Eprod\ A0\ A1) \quad (13)$$

Let  $c\_Erealax\_Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_Erealax\_Ereal\_REP\_CLASS \in ((2^{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)})^{ty\_Erealax}) \quad (14)$$

**Definition 13** We define  $c\_Emin\_E40$  to be  $\lambda A.\lambda P \in 2^A.if\ (\exists x \in A.p\ (ap\ P\ x))\ then\ (the\ (\lambda x.x \in A)\ of\ type\ \iota \Rightarrow \iota$ .

**Definition 14** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_2Emin\_2E40 (t$

Let  $c\_2Erealax\_2Etrealm\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_neg \in ((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) \quad (15)$$

Let  $c\_2Erealax\_2Etrealm\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_eq \in ((2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)})(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) \quad (16)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)}} \quad (17)$$

**Definition 15** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty$

**Definition 16** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap c\_2Erealax\_2Ereal$

Let  $c\_2Erealax\_2Etrealm\_inv : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_inv \in ((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)) \quad (18)$$

**Definition 17** We define  $c\_2Erealax\_2Einv$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap c\_2Erealax\_2Ereal\_ABS$

Let  $c\_2Erealax\_2Etrealm\_mul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_mul \in (((ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal))^{(ty\_2Epair\_2Eprod ty\_2Ehreal\_2Ehreal ty\_2Ehreal\_2Ehreal)}) \quad (19)$$

**Definition 18** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax$

**Definition 19** We define  $c\_2Ereal\_2E2F$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.($

Let  $c\_2Earithmetic\_2EEVEN : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Eenum\_2Eenum}) \quad (20)$$

**Definition 20** We define  $c\_2Ebool\_2E2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E21 2) (\lambda V2t$

**Definition 21** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b} A\_27a})}) \quad (21)$$

**Definition 22** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Ereal\_2Esum : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Esum \in ((ty\_2Erealax\_2Ereal^{(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum)})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum)} \quad (22)$$

**Definition 23** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40$

**Definition 24** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 25** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 26** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

**Definition 27** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_2Erealax\_2Etrealm\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)} \quad (23)$$

**Definition 28** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 29** We define  $c\_2Ereal\_2Ereal\_sub$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

Let  $c\_2Erealax\_2Etrealm\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)} \quad (24)$$

**Definition 30** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 31** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

**Definition 32** We define  $c\_2Ereal\_2Eabs$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (ap\ (c\_2Ebool\_2ECONJ$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (25)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (26)$$

**Definition 33** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A-27$



**Definition 42** We define `c_2Etransc_2Epi` to be  $(ap (ap c\_2Erealax\_2Ereal\_mul (ap c\_2Ereal\_2Ereal\_of\_num$

**Definition 43** We define `c_2Etransc_2Eatn` to be  $\lambda V0y \in ty\_2Erealax\_2Ereal.(ap (c\_2Emin\_2E40 ty\_2Ereal$

Assume the following.

$$True \quad (34)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (35)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (36) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg( \\ & p V0t)))))) \quad (37) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0y \in ty\_2Erealax\_2Ereal.((p (ap (ap c\_2Erealax\_2Ereal\_lt \\ & (ap c\_2Erealax\_2Ereal\_neg (ap (ap c\_2Ereal\_2E2F c\_2Etransc\_2Epi) \\ & (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL ( \\ & ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO)))))) (ap c\_2Etransc\_2Eatn \\ & V0y)) \wedge ((p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Etransc\_2Eatn \\ & V0y)) (ap (ap c\_2Ereal\_2E2F c\_2Etransc\_2Epi) (ap c\_2Ereal\_2Ereal\_of\_num \\ & (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO)))))) \wedge \\ & ((ap c\_2Etransc\_2Etan (ap c\_2Etransc\_2Eatn V0y)) = V0y))) \quad (38) \end{aligned}$$

**Theorem 1**

$$\begin{aligned} & (\forall V0y \in ty\_2Erealax\_2Ereal.((p (ap (ap c\_2Erealax\_2Ereal\_lt \\ & (ap c\_2Erealax\_2Ereal\_neg (ap (ap c\_2Ereal\_2E2F c\_2Etransc\_2Epi) \\ & (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL ( \\ & ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO)))))) (ap c\_2Etransc\_2Eatn \\ & V0y)) \wedge ((p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Etransc\_2Eatn \\ & V0y)) (ap (ap c\_2Ereal\_2E2F c\_2Etransc\_2Epi) (ap c\_2Ereal\_2Ereal\_of\_num \\ & (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))))))))) \end{aligned}$$