

# thm\_2Etransc\_2ECOS\_2 (TMLk3AgwKT4X8NCBnZNrfwdMZzsmXQAhmZk)

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Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o\ (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A\_27a}))\ (\lambda V1P \in 2.V1P))\ (\lambda V2P \in 2.V2P))$

**Definition 4** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2EF))$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in 2.V2t))\ (\lambda V3t \in 2.V3t))\ (\lambda V4t \in 2.V4t))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{2}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{3}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{4}$$

**Definition 8** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ m)$

**Definition 9** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if\ (\exists x \in A.p\ (ap\ P\ x))\ then\ (the\ (\lambda x.x \in A \wedge p\ x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 10** We define  $c\_Ebool\_E\_3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ V0P\ (ap\ (c\_Emin\_E\_40$

**Definition 11** We define  $c\_Eprim\_rec\_E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 12** We define  $c\_Earithmic\_E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 13** We define  $c\_Ebool\_E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_Ebool\_E\_21\ 2)\ (\lambda V2t \in$

**Definition 14** We define  $c\_Earithmic\_E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_Earithmetic\_EEXP : \iota$  be given. Assume the following.

$$c\_Earithmetic\_EEXP \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (5)$$

Let  $c\_Earithmetic\_E\_2D : \iota$  be given. Assume the following.

$$c\_Earithmetic\_E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (6)$$

**Definition 15** We define  $c\_Enumeral\_EiiSUC$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap\ c\_Enum\_2ESUC\ (ap$

Let  $c\_Earithmetic\_EEVEN : \iota$  be given. Assume the following.

$$c\_Earithmetic\_EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (7)$$

Let  $c\_Enumeral\_Eonecount : \iota$  be given. Assume the following.

$$c\_Enumeral\_Eonecount \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (8)$$

Let  $c\_Enumeral\_Eexactlog : \iota$  be given. Assume the following.

$$c\_Enumeral\_Eexactlog \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}) \quad (9)$$

Let  $c\_Enum\_EZERO\_REP : \iota$  be given. Assume the following.

$$c\_Enum\_EZERO\_REP \in \omega \quad (10)$$

**Definition 16** We define  $c\_Enum\_E0$  to be  $(ap\ c\_Enum\_2EABS\_num\ c\_Enum\_EZERO\_REP)$ .

**Definition 17** We define  $c\_Ebool\_ECOND$  to be  $\lambda A\_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A\_27a. (\lambda V2t2 \in A\_27a. ($

**Definition 18** We define  $c\_Eprim\_rec\_EPRE$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. (ap\ (ap\ (ap\ (c\_Ebool\_2E$

**Definition 19** We define  $c\_Earithmetic\_EZERO$  to be  $c\_Enum\_E0$ .

Let  $c\_Earithmetic\_E\_2B : \iota$  be given. Assume the following.

$$c\_Earithmetic\_E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (11)$$

**Definition 20** We define  $c\_Earithmetic\_EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap\ (ap\ c\_Earithmetic$

**Definition 21** We define `c_2Earithmetic_2ENUMERAL` to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let `c_2Earithmetic_2EDIV` :  $\iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (12)$$

**Definition 22** We define `c_2Earithmetic_2EDIV2` to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2EDIV) n)$

Let `c_2Enumeral_2Eexp_help` :  $\iota$  be given. Assume the following.

$$c\_2Enumeral\_2Eexp\_help \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (13)$$

Let `c_2Earithmetic_2EODD` :  $\iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EODD \in (2^{ty\_2Enum\_2Enum}) \quad (14)$$

**Definition 23** We define `c_2Ebool_2ELET` to be  $\lambda A.27a : \iota.\lambda A.27b : \iota.(\lambda V0f \in (A.27b^{A.27a}).(\lambda V1x \in A.27b.f x))$

**Definition 24** We define `c_2Enumeral_2EiDUB` to be  $\lambda V0x \in ty\_2Enum\_2Enum.(ap (ap c\_2Earithmetic\_2EDIV2) x)$

**Definition 25** We define `c_2Enumeral_2EiZ` to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let `c_2Earithmetic_2E_2A` :  $\iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (15)$$

**Definition 26** We define `c_2Enumeral_2Einternal_mult` to be `c_2Earithmetic_2E_2A`.

Let `ty_2Ehreal_2Ehreal` :  $\iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (16)$$

Let `ty_2Epair_2Eprod` :  $\iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (17)$$

Let `ty_2Erealax_2Ereal` :  $\iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (18)$$

Let `c_2Erealax_2Ereal__REP__CLASS` :  $\iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal}) \quad (19)$$

**Definition 27** We define `c_2Erealax_2Ereal__REP` to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_2Emin\_2E40) a)$

Let  $c\_2Erealax\_2Etreal\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)) \quad (20)$$

Let  $c\_2Erealax\_2Etreal\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)) \quad (21)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}})) \quad (22)$$

**Definition 28** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 29** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal\_neg\ T1)$

Let  $c\_2Erealax\_2Etreal\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal))(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)) \quad (23)$$

**Definition 30** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal.$

**Definition 31** We define  $c\_2Ereal\_2Ereal\_sub$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.$

**Definition 32** We define  $c\_2Earithmatic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.$

Let  $c\_2Erealax\_2Etreal\_inv : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_inv \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)) \quad (24)$$

**Definition 33** We define  $c\_2Erealax\_2Einv$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal\_ABS\ T1)$

Let  $c\_2Erealax\_2Etreal\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)) \quad (25)$$

**Definition 34** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal.$

**Definition 35** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (26)$$

**Definition 36** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in A\_27a. \lambda V1y \in A\_27b. (ap (c\_2Ereal\_2Esum : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Esum \in ((ty\_2Erealax\_2Ereal^{(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum)}) \quad (27)$$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (28)$$

**Definition 37** We define  $c\_2Ereal\_2Eabs$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal. (ap (ap (ap (c\_2Ebool\_2ECOND$  Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND \\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \end{aligned} \quad (29)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST \\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \end{aligned} \quad (30)$$

**Definition 38** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda A\_27c : \iota. \lambda V0f \in ((A\_27c^{A\_27a \rightarrow A\_27b})$  Let  $ty\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty\_2Emetric\_2Emetric\ A0) \quad (31)$$

Let  $c\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Emetric\ A\_27a \in ((ty\_2Emetric\_2Emetric \\ A\_27a)^{(ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})}) \end{aligned} \quad (32)$$

**Definition 39** We define  $c\_2Emetric\_2Emr1$  to be  $(ap (c\_2Emetric\_2Emetric\ ty\_2Erealax\_2Ereal) (ap (c\_2Emetric\_2Edist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Edist\ A\_27a \in ((ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}) \quad (33)$$

Let  $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \quad (34)$$

Let  $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a. nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in \\ ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^{A\_27a})})}) \end{aligned} \quad (35)$$

**Definition 40** We define  $c\_Emetric\_Emtop$  to be  $\lambda A\_27a : \iota. \lambda V0m \in (ty\_Emetric\_Emetric\ A\_27a). (ap$

Let  $c\_Enets\_Etends : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} & \forall A\_27a. nonempty\ A\_27a \Rightarrow \forall A\_27b. nonempty\ A\_27b \Rightarrow c\_Enets\_Etends \\ & A\_27a\ A\_27b \in (((2^{(ty\_Epair\_Eprod\ (ty\_Etopology\_Etopology\ A\_27a)\ ((2^{A\_27b})^{A\_27b}))})^{A\_27a})^{(A\_27a)^{A\_27b}}) \end{aligned} \quad (36)$$

**Definition 41** We define  $c\_Eseq\_E\_2D\_2D\_3E$  to be  $\lambda V0x \in (ty\_Erealx\_Ereal^{ty\_Eenum\_Eenum}). \lambda V1x$

**Definition 42** We define  $c\_Eseq\_Esums$  to be  $\lambda V0f \in (ty\_Erealx\_Ereal^{ty\_Eenum\_Eenum}). \lambda V1s \in ty\_E$

**Definition 43** We define  $c\_Eseq\_Esuminf$  to be  $\lambda V0f \in (ty\_Erealx\_Ereal^{ty\_Eenum\_Eenum}). (ap\ (c\_E$

**Definition 44** We define  $c\_Eseq\_Esummable$  to be  $\lambda V0f \in (ty\_Erealx\_Ereal^{ty\_Eenum\_Eenum}). (ap\ (c\_E$

Let  $c\_Ereal\_Epow : \iota$  be given. Assume the following.

$$c\_Ereal\_Epow \in ((ty\_Erealx\_Ereal^{ty\_Eenum\_Eenum})^{ty\_Erealx\_Ereal}) \quad (37)$$

Let  $c\_Earithmetic\_EFACT : \iota$  be given. Assume the following.

$$c\_Earithmetic\_EFACT \in (ty\_Eenum\_Eenum^{ty\_Eenum\_Eenum}) \quad (38)$$

**Definition 45** We define  $c\_Earithmetic\_EBIT1$  to be  $\lambda V0n \in ty\_Eenum\_Eenum. (ap\ (ap\ c\_Earithmetic$

Let  $c\_Erealx\_Etrealmul : \iota$  be given. Assume the following.

$$c\_Erealx\_Etrealmul \in (((ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)^{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)})^{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)}) \quad (39)$$

**Definition 46** We define  $c\_Erealx\_Ereal\_mul$  to be  $\lambda V0T1 \in ty\_Erealx\_Ereal. \lambda V1T2 \in ty\_Erealx\_Ereal.$

**Definition 47** We define  $c\_Ereal\_E\_2F$  to be  $\lambda V0x \in ty\_Erealx\_Ereal. \lambda V1y \in ty\_Erealx\_Ereal. ($

**Definition 48** We define  $c\_Etransc\_Ecos$  to be  $\lambda V0x \in ty\_Erealx\_Ereal. (ap\ c\_Eseq\_Esuminf\ (\lambda V1n$

Assume the following.

$$((ap\ c\_Earithmetic\_ENUMERAL\ (ap\ c\_Earithmetic\_EBIT1\ c\_Earithmetic\_EZERO)) = (ap\ c\_Eenum\_ESUC\ c\_Eenum\_E0)) \quad (40)$$

Assume the following.

$$((ap\ c\_Earithmetic\_ENUMERAL\ (ap\ c\_Earithmetic\_EBIT2\ c\_Earithmetic\_EZERO)) = (ap\ c\_Eenum\_ESUC\ (ap\ c\_Earithmetic\_ENUMERAL\ (ap\ c\_Earithmetic\_EBIT1\ c\_Earithmetic\_EZERO)))) \quad (41)$$

Assume the following.

$$((ap \ c\_2Earithmetic\_2ENUMERAL \ c\_2Earithmetic\_2EZERO) = c\_2Enum\_2E0) \quad (42)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\ & ((ap \ (ap \ c\_2Earithmetic\_2E\_2B \ c\_2Enum\_2E0) \ V0m) = V0m) \wedge (((ap \ ( \\ & ap \ c\_2Earithmetic\_2E\_2B \ V0m) \ c\_2Enum\_2E0) = V0m) \wedge (((ap \ (ap \ c\_2Earithmetic\_2E\_2B \\ & (ap \ c\_2Enum\_2ESUC \ V0m)) \ V1n) = (ap \ c\_2Enum\_2ESUC \ (ap \ (ap \ c\_2Earithmetic\_2E\_2B \\ & V0m) \ V1n))) \wedge ((ap \ (ap \ c\_2Earithmetic\_2E\_2B \ V0m) \ (ap \ c\_2Enum\_2ESUC \\ & V1n)) = (ap \ c\_2Enum\_2ESUC \ (ap \ (ap \ c\_2Earithmetic\_2E\_2B \ V0m) \ V1n))))))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\ & (p \ (ap \ (ap \ c\_2Eprim\_rec\_2E\_3C \ (ap \ c\_2Enum\_2ESUC \ V0m)) \ (ap \ c\_2Enum\_2ESUC \\ & V1n))) \Leftrightarrow (p \ (ap \ (ap \ c\_2Eprim\_rec\_2E\_3C \ V0m) \ V1n)))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. ((ap \ c\_2Enum\_2ESUC \ V0m) = (ap \ (ap \\ & c\_2Earithmetic\_2E\_2B \ V0m) \ (ap \ c\_2Earithmetic\_2ENUMERAL \ (ap \ c\_2Earithmetic\_2EBIT1 \\ & c\_2Earithmetic\_2EZERO)))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\ & ((ap \ (ap \ c\_2Earithmetic\_2E\_2A \ c\_2Enum\_2E0) \ V0m) = c\_2Enum\_2E0) \wedge \\ & (((ap \ (ap \ c\_2Earithmetic\_2E\_2A \ V0m) \ c\_2Enum\_2E0) = c\_2Enum\_2E0) \wedge \\ & (((ap \ (ap \ c\_2Earithmetic\_2E\_2A \ (ap \ c\_2Earithmetic\_2ENUMERAL \\ & (ap \ c\_2Earithmetic\_2EBIT1 \ c\_2Earithmetic\_2EZERO)) \ V0m) = V0m) \wedge \\ & (((ap \ (ap \ c\_2Earithmetic\_2E\_2A \ V0m) \ (ap \ c\_2Earithmetic\_2ENUMERAL \\ & (ap \ c\_2Earithmetic\_2EBIT1 \ c\_2Earithmetic\_2EZERO)) = V0m) \wedge ( \\ & ((ap \ (ap \ c\_2Earithmetic\_2E\_2A \ (ap \ c\_2Enum\_2ESUC \ V0m)) \ V1n) = (ap \\ & (ap \ c\_2Earithmetic\_2E\_2B \ (ap \ (ap \ c\_2Earithmetic\_2E\_2A \ V0m) \ V1n)) \\ & V1n)) \wedge ((ap \ (ap \ c\_2Earithmetic\_2E\_2A \ V0m) \ (ap \ c\_2Enum\_2ESUC \ V1n)) = \\ & (ap \ (ap \ c\_2Earithmetic\_2E\_2B \ V0m) \ (ap \ (ap \ c\_2Earithmetic\_2E\_2A \\ & V0m) \ V1n))))))))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\ & (p \ (ap \ (ap \ c\_2Eprim\_rec\_2E\_3C \ c\_2Enum\_2E0) \ V1n)) \Rightarrow ((V0m = (ap \ c\_2Eprim\_rec\_2EPRE \\ & V1n)) \Leftrightarrow ((ap \ c\_2Enum\_2ESUC \ V0m) = V1n)))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned}
&(((ap \ c\_2Earithmetic\_2EFACT \ c\_2Enum\_2E0) = (ap \ c\_2Earithmetic\_2ENUMERAL \\
&\quad (ap \ c\_2Earithmetic\_2EBIT1 \ c\_2Earithmetic\_2EZERO))) \wedge (\forall V0n \in \\
&\quad ty\_2Enum\_2Enum. ((ap \ c\_2Earithmetic\_2EFACT \ (ap \ c\_2Enum\_2ESUC \\
&\quad V0n)) = (ap \ (ap \ c\_2Earithmetic\_2E\_2A \ (ap \ c\_2Enum\_2ESUC \ V0n)) \ (ap \\
&\quad \quad c\_2Earithmetic\_2EFACT \ V0n))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
&(\forall V0n \in ty\_2Enum\_2Enum. (p \ (ap \ (ap \ c\_2Eprim\_rec\_2E\_3C \ c\_2Enum\_2E0) \\
&\quad (ap \ c\_2Earithmetic\_2EFACT \ V0n))))
\end{aligned} \tag{49}$$

Assume the following.

$$True \tag{50}$$

Assume the following.

$$\begin{aligned}
&(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p \ V0t1) \Rightarrow (p \ V1t2)) \Rightarrow (((p \\
&\quad V1t2) \Rightarrow (p \ V0t1)) \Rightarrow ((p \ V0t1) \Leftrightarrow (p \ V1t2))))))
\end{aligned} \tag{51}$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p \ V0t))) \tag{52}$$

Assume the following.

$$\begin{aligned}
&(\forall V0t \in 2. (((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow \\
&(p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge \\
&\quad (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
&(\forall V0t \in 2. (((True \vee (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \vee True) \Leftrightarrow True) \wedge \\
&(((False \vee (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee False) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \vee \\
&\quad (p \ V0t)) \Leftrightarrow (p \ V0t))))))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
&((\forall V0t \in 2. ((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\
&\quad ((\neg False) \Leftrightarrow True)))
\end{aligned} \tag{55}$$

Assume the following.

$$\forall A\_27a. nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \tag{56}$$

Assume the following.

$$\forall A\_27a. nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{57}$$



Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p \ V0t)))))) \quad (58)$$

Assume the following.

$$\forall A\_27a. nonempty \ A\_27a \Rightarrow (\forall V0t1 \in A\_27a. (\forall V1t2 \in A\_27a. (((ap \ (ap \ (ap \ (c\_2Ebool\_2ECOND \ A\_27a) \ c\_2Ebool\_2ET) \ V0t1) \ V1t2) = V0t1) \wedge ((ap \ (ap \ (ap \ (c\_2Ebool\_2ECOND \ A\_27a) \ c\_2Ebool\_2EF) \ V0t1) \ V1t2) = V1t2)))))) \quad (59)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum. (\neg((ap \ c\_2Enum\_2ESUC \ V0n) = c\_2Enum\_2E0))) \quad (60)$$

Assume the following.

$$(((ap \ c\_2Enum\_2ESUC \ c\_2Earithmetic\_2EZERO) = (ap \ c\_2Earithmetic\_2EBIT1 \ c\_2Earithmetic\_2EZERO)) \wedge ((\forall V0n \in ty\_2Enum\_2Enum. ((ap \ c\_2Enum\_2ESUC \ (ap \ c\_2Earithmetic\_2EBIT1 \ V0n)) = (ap \ c\_2Earithmetic\_2EBIT2 \ V0n))) \wedge (\forall V1n \in ty\_2Enum\_2Enum. ((ap \ c\_2Enum\_2ESUC \ (ap \ c\_2Earithmetic\_2EBIT2 \ V1n)) = (ap \ c\_2Earithmetic\_2EBIT1 \ (ap \ c\_2Enum\_2ESUC \ V1n)))))) \quad (61)$$

$$\begin{aligned}
& ((\forall V0n \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmic\_2E\_2B \\
& \quad c\_2Enum\_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty\_2Enum\_2Enum. ((ap \\
& \quad (ap c\_2Earithmic\_2E\_2B V1n) c\_2Enum\_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty\_2Enum\_2Enum. (\forall V3m \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmic\_2E\_2B \\
& \quad (ap c\_2Earithmic\_2ENUMERAL V2n)) (ap c\_2Earithmic\_2ENUMERAL \\
& \quad V3m)) = (ap c\_2Earithmic\_2ENUMERAL (ap c\_2Enumeral\_2EiZ (ap \\
& \quad (ap c\_2Earithmic\_2E\_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmic\_2E\_2A c\_2Enum\_2E0) V4n) = c\_2Enum\_2E0)) \wedge \\
& \quad ((\forall V5n \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmic\_2E\_2A \\
& \quad V5n) c\_2Enum\_2E0) = c\_2Enum\_2E0)) \wedge ((\forall V6n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V7m \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmic\_2E\_2A ( \\
& \quad ap c\_2Earithmic\_2ENUMERAL V6n)) (ap c\_2Earithmic\_2ENUMERAL \\
& \quad V7m)) = (ap c\_2Earithmic\_2ENUMERAL (ap (ap c\_2Earithmic\_2E\_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmic\_2E\_2D \\
& \quad c\_2Enum\_2E0) V8n) = c\_2Enum\_2E0)) \wedge ((\forall V9n \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmic\_2E\_2D V9n) c\_2Enum\_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty\_2Enum\_2Enum. (\forall V11m \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmic\_2E\_2D \\
& \quad (ap c\_2Earithmic\_2ENUMERAL V10n)) (ap c\_2Earithmic\_2ENUMERAL \\
& \quad V11m)) = (ap c\_2Earithmic\_2ENUMERAL (ap (ap c\_2Earithmic\_2E\_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmic\_2EEXP \\
& \quad c\_2Enum\_2E0) (ap c\_2Earithmic\_2ENUMERAL (ap c\_2Earithmic\_2EBIT1 \\
& \quad V12n))) = c\_2Enum\_2E0)) \wedge ((\forall V13n \in ty\_2Enum\_2Enum. ((ap \\
& \quad (ap c\_2Earithmic\_2EEXP c\_2Enum\_2E0) (ap c\_2Earithmic\_2ENUMERAL \\
& \quad (ap c\_2Earithmic\_2EBIT2 V13n))) = c\_2Enum\_2E0)) \wedge ((\forall V14n \in \\
& \quad ty\_2Enum\_2Enum. ((ap (ap c\_2Earithmic\_2EEXP V14n) c\_2Enum\_2E0) = \\
& \quad (ap c\_2Earithmic\_2ENUMERAL (ap c\_2Earithmic\_2EBIT1 c\_2Earithmic\_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty\_2Enum\_2Enum. (\forall V16m \in ty\_2Enum\_2Enum. \\
& \quad ((ap (ap c\_2Earithmic\_2EEXP (ap c\_2Earithmic\_2ENUMERAL V15n)) \\
& \quad (ap c\_2Earithmic\_2ENUMERAL V16m)) = (ap c\_2Earithmic\_2ENUMERAL \\
& \quad (ap (ap c\_2Earithmic\_2EEXP V15n) V16m)))))) \wedge (((ap c\_2Enum\_2ESUC \\
& \quad c\_2Enum\_2E0) = (ap c\_2Earithmic\_2ENUMERAL (ap c\_2Earithmic\_2EBIT1 \\
& \quad c\_2Earithmic\_2EZERO))) \wedge ((\forall V17n \in ty\_2Enum\_2Enum. ( \\
& \quad (ap c\_2Enum\_2ESUC (ap c\_2Earithmic\_2ENUMERAL V17n)) = (ap c\_2Earithmic\_2ENUMERAL \\
& \quad (ap c\_2Enum\_2ESUC V17n)))))) \wedge (((ap c\_2Eprim\_rec\_2EPRE c\_2Enum\_2E0) = \\
& \quad c\_2Enum\_2E0) \wedge ((\forall V18n \in ty\_2Enum\_2Enum. ((ap c\_2Eprim\_rec\_2EPRE \\
& \quad (ap c\_2Earithmic\_2ENUMERAL V18n)) = (ap c\_2Earithmic\_2ENUMERAL \\
& \quad (ap c\_2Eprim\_rec\_2EPRE V18n)))))) \wedge ((\forall V19n \in ty\_2Enum\_2Enum. \\
& \quad (((ap c\_2Earithmic\_2ENUMERAL V19n) = c\_2Enum\_2E0) \Leftrightarrow (V19n = c\_2Earithmic\_2EZERO)))))) \wedge \\
& \quad ((\forall V20n \in ty\_2Enum\_2Enum. ((c\_2Enum\_2E0 = (ap c\_2Earithmic\_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c\_2Earithmic\_2EZERO))) \wedge ((\forall V21n \in ty\_2Enum\_2Enum. \\
& \quad (\forall V22m \in ty\_2Enum\_2Enum. ((ap c\_2Earithmic\_2ENUMERAL \\
& \quad V21n) = (ap c\_2Earithmic\_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))))) \wedge \\
& \quad ((\forall V23n \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V23n) c\_2Enum\_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) (ap c\_2Earithmic\_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmic\_2EZERO) \\
& \quad V24n)))))) \wedge ((\forall V25n \in ty\_2Enum\_2Enum. (\forall V26m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Eprim\_rec\_2E\_3C (ap c\_2Earithmic\_2ENUMERAL \\
& \quad V25n)) (ap c\_2Earithmic\_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmic\_2E\_3E \\
& \quad c\_2Enum\_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmic\_2E\_3E (ap c\_2Earithmic\_2ENUMERAL \\
& \quad V28n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Earithmic\_2EZERO) \\
& \quad V28n)))))) \wedge ((\forall V29n \in ty\_2Enum\_2Enum. (\forall V30m \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmic\_2E\_3E (ap c\_2Earithmic\_2ENUMERAL \\
& \quad V29n)) (ap c\_2Earithmic\_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmic\_2E\_3C\_3D \\
& \quad c\_2Enum\_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmic\_2E\_3C\_3D (ap c\_2Earithmic\_2ENUMERAL \\
& \quad V32n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C\_3D c\_2Earithmic\_2EZERO) \\
& \quad V32n)))))) \wedge ((\forall V33n \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmic\_2E\_3C\_3D \\
& \quad (ap c\_2Earithmic\_2ENUMERAL V33n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C\_3D \\
& \quad (ap c\_2Earithmic\_2ENUMERAL V33n)) c\_2Enum\_2E0)))))) \wedge ((\forall V34n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmic\_2E\_3C\_3D (ap c\_2Earithmic\_2ENUMERAL V34n)) c\_2Enum\_2E0)) \\
& \quad \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C\_3D (ap c\_2Earithmic\_2ENUMERAL V34n)) c\_2Enum\_2E0)))))) \\
& \quad \wedge ((\forall V35n \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmic\_2E\_3C\_3D (ap c\_2Earithmic\_2ENUMERAL \\
& \quad V35n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C\_3D (ap c\_2Earithmic\_2ENUMERAL \\
& \quad V35n)) c\_2Enum\_2E0)))))) \wedge ((\forall V36n \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmic\_2E\_3C\_3D \\
& \quad (ap c\_2Earithmic\_2ENUMERAL V36n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C\_3D \\
& \quad (ap c\_2Earithmic\_2ENUMERAL V36n)) c\_2Enum\_2E0)))))) \wedge ((\forall V37n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmic\_2E\_3C\_3D (ap c\_2Earithmic\_2ENUMERAL V37n)) c\_2Enum\_2E0)) \\
& \quad \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C\_3D (ap c\_2Earithmic\_2ENUMERAL V37n)) c\_2Enum\_2E0)))))) \\
& \quad \wedge ((\forall V38n \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmic\_2E\_3C\_3D (ap c\_2Earithmic\_2ENUMERAL \\
& \quad V38n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C\_3D (ap c\_2Earithmic\_2ENUMERAL \\
& \quad V38n)) c\_2Enum\_2E0)))))) \wedge ((\forall V39n \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmic\_2E\_3C\_3D \\
& \quad (ap c\_2Earithmic\_2ENUMERAL V39n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C\_3D \\
& \quad (ap c\_2Earithmic\_2ENUMERAL V39n)) c\_2Enum\_2E0)))))) \wedge ((\forall V40n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmic\_2E\_3C\_3D (ap c\_2Earithmic\_2ENUMERAL V40n)) c\_2Enum\_2E0)) \\
& \quad \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C\_3D (ap c\_2Earithmic\_2ENUMERAL V40n)) c\_2Enum\_2E0)))))) \\
& \quad \wedge ((\forall V41n \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmic\_2E\_3C\_3D (ap c\_2Earithmic\_2ENUMERAL \\
& \quad V41n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C\_3D (ap c\_2Earithmic\_2ENUMERAL \\
& \quad V41n)) c\_2Enum\_2E0)))))) \wedge ((\forall V42n \in ty\_2Enum\_2Enum. ((p (ap (ap c\_2Earithmic\_2E\_3C\_3D \\
& \quad (ap c\_2Earithmic\_2ENUMERAL V42n)) c\_2Enum\_2E0)) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C\_3D \\
& \quad (ap c\_2Earithmic\_2ENUMERAL V42n)) c\_2Enum\_2E0)))))) \wedge ((\forall V43n \in ty\_2Enum\_2Enum. \\
& \quad ((p (ap (ap c\_2Earithmic\_2E\_3C\_3D (ap c\_2Earithmic\_2ENUMERAL V4$$

[illegible]

(63)

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((c\_2Earithmic\_2EZERO = (ap\ c\_2Earithmic\_2EBIT1\ V0n)) \Leftrightarrow False) \wedge \\
& (((ap\ c\_2Earithmic\_2EBIT1\ V0n) = c\_2Earithmic\_2EZERO) \Leftrightarrow \\
& False) \wedge (((c\_2Earithmic\_2EZERO = (ap\ c\_2Earithmic\_2EBIT2\ \\
& V0n)) \Leftrightarrow False) \wedge (((ap\ c\_2Earithmic\_2EBIT2\ V0n) = c\_2Earithmic\_2EZERO) \Leftrightarrow \\
& False) \wedge (((ap\ c\_2Earithmic\_2EBIT1\ V0n) = (ap\ c\_2Earithmic\_2EBIT2\ \\
& V1m)) \Leftrightarrow False) \wedge (((ap\ c\_2Earithmic\_2EBIT2\ V0n) = (ap\ c\_2Earithmic\_2EBIT1\ \\
& V1m)) \Leftrightarrow False) \wedge (((ap\ c\_2Earithmic\_2EBIT1\ V0n) = (ap\ c\_2Earithmic\_2EBIT1\ \\
& V1m)) \Leftrightarrow (V0n = V1m)) \wedge (((ap\ c\_2Earithmic\_2EBIT2\ V0n) = (ap\ c\_2Earithmic\_2EBIT2\ \\
& V1m)) \Leftrightarrow (V0n = V1m))))))))) \\
& \tag{64}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ c\_2Earithmic\_2EZERO)\ (ap\ c\_2Earithmic\_2EBIT1\ \\
& V0n))) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ c\_2Earithmic\_2EZERO)\ \\
& (ap\ c\_2Earithmic\_2EBIT2\ V0n))) \Leftrightarrow True) \wedge (((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ \\
& V0n)\ c\_2Earithmic\_2EZERO)) \Leftrightarrow False) \wedge (((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ \\
& (ap\ c\_2Earithmic\_2EBIT1\ V0n))\ (ap\ c\_2Earithmic\_2EBIT1\ V1m))) \Leftrightarrow \\
& (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ \\
& (ap\ c\_2Earithmic\_2EBIT2\ V0n))\ (ap\ c\_2Earithmic\_2EBIT2\ V1m))) \Leftrightarrow \\
& (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0n)\ V1m))) \wedge (((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ \\
& (ap\ c\_2Earithmic\_2EBIT1\ V0n))\ (ap\ c\_2Earithmic\_2EBIT2\ V1m))) \Leftrightarrow \\
& (\neg(p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V1m)\ V0n)))) \wedge ((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ \\
& (ap\ c\_2Earithmic\_2EBIT2\ V0n))\ (ap\ c\_2Earithmic\_2EBIT1\ V1m))) \Leftrightarrow \\
& (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0n)\ V1m))))))))) \\
& \tag{65}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (((ap\ c\_2Eprim\_rec\_2EPRE\ c\_2Earithmic\_2EZERO) = c\_2Earithmic\_2EZERO) \wedge \\
& (((ap\ c\_2Eprim\_rec\_2EPRE\ (ap\ c\_2Earithmic\_2EBIT1\ c\_2Earithmic\_2EZERO)) = \\
& c\_2Earithmic\_2EZERO) \wedge ((\forall V0n \in ty\_2Enum\_2Enum. ((ap\ \\
& c\_2Eprim\_rec\_2EPRE\ (ap\ c\_2Earithmic\_2EBIT1\ (ap\ c\_2Earithmic\_2EBIT1\ \\
& V0n))) = (ap\ c\_2Earithmic\_2EBIT2\ (ap\ c\_2Eprim\_rec\_2EPRE\ (ap\ \\
& c\_2Earithmic\_2EBIT1\ V0n)))))) \wedge ((\forall V1n \in ty\_2Enum\_2Enum. \\
& ((ap\ c\_2Eprim\_rec\_2EPRE\ (ap\ c\_2Earithmic\_2EBIT1\ (ap\ c\_2Earithmic\_2EBIT2\ \\
& V1n))) = (ap\ c\_2Earithmic\_2EBIT2\ (ap\ c\_2Earithmic\_2EBIT1\ \\
& V1n)))) \wedge ((\forall V2n \in ty\_2Enum\_2Enum. ((ap\ c\_2Eprim\_rec\_2EPRE\ \\
& (ap\ c\_2Earithmic\_2EBIT2\ V2n)) = (ap\ c\_2Earithmic\_2EBIT1\ V2n))))))))) \\
& \tag{66}
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (((ap\ c\_2Enumeral\_2EiDUB\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n)) = (ap\ c\_2Earithmetic\_2EBIT2\ (ap\ c\_2Enumeral\_2EiDUB\ V0n))) \wedge \\
& \quad (((ap\ c\_2Enumeral\_2EiDUB\ (ap\ c\_2Earithmetic\_2EBIT2\ V0n)) = (ap\ c\_2Earithmetic\_2EBIT2\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n))) \wedge ((ap\ c\_2Enumeral\_2EiDUB\ c\_2Earithmetic\_2EZERO) = c\_2Earithmetic\_2EZERO)))) \\
& \hspace{15em} (67)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. ((p\ (ap\ c\_2Earithmetic\_2EEVEN\ c\_2Earithmetic\_2EZERO)) \wedge \\
& \quad ((p\ (ap\ c\_2Earithmetic\_2EEVEN\ (ap\ c\_2Earithmetic\_2EBIT2\ V0n))) \wedge \\
& \quad ((\neg(p\ (ap\ c\_2Earithmetic\_2EEVEN\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n)))) \wedge \\
& \quad ((\neg(p\ (ap\ c\_2Earithmetic\_2EODD\ c\_2Earithmetic\_2EZERO))) \wedge ((\neg(p\ (ap\ c\_2Earithmetic\_2EODD\ (ap\ c\_2Earithmetic\_2EBIT2\ V0n)))) \wedge \\
& \quad (p\ (ap\ c\_2Earithmetic\_2EODD\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n))))))))) \\
& \hspace{15em} (68)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (((ap\ c\_2Earithmetic\_2EFACT\ c\_2Enum\_2E0) = (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO))) \wedge ((\forall V0n \in ty\_2Enum\_2Enum. ((ap\ c\_2Earithmetic\_2EFACT\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n))) = (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n))) (ap\ c\_2Earithmetic\_2EFACT\ (ap\ c\_2Eprim\_rec\_2EPRE\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ V0n)))))) \wedge ((\forall V1n \in ty\_2Enum\_2Enum. ((ap\ c\_2Earithmetic\_2EFACT\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT2\ V1n))) = (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT2\ V1n))) (ap\ c\_2Earithmetic\_2EFACT\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ V1n))))))))) \\
& \hspace{15em} (69)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0acc \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. \\
& \quad (((ap\ (ap\ c\_2Enumeral\_2Eexp\_help\ c\_2Earithmetic\_2EZERO\ V0acc) = (ap\ c\_2Earithmetic\_2EBIT2\ V0acc)) \wedge (((ap\ (ap\ c\_2Enumeral\_2Eexp\_help\ (ap\ c\_2Earithmetic\_2EBIT1\ V1n))\ V0acc) = (ap\ (ap\ c\_2Enumeral\_2Eexp\_help\ (ap\ c\_2Eprim\_rec\_2EPRE\ (ap\ c\_2Earithmetic\_2EBIT1\ V1n)))\ (ap\ c\_2Earithmetic\_2EBIT1\ V0acc))) \wedge ((ap\ (ap\ c\_2Enumeral\_2Eexp\_help\ (ap\ c\_2Earithmetic\_2EBIT2\ V1n))\ V0acc) = (ap\ (ap\ c\_2Enumeral\_2Eexp\_help\ (ap\ c\_2Earithmetic\_2EBIT1\ V1n))\ (ap\ c\_2Earithmetic\_2EBIT1\ V0acc)))))) \\
& \hspace{15em} (70)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0x \in ty\_2Enum\_2Enum. ((ap (ap c\_2Enumeral\_2Eonecount \\
& c\_2Earithmetic\_2EZERO) V0x) = V0x)) \wedge (\forall V1n \in ty\_2Enum\_2Enum. \\
& (\forall V2x \in ty\_2Enum\_2Enum. ((ap (ap c\_2Enumeral\_2Eonecount \\
& (ap c\_2Earithmetic\_2EBIT1 V1n)) V2x) = (ap (ap c\_2Enumeral\_2Eonecount \\
& V1n) (ap c\_2Enum\_2ESUC V2x)))))) \wedge (\forall V3n \in ty\_2Enum\_2Enum. \\
& (\forall V4x \in ty\_2Enum\_2Enum. ((ap (ap c\_2Enumeral\_2Eonecount \\
& (ap c\_2Earithmetic\_2EBIT2 V3n)) V4x) = c\_2Earithmetic\_2EZERO))))))
\end{aligned} \tag{71}$$

Assume the following.

$$\begin{aligned}
& (((ap c\_2Enumeral\_2Exactlog c\_2Earithmetic\_2EZERO) = c\_2Earithmetic\_2EZERO) \wedge \\
& ((\forall V0n \in ty\_2Enum\_2Enum. ((ap c\_2Enumeral\_2Exactlog ( \\
& ap c\_2Earithmetic\_2EBIT1 V0n)) = c\_2Earithmetic\_2EZERO)) \wedge (\forall V1n \in \\
& ty\_2Enum\_2Enum. ((ap c\_2Enumeral\_2Exactlog (ap c\_2Earithmetic\_2EBIT2 \\
& V1n)) = (ap (ap (c\_2Ebool\_2ELET ty\_2Enum\_2Enum ty\_2Enum\_2Enum) \\
& (\lambda V2x \in ty\_2Enum\_2Enum. (ap (ap (ap (c\_2Ebool\_2ECOND ty\_2Enum\_2Enum) \\
& (ap (ap (c\_2Emin\_2E\_3D ty\_2Enum\_2Enum) V2x) c\_2Earithmetic\_2EZERO)) \\
& c\_2Earithmetic\_2EZERO) (ap c\_2Earithmetic\_2EBIT1 V2x)))) (ap \\
& (ap c\_2Enumeral\_2Eonecount V1n) c\_2Earithmetic\_2EZERO))))))
\end{aligned} \tag{72}$$

Assume the following.

$$(\forall V0x \in ty\_2Enum\_2Enum. ((ap c\_2Earithmetic\_2EDIV2 (ap \\ c\_2Earithmetic\_2EBIT1 V0x)) = V0x)) \tag{73}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1x \in ty\_2Enum\_2Enum. ( \\
& \forall V2y \in ty\_2Enum\_2Enum. (((ap (ap c\_2Earithmetic\_2E\_2A c\_2Earithmetic\_2EZERO) \\
& V0n) = c\_2Earithmetic\_2EZERO) \wedge (((ap (ap c\_2Earithmetic\_2E\_2A \\
& V0n) c\_2Earithmetic\_2EZERO) = c\_2Earithmetic\_2EZERO) \wedge ((ap \\
& (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2EBIT1 V1x)) (ap \\
& c\_2Earithmetic\_2EBIT1 V2y)) = (ap (ap c\_2Enumeral\_2Einternal\_mult \\
& (ap c\_2Earithmetic\_2EBIT1 V1x)) (ap c\_2Earithmetic\_2EBIT1 V2y))) \wedge \\
& (((ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2EBIT1 V1x)) \\
& (ap c\_2Earithmetic\_2EBIT2 V2y)) = (ap (ap (c\_2Ebool\_2ELET ty\_2Enum\_2Enum \\
& ty\_2Enum\_2Enum) (\lambda V3n \in ty\_2Enum\_2Enum. (ap (ap (ap (c\_2Ebool\_2ECOND \\
& ty\_2Enum\_2Enum) (ap c\_2Earithmetic\_2EODD V3n)) (ap (ap c\_2Enumeral\_2Eexp\_help \\
& (ap c\_2Earithmetic\_2EDIV2 V3n)) (ap c\_2Eprim\_rec\_2EPRE (ap c\_2Earithmetic\_2EBIT1 \\
& V1x)))) (ap (ap c\_2Enumeral\_2Einternal\_mult (ap c\_2Earithmetic\_2EBIT1 \\
& V1x)) (ap c\_2Earithmetic\_2EBIT2 V2y)))) (ap c\_2Enumeral\_2Eexactlog \\
& (ap c\_2Earithmetic\_2EBIT2 V2y)))) \wedge (((ap (ap c\_2Earithmetic\_2E\_2A \\
& (ap c\_2Earithmetic\_2EBIT2 V1x)) (ap c\_2Earithmetic\_2EBIT1 V2y)) = \\
& (ap (ap (c\_2Ebool\_2ELET ty\_2Enum\_2Enum ty\_2Enum\_2Enum) (\lambda V4m \in \\
& ty\_2Enum\_2Enum. (ap (ap (ap (c\_2Ebool\_2ECOND ty\_2Enum\_2Enum) \\
& (ap c\_2Earithmetic\_2EODD V4m)) (ap (ap c\_2Enumeral\_2Eexp\_help \\
& (ap c\_2Earithmetic\_2EDIV2 V4m)) (ap c\_2Eprim\_rec\_2EPRE (ap c\_2Earithmetic\_2EBIT1 \\
& V2y)))) (ap (ap c\_2Enumeral\_2Einternal\_mult (ap c\_2Earithmetic\_2EBIT2 \\
& V1x)) (ap c\_2Earithmetic\_2EBIT1 V2y)))) (ap c\_2Enumeral\_2Eexactlog \\
& (ap c\_2Earithmetic\_2EBIT2 V1x)))) \wedge ((ap (ap c\_2Earithmetic\_2E\_2A \\
& (ap c\_2Earithmetic\_2EBIT2 V1x)) (ap c\_2Earithmetic\_2EBIT2 V2y)) = \\
& (ap (ap (c\_2Ebool\_2ELET ty\_2Enum\_2Enum ty\_2Enum\_2Enum) (\lambda V5m \in \\
& ty\_2Enum\_2Enum. (ap (ap (c\_2Ebool\_2ELET ty\_2Enum\_2Enum ty\_2Enum\_2Enum) \\
& (\lambda V6n \in ty\_2Enum\_2Enum. (ap (ap (ap (c\_2Ebool\_2ECOND ty\_2Enum\_2Enum) \\
& (ap c\_2Earithmetic\_2EODD V5m)) (ap (ap c\_2Enumeral\_2Eexp\_help \\
& (ap c\_2Earithmetic\_2EDIV2 V5m)) (ap c\_2Eprim\_rec\_2EPRE (ap c\_2Earithmetic\_2EBIT2 \\
& V2y)))) (ap (ap (ap (c\_2Ebool\_2ECOND ty\_2Enum\_2Enum) (ap c\_2Earithmetic\_2EODD \\
& V6n)) (ap (ap c\_2Enumeral\_2Eexp\_help (ap c\_2Earithmetic\_2EDIV2 \\
& V6n)) (ap c\_2Eprim\_rec\_2EPRE (ap c\_2Earithmetic\_2EBIT2 V1x)))) \\
& (ap (ap c\_2Enumeral\_2Einternal\_mult (ap c\_2Earithmetic\_2EBIT2 \\
& V1x)) (ap c\_2Earithmetic\_2EBIT2 V2y)))))) (ap c\_2Enumeral\_2Eexactlog \\
& (ap c\_2Earithmetic\_2EBIT2 V2y)))) (ap c\_2Enumeral\_2Eexactlog \\
& (ap c\_2Earithmetic\_2EBIT2 V1x))))))))) (74)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (\forall V1m \in ty\_2Enum\_2Enum. ( \\
& ((ap (ap c\_2Enumeral\_2Einternal\_mult c\_2Earithmetric\_2EZERO) \\
& V0n) = c\_2Earithmetric\_2EZERO) \wedge (((ap (ap c\_2Enumeral\_2Einternal\_mult \\
& V0n) c\_2Earithmetric\_2EZERO) = c\_2Earithmetric\_2EZERO) \wedge (((ap \\
& (ap c\_2Enumeral\_2Einternal\_mult (ap c\_2Earithmetric\_2EBIT1 \\
& V0n)) V1m) = (ap c\_2Enumeral\_2EiZ (ap (ap c\_2Earithmetric\_2E\_2B \\
& (ap c\_2Enumeral\_2EiDUB (ap (ap c\_2Enumeral\_2Einternal\_mult \\
& V0n) V1m))) V1m))) \wedge ((ap (ap c\_2Enumeral\_2Einternal\_mult (ap \\
& c\_2Earithmetric\_2EBIT2 V0n)) V1m) = (ap c\_2Enumeral\_2EiDUB (ap \\
& c\_2Enumeral\_2EiZ (ap (ap c\_2Earithmetric\_2E\_2B (ap (ap c\_2Enumeral\_2Einternal\_mult \\
& V0n) V1m)) V1m))))))))))
\end{aligned} \tag{75}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) \\
& (ap c\_2Enum\_2ESUC V0n))))
\end{aligned} \tag{76}$$

Assume the following.

$$\begin{aligned}
& (\neg((ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetric\_2ENUMERAL \\
& (ap c\_2Earithmetric\_2EBIT1 c\_2Earithmetric\_2EZERO))) = (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)))
\end{aligned} \tag{77}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2z \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_add \\
& V0x) (ap (ap c\_2Erealax\_2Ereal\_add V1y) V2z)) = (ap (ap c\_2Erealax\_2Ereal\_add \\
& (ap (ap c\_2Erealax\_2Ereal\_add V0x) V1y)) V2z))))))
\end{aligned} \tag{78}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_add \\
& (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) V0x) = V0x))
\end{aligned} \tag{79}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_add \\
& (ap c\_2Erealax\_2Ereal\_neg V0x)) V0x) = (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)))
\end{aligned} \tag{80}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2z \in ty\_2Erealax\_2Ereal. (((p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& V0x) V1y)) \wedge (p (ap (ap c\_2Erealax\_2Ereal\_lt V1y) V2z))) \Rightarrow (p (ap \\
& (ap c\_2Erealax\_2Ereal\_lt V0x) V2z))))))
\end{aligned} \tag{81}$$



Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_mul V0x) V1y) = (ap (ap c\_2Erealax\_2Ereal\_mul V1y) V0x)))) \quad (82)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. (\forall V2z \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_mul V0x) (ap (ap c\_2Erealax\_2Ereal\_mul V1y) V2z)) = (ap (ap c\_2Erealax\_2Ereal\_mul (ap (ap c\_2Erealax\_2Ereal\_mul V0x) V1y)) V2z)))))) \quad (83)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_mul (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) V0x) = V0x)) \quad (84)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Erealax\_2Ereal\_mul V0x) (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) = V0x)) \quad (85)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal. ((\neg (V0x = (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0))) \Rightarrow ((ap (ap c\_2Erealax\_2Ereal\_mul V0x) (ap c\_2Erealax\_2Einv V0x)) = (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \quad (86)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. ((ap c\_2Erealax\_2Ereal\_neg (ap (ap c\_2Erealax\_2Ereal\_mul V0x) V1y)) = (ap (ap c\_2Erealax\_2Ereal\_mul (ap c\_2Erealax\_2Ereal\_neg V0x)) V1y)))) \quad (87)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. ((ap c\_2Erealax\_2Ereal\_neg (ap (ap c\_2Erealax\_2Ereal\_mul V0x) V1y)) = (ap (ap c\_2Erealax\_2Ereal\_mul V0x) (ap c\_2Erealax\_2Ereal\_neg V1y)))))) \quad (88)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal. ((ap c\_2Erealax\_2Ereal\_neg (ap c\_2Erealax\_2Ereal\_neg V0x)) = V0x)) \quad (89)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (((ap (ap c\_2Erealax\_2Ereal\_mul V0x) V1y) = (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad c\_2Enum\_2E0)) \Leftrightarrow ((V0x = (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) \vee \\
& \quad (V1y = (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0))))))
\end{aligned} \tag{90}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& \quad (ap c\_2Erealax\_2Ereal\_neg V0x)) (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad c\_2Enum\_2E0))) \Leftrightarrow (p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad c\_2Enum\_2E0)) V0x))))
\end{aligned} \tag{91}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad c\_2Enum\_2E0)) (ap (ap c\_2Ereal\_2Ereal\_sub V0x) V1y))) \Leftrightarrow (p (ap \\
& \quad (ap c\_2Erealax\_2Ereal\_lt V1y) V0x))))))
\end{aligned} \tag{92}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap c\_2Erealax\_2Ereal\_neg \\
& \quad V0x) = (ap (ap c\_2Erealax\_2Ereal\_mul (ap c\_2Erealax\_2Ereal\_neg \\
& \quad (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL ( \\
& \quad \quad ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) V0x)))
\end{aligned} \tag{93}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& \quad (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) V0x)) \Rightarrow (p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& \quad (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) (ap c\_2Erealax\_2Einv \\
& \quad \quad V0x))))))
\end{aligned} \tag{94}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& \quad (\forall V2z \in ty\_2Erealax\_2Ereal. (((p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& \quad V0x) V1y)) \wedge (p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad \quad c\_2Enum\_2E0)) V2z))) \Rightarrow (p (ap (ap c\_2Erealax\_2Ereal\_lt (ap (ap \\
& \quad \quad c\_2Erealax\_2Ereal\_mul V0x) V2z)) (ap (ap c\_2Erealax\_2Ereal\_mul \\
& \quad \quad \quad V1y) V2z)))))))
\end{aligned} \tag{95}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \quad (p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad V0m)) (ap c\_2Ereal\_2Ereal\_of\_num V1n))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& \quad \quad V0m) V1n))))))
\end{aligned} \tag{96}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \quad (p (ap (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad V0m)) (ap c\_2Ereal\_2Ereal\_of\_num V1n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& \quad V0m) V1n))))))
\end{aligned} \tag{97}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \quad ((ap c\_2Ereal\_2Ereal\_of\_num V0m) = (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad V1n)) \Leftrightarrow (V0m = V1n))))
\end{aligned} \tag{98}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \quad (ap (ap c\_2Erealax\_2Ereal\_add (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad V0m)) (ap c\_2Ereal\_2Ereal\_of\_num V1n)) = (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n))))))
\end{aligned} \tag{99}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& \quad (ap (ap c\_2Erealax\_2Ereal\_mul (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad V0m)) (ap c\_2Ereal\_2Ereal\_of\_num V1n)) = (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n))))))
\end{aligned} \tag{100}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((\neg (V0x = (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad c\_2Enum\_2E0))) \Rightarrow ((ap (ap c\_2Ereal\_2E\_2F V0x) V0x) = (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))))))
\end{aligned} \tag{101}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& \quad (((\neg (V0x = (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0))) \wedge (\neg (V1y = \\
& \quad (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)))) \Rightarrow ((ap c\_2Erealax\_2Einv \\
& \quad (ap (ap c\_2Erealax\_2Ereal\_mul V0x) V1y)) = (ap (ap c\_2Erealax\_2Ereal\_mul \\
& \quad (ap c\_2Erealax\_2Einv V0x)) (ap c\_2Erealax\_2Einv V1y))))))
\end{aligned} \tag{102}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& \quad (((p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad c\_2Enum\_2E0)) V0x)) \wedge (p (ap (ap c\_2Erealax\_2Ereal\_lt V0x) V1y))) \Rightarrow \\
& \quad (p (ap (ap c\_2Erealax\_2Ereal\_lt (ap (ap c\_2Ereal\_2E\_2F V0x) V1y)) \\
& \quad (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL ( \\
& \quad \quad ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))))
\end{aligned} \tag{103}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) V0x)) \Rightarrow (\neg (V0x = (ap \\
& c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0))))))
\end{aligned} \tag{104}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2z \in ty\_2Erealax\_2Ereal. (((\neg (V0x = (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0))) \wedge ((ap (ap c\_2Erealax\_2Ereal\_mul V0x) V1y) = (ap \\
& (ap c\_2Erealax\_2Ereal\_mul V0x) V2z))) \Rightarrow (V1y = V2z))))))
\end{aligned} \tag{105}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0x \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Ereal\_2Epow V0x) \\
& c\_2Enum\_2E0) = (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \wedge (\forall V1x \in \\
& ty\_2Erealax\_2Ereal. (\forall V2n \in ty\_2Enum\_2Enum. ((ap (ap c\_2Ereal\_2Epow \\
& V1x) (ap c\_2Enum\_2ESUC V2n)) = (ap (ap c\_2Erealax\_2Ereal\_mul V1x) \\
& (ap (ap c\_2Ereal\_2Epow V1x) V2n))))))
\end{aligned} \tag{106}$$

Assume the following.

$$\begin{aligned}
& (\forall V0c \in ty\_2Erealax\_2Ereal. (\forall V1m \in ty\_2Enum\_2Enum. \\
& (\forall V2n \in ty\_2Enum\_2Enum. ((ap (ap c\_2Ereal\_2Epow V0c) (ap \\
& (ap c\_2Earithmetic\_2E\_2B V1m) V2n)) = (ap (ap c\_2Erealax\_2Ereal\_mul \\
& (ap (ap c\_2Ereal\_2Epow V0c) V1m)) (ap (ap c\_2Ereal\_2Epow V0c) V2n))))))
\end{aligned} \tag{107}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Ereal\_2Epow V0x) \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) = \\
& V0x))
\end{aligned} \tag{108}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Ereal\_2Epow V0x) \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO)))) = \\
& (ap (ap c\_2Erealax\_2Ereal\_mul V0x) V0x)))
\end{aligned} \tag{109}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1n \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)) V0x)) \Rightarrow (p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)) (ap (ap c\_2Ereal\_2Epow V0x) (ap c\_2Enum\_2ESUC V1n))))))
\end{aligned} \tag{110}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty\_2Enum\_2Enum. ((ap (ap c\_2Ereal\_2Epow (ap c\_2Erealax\_2Ereal\_neg \\
& \quad (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL ( \\
& \quad \quad ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) (ap (ap \\
& \quad c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 \\
& \quad \quad c\_2Earithmetic\_2EZERO))) V0n)) = (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))))))
\end{aligned} \tag{111}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty\_2Enum\_2Enum. (\forall V1f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). \\
& \quad ((ap (ap c\_2Ereal\_2Esum (ap (ap (c\_2Epair\_2E\_2C ty\_2Enum\_2Enum \\
& \quad ty\_2Enum\_2Enum) V0n) c\_2Enum\_2E0)) V1f) = (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad \quad c\_2Enum\_2E0)))) \wedge (\forall V2n \in ty\_2Enum\_2Enum. (\forall V3m \in \\
& \quad ty\_2Enum\_2Enum. (\forall V4f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). \\
& \quad ((ap (ap c\_2Ereal\_2Esum (ap (ap (c\_2Epair\_2E\_2C ty\_2Enum\_2Enum \\
& \quad ty\_2Enum\_2Enum) V2n) (ap c\_2Enum\_2ESUC V3m))) V4f) = (ap (ap c\_2Erealax\_2Ereal\_add \\
& \quad \quad (ap (ap c\_2Ereal\_2Esum (ap (ap (c\_2Epair\_2E\_2C ty\_2Enum\_2Enum \\
& \quad \quad ty\_2Enum\_2Enum) V2n) V3m)) V4f)) (ap V4f (ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad \quad \quad V2n) V3m))))))))))
\end{aligned} \tag{112}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). (\forall V1n \in \\
& \quad ty\_2Enum\_2Enum. ((ap (ap c\_2Ereal\_2Esum (ap (ap (c\_2Epair\_2E\_2C \\
& \quad ty\_2Enum\_2Enum ty\_2Enum\_2Enum) V1n) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad \quad (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO)))) V0f) = ( \\
& \quad ap (ap c\_2Erealax\_2Ereal\_add (ap V0f V1n)) (ap V0f (ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad \quad V1n) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\
& \quad \quad \quad c\_2Earithmetic\_2EZERO))))))))))
\end{aligned} \tag{113}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). (\forall V1l \in \\
& \quad ty\_2Erealax\_2Ereal. ((p (ap (ap c\_2Eseq\_2Esums V0f) V1l)) \Rightarrow (p ( \\
& \quad \quad ap c\_2Eseq\_2Esumable V0f))))))
\end{aligned} \tag{114}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). (\forall V1x \in \\
& \quad ty\_2Erealax\_2Ereal. ((p (ap (ap c\_2Eseq\_2Esums V0f) V1x)) \Rightarrow (V1x = \\
& \quad \quad (ap c\_2Eseq\_2Esuminf V0f))))))
\end{aligned} \tag{115}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V1n \in \\
& \quad ty\_2Enum\_2Enum.(((p (ap c\_2Eseq\_2Esummable V0f)) \wedge (\forall V2d \in \\
& \quad ty\_2Enum\_2Enum.(p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad c\_2Enum\_2E0)) (ap (ap c\_2Erealax\_2Ereal\_add (ap V0f (ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad V1n) (ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) V2d)))) \\
& \quad (ap V0f (ap (ap c\_2Earithmetic\_2E\_2B V1n) (ap (ap c\_2Earithmetic\_2E\_2B \\
& \quad (ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2ENUMERAL (ap \\
& \quad c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) V2d)) (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))))))))) \Rightarrow \\
& \quad (p (ap (ap c\_2Erealax\_2Ereal\_lt (ap (ap c\_2Ereal\_2Esum (ap (ap \\
& \quad (c\_2Epair\_2E\_2C ty\_2Enum\_2Enum ty\_2Enum\_2Enum) c\_2Enum\_2E0) \\
& \quad V1n)) V0f)) (ap c\_2Eseq\_2Esuminf V0f))))))
\end{aligned} \tag{116}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V1x0 \in \\
& \quad ty\_2Erealax\_2Ereal.((p (ap (ap c\_2Eseq\_2Esums V0x) V1x0)) \Rightarrow (p \\
& \quad (ap (ap c\_2Eseq\_2Esums (\lambda V2n \in ty\_2Enum\_2Enum.(ap c\_2Erealax\_2Ereal\_neg \\
& \quad (ap V0x V2n)))) (ap c\_2Erealax\_2Ereal\_neg V1x0))))))
\end{aligned} \tag{117}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal.(p (ap (ap c\_2Eseq\_2Esums (\lambda V1n \in \\
& \quad ty\_2Enum\_2Enum.(ap (ap c\_2Erealax\_2Ereal\_mul (ap (ap c\_2Ereal\_2E\_2F \\
& \quad (ap (ap c\_2Ereal\_2Epow (ap c\_2Erealax\_2Ereal\_neg (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) \\
& \quad V1n)) (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2EFACT \\
& \quad (ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2ENUMERAL (ap \\
& \quad c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) V1n)))) (ap \\
& \quad (ap c\_2Ereal\_2Epow V0x) (ap (ap c\_2Earithmetic\_2E\_2A (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO))) V1n)))) \\
& \quad (ap c\_2Etransc\_2Ecos V0x))))
\end{aligned} \tag{118}$$

**Theorem 1**

$$\begin{aligned}
& (p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Etransc\_2Ecos (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO)))))) \\
& \quad (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)))
\end{aligned}$$