

thm\_2Etransc\_2ECOS\_\_ISZERO  
(TMM2yxWgBTxmJi4Cf3kiyczEosiPCf9NoWd)

October 26, 2020

**Definition 1** We define `c_2Emin_2E_3D` to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 3** We define `c_2Ebool_2E_2T` to be  $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define `c_2Ebool_2E_21` to be  $\lambda A_{27a} : \iota.(\lambda V0P \in (2^{A_{27a}}).(ap (ap (c_2Emin_2E_3D (2^{A_{27a}}))$

**Definition 5** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

**Definition 6** We define `c_2Emin_2E_40` to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge p (ap P x)))$  of type  $\iota \Rightarrow \iota$ .

**Definition 7** We define `c_2Ebool_2E_3F` to be  $\lambda A_{27a} : \iota.(\lambda V0P \in (2^{A_{27a}}).(ap V0P (ap (c_2Emin_2E_40 A_{27a} P)))$

**Definition 8** We define `c_2Ebool_2E_3F_21` to be  $\lambda A_{27a} : \iota.(\lambda V0P \in (2^{A_{27a}}).(ap (ap c_2Ebool_2E_2F_5C A_{27a} P)))$

Let `c_2Enum_2EZERO__REP` :  $\iota$  be given. Assume the following.

$$c_2Enum_2EZERO__REP \in \omega \tag{1}$$

Let `ty_2Enum_2Enum` :  $\iota$  be given. Assume the following.

$$nonempty \ ty_2Enum_2Enum \tag{2}$$

Let `c_2Enum_2EABS__num` :  $\iota$  be given. Assume the following.

$$c_2Enum_2EABS__num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

**Definition 9** We define `c_2Enum_2E0` to be  $(ap c_2Enum_2EABS__num c_2Enum_2EZERO__REP)$ .

Let  $ty\_2Erealx\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealx\_2Ereal \quad (4)$$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealx\_2Ereal^{ty\_2Enum\_2Enum}) \quad (5)$$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (6)$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (7)$$

Let  $c\_2Erealx\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealx\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealx\_2Ereal}) \quad (8)$$

**Definition 10** We define  $c\_2Erealx\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealx\_2Ereal.(ap\ (c\_2Emin\_2E\_40\ ty\_2Erealx\_2Ereal\_REP\_CLASS)\ a)$

Let  $c\_2Erealx\_2Etreall\_add : \iota$  be given. Assume the following.

$$c\_2Erealx\_2Etreall\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (9)$$

Let  $c\_2Erealx\_2Etreall\_eq : \iota$  be given. Assume the following.

$$c\_2Erealx\_2Etreall\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (10)$$

Let  $c\_2Erealx\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealx\_2Ereal\_ABS\_CLASS \in (ty\_2Erealx\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})}) \quad (11)$$

**Definition 11** We define  $c\_2Erealx\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal).c\_2Erealx\_2Ereal\_ABS\_CLASS\ r$

**Definition 12** We define  $c\_2Erealx\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealx\_2Ereal.\lambda V1T2 \in ty\_2Erealx\_2Ereal.c\_2Etreall\_add\ T1\ T2$

Let  $c\_2Erealx\_2Etreall\_neg : \iota$  be given. Assume the following.

$$c\_2Erealx\_2Etreall\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal}) \quad (12)$$

**Definition 13** We define  $c\_2Erealx\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealx\_2Ereal.(ap\ c\_2Erealx\_2Ereal\_neg\ T1)$

**Definition 14** We define  $c\_2Ereal\_2Ereal\_sub$  to be  $\lambda V0x \in ty\_2Erealx\_2Ereal.\lambda V1y \in ty\_2Erealx\_2Ereal.c\_2Erealx\_2Ereal\_neg\ (c\_2Erealx\_2Ereal\_add\ x\ y)$

Let  $c\_2Erealax\_2Etreal\_inv : \iota$  be given. Assume the following.

$$\begin{aligned} c\_2Erealax\_2Etreal\_inv \in & ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal \\ & ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \end{aligned} \quad (13)$$

**Definition 15** We define  $c\_2Erealax\_2Einv$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal\_ABS$

Let  $c\_2Erealax\_2Etreal\_mul : \iota$  be given. Assume the following.

$$\begin{aligned} c\_2Erealax\_2Etreal\_mul \in & (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal \\ & ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \end{aligned} \quad (14)$$

**Definition 16** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal.$

**Definition 17** We define  $c\_2Ereal\_2E\_2F$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.$

Let  $c\_2Erealax\_2Etreal\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (15)$$

**Definition 18** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal.$

**Definition 19** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 20** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E\_7E))$

**Definition 21** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.$

**Definition 22** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.$

**Definition 23** We define  $c\_2Ereal\_2Eabs$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND \\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \end{aligned} \quad (16)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST \\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \end{aligned} \quad (17)$$

**Definition 24** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27a}$

Let  $ty\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Emetric\_2Emetric A0) \quad (18)$$

Let  $c\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emetric\_2Emetric A\_27a \in ((ty\_2Emetric\_2Emetric A\_27a)^{(ty\_2Erealax\_2Ereal)^{(ty\_2Epair\_2Eprod A\_27a A\_27a)}}) \quad (19)$$

**Definition 25** We define  $c\_2Emetric\_2Emr1$  to be  $(ap (c\_2Emetric\_2Emetric ty\_2Erealax\_2Ereal) (ap (c\_2Epair\_2EABS\_prod$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (20)$$

**Definition 26** We define  $c\_2Epair\_2E2C$  to be  $\lambda A\_27a : \iota. \lambda A\_27b : \iota. \lambda V0x \in A\_27a. \lambda V1y \in A\_27b. (ap (c\_2Epair\_2EABS\_prod$

Let  $c\_2Enets\_2Etendsto : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Enets\_2Etendsto A\_27a \in (((2^{A\_27a})^{A\_27a})^{(ty\_2Epair\_2Eprod (ty\_2Emetric\_2Emetric A\_27a A\_27a))}) \quad (21)$$

Let  $c\_2Emetric\_2Edist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emetric\_2Edist A\_27a \in ((ty\_2Erealax\_2Ereal)^{(ty\_2Epair\_2Eprod A\_27a A\_27a)}) \quad (22)$$

Let  $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Etopology\_2Etopology A0) \quad (23)$$

Let  $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Etopology\_2Etopology A\_27a \in ((ty\_2Etopology\_2Etopology A\_27a)^{(2^{(2^{A\_27a})})}) \quad (24)$$

**Definition 27** We define  $c\_2Emetric\_2Emtop$  to be  $\lambda A\_27a : \iota. \lambda V0m \in (ty\_2Emetric\_2Emetric A\_27a). (ap (c\_2Epair\_2EABS\_prod$

Let  $c\_2Enets\_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Enets\_2Etends A\_27a A\_27b \in (((2^{(ty\_2Epair\_2Eprod (ty\_2Etopology\_2Etopology A\_27a))})^{(2^{A\_27b})^{A\_27b}}))^{A\_27a})^{(A\_27a)^{A\_27b}} \quad (25)$$

**Definition 28** We define  $c\_2Elim\_2Etends\_real\_real$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).$

**Definition 29** We define  $c\_2Elim\_2Ediff$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). \lambda V1l \in ty\_2Erealax\_2Ereal.$

**Definition 30** We define  $c\_2Elim\_2Edifferentiable$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}). \lambda V1l \in ty\_2Erealax\_2Ereal.$

**Definition 31** We define  $c\_2Elim\_2Econtl$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).\lambda V1x \in ty$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (26)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (27)$$

**Definition 32** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num$

**Definition 33** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 34** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

**Definition 35** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (28)$$

**Definition 36** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Let  $c\_2Ereal\_2Epow : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})^{ty\_2Erealax\_2Ereal}) \quad (29)$$

Let  $c\_2Earithmetic\_2EFACT : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EFACT \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}) \quad (30)$$

**Definition 37** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

**Definition 38** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

**Definition 39** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (31)$$

Let  $c\_2Earithmetic\_2EDIV : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (32)$$

Let  $c\_2Earithmetic\_2EEVEN : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (33)$$

Let  $c\_2Ereal\_2Esum : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Esum \in ((ty\_2Erealax\_2Ereal^{(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum)}) \quad (34)$$

**Definition 40** We define  $c\_2Earithmic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 41** We define  $c\_2Earithmic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 42** We define  $c\_2Eseq\_2E\_2D\_2D\_3E$  to be  $\lambda V0x \in (ty\_2Erealx\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1x \in ty\_2Erealx\_2Ereal^{ty\_2Enum\_2Enum}$

**Definition 43** We define  $c\_2Eseq\_2Esums$  to be  $\lambda V0f \in (ty\_2Erealx\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1s \in ty\_2Erealx\_2Ereal^{ty\_2Enum\_2Enum}$

**Definition 44** We define  $c\_2Eseq\_2Esuminf$  to be  $\lambda V0f \in (ty\_2Erealx\_2Ereal^{ty\_2Enum\_2Enum}).(ap (c\_2Eseq\_2Esums (ap c\_2Eseq\_2Esums f)))$

**Definition 45** We define  $c\_2Etransc\_2Esin$  to be  $\lambda V0x \in ty\_2Erealx\_2Ereal.(ap c\_2Eseq\_2Esuminf (\lambda V1n \in ty\_2Enum\_2Enum. (ap c\_2Eseq\_2Esums (ap c\_2Eseq\_2Esums f))))$

**Definition 46** We define  $c\_2Etransc\_2Ecos$  to be  $\lambda V0x \in ty\_2Erealx\_2Ereal.(ap c\_2Eseq\_2Esuminf (\lambda V1n \in ty\_2Enum\_2Enum. (ap c\_2Eseq\_2Esums (ap c\_2Eseq\_2Esums f))))$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p (ap (ap c\_2Earithmic\_2E\_3C\_3D c\_2Enum\_2E0) V0n))) \quad (35)$$

Assume the following.

$$True \quad (36)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (37)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (38)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (39)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \wedge ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3)))))) \quad (40)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \quad (41)$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \vee True) \Leftrightarrow True) \wedge (((False \vee (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee False) \Leftrightarrow (p V0t)) \wedge (((p V0t) \vee (p V0t)) \Leftrightarrow (p V0t)))))) \quad (42)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\
& True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\
& (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\
& p V0t))))))
\end{aligned} \tag{45}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.((\neg((p V0A) \Rightarrow (p V1B))) \Leftrightarrow ((p V0A) \wedge (\neg(p V1B)))))) \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1l \in \\
& ty\_2Erealax\_2Ereal.(\forall V2m \in ty\_2Erealax\_2Ereal.(\forall V3x \in \\
& ty\_2Erealax\_2Ereal.(((p (ap (ap (ap c\_2Elim\_2Ediff1 V0f) V1l) \\
& V3x)) \wedge (p (ap (ap (ap c\_2Elim\_2Ediff1 V0f) V2m) V3x))) \Rightarrow (V1l = V2m))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1l \in \\
& ty\_2Erealax\_2Ereal.(\forall V2x \in ty\_2Erealax\_2Ereal.((p (ap \\
& (ap (ap c\_2Elim\_2Ediff1 V0f) V1l) V2x)) \Rightarrow (p (ap (ap c\_2Elim\_2Econt1 \\
& V0f) V2x))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1a \in \\
& ty\_2Erealax\_2Ereal.(\forall V2b \in ty\_2Erealax\_2Ereal.(\forall V3y \in \\
& ty\_2Erealax\_2Ereal.(((p (ap (ap c\_2Ereal\_2Ereal\_lte V1a) V2b)) \wedge \\
& (((p (ap (ap c\_2Ereal\_2Ereal\_lte (ap V0f V2b)) V3y)) \wedge (p (ap (ap \\
& c\_2Ereal\_2Ereal\_lte V3y) (ap V0f V1a)))) \wedge (\forall V4x \in ty\_2Erealax\_2Ereal. \\
& (((p (ap (ap c\_2Ereal\_2Ereal\_lte V1a) V4x)) \wedge (p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& V4x) V2b))) \Rightarrow (p (ap (ap c\_2Elim\_2Econt1 V0f) V4x)))))) \Rightarrow (\exists V5x \in \\
& ty\_2Erealax\_2Ereal.((p (ap (ap c\_2Ereal\_2Ereal\_lte V1a) V5x)) \wedge \\
& ((p (ap (ap c\_2Ereal\_2Ereal\_lte V5x) V2b)) \wedge ((ap V0f V5x) = V3y))))))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1a \in \\
& \quad ty\_2Erealax\_2Ereal.(\forall V2b \in ty\_2Erealax\_2Ereal.(((p ( \\
& \quad ap (ap c\_2Erealax\_2Ereal\_lt V1a) V2b)) \wedge ((ap V0f V1a) = (ap V0f \\
& \quad V2b)) \wedge ((\forall V3x \in ty\_2Erealax\_2Ereal.(((p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& \quad V1a) V3x)) \wedge (p (ap (ap c\_2Ereal\_2Ereal\_lte V3x) V2b))) \Rightarrow (p (ap ( \\
& \quad ap c\_2Elim\_2Econtl V0f) V3x)))) \wedge (\forall V4x \in ty\_2Erealax\_2Ereal. \\
& \quad (((p (ap (ap c\_2Erealax\_2Ereal\_lt V1a) V4x)) \wedge (p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& \quad V4x) V2b))) \Rightarrow (p (ap (ap c\_2Elim\_2Edifferentiable V0f) V4x)))))) \Rightarrow \\
& \quad (\exists V5z \in ty\_2Erealax\_2Ereal.((p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& \quad V1a) V5z)) \wedge ((p (ap (ap c\_2Erealax\_2Ereal\_lt V5z) V2b)) \wedge (p (ap \\
& \quad (ap (ap c\_2Elim\_2Ediff1 V0f) (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0) \\
& \quad V5z))))))))))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. \\
& \quad ((V0x = V1y) \vee ((p (ap (ap c\_2Erealax\_2Ereal\_lt V0x) V1y)) \vee (p (ap \\
& \quad (ap c\_2Erealax\_2Ereal\_lt V1y) V0x))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. \\
& \quad ((p (ap (ap c\_2Erealax\_2Ereal\_lt V0x) V1y)) \Rightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& \quad V0x) V1y))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. \\
& \quad (\forall V2z \in ty\_2Erealax\_2Ereal.(((p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& \quad V0x) V1y)) \wedge (p (ap (ap c\_2Ereal\_2Ereal\_lte V1y) V2z))) \Rightarrow (p (ap ( \\
& \quad ap c\_2Erealax\_2Ereal\_lt V0x) V2z))))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. \\
& \quad (\forall V2z \in ty\_2Erealax\_2Ereal.(((p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& \quad V0x) V1y)) \wedge (p (ap (ap c\_2Erealax\_2Ereal\_lt V1y) V2z))) \Rightarrow (p (ap \\
& \quad (ap c\_2Erealax\_2Ereal\_lt V0x) V2z))))))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& (p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad c\_2Enum\_2E0) (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL \\
& \quad (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))))))
\end{aligned} \tag{55}$$



Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal.(((ap\ c\_2Erealax\_2Ereal\_neg \\
V0x) = (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0)) \Leftrightarrow (V0x = (ap\ c\_2Ereal\_2Ereal\_of\_num \\
& \qquad \qquad \qquad c\_2Enum\_2E0))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\
& (p\ (ap\ (ap\ c\_2Ereal\_2Ereal\_lte\ (ap\ c\_2Ereal\_2Ereal\_of\_num \\
V0m))\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ c\_2Earithmic\_2E\_3C\_3D \\
& \qquad \qquad \qquad V0m)\ V1n))))))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt \\
& (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0))\ V0x)) \Rightarrow (\neg(V0x = (ap \\
& \qquad \qquad \qquad c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0))))))
\end{aligned} \tag{58}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{59}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \tag{60}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\
& ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False))))))
\end{aligned} \tag{62}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \tag{63}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow ( \\
& (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee (\neg( \\
& p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\
& \qquad \qquad \qquad ((\neg(p\ V1q)) \vee (\neg(p\ V0p))))))))))
\end{aligned} \tag{64}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))) \quad (65)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal. (p (ap (ap (ap c\_2Elim\_2Ediff c\_2Etransc\_2Ecos) (ap c\_2Erealax\_2Ereal\_neg (ap c\_2Etransc\_2Esin V0x))) V0x))) \quad (66)$$

Assume the following.

$$((ap c\_2Etransc\_2Ecos (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) = (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))) \quad (67)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal. (((p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) V0x)) \wedge (p (ap (ap c\_2Erealax\_2Ereal\_lt V0x) (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO)))))) \Rightarrow (p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) (ap c\_2Etransc\_2Esin V0x)))))) \quad (68)$$

Assume the following.

$$(p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Etransc\_2Ecos (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO)))))) (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0))) \quad (69)$$

### Theorem 1

$$(p (ap (c\_2Ebool\_2E\_3F\_21 ty\_2Erealax\_2Ereal) (\lambda V0x \in ty\_2Erealax\_2Ereal. (ap (ap c\_2Ebool\_2E\_2F\_5C (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) V0x)) (ap (ap c\_2Ebool\_2E\_2F\_5C (ap (ap c\_2Ereal\_2Ereal\_lte V0x) (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO)))))) (ap (ap c\_2Emin\_2E\_3D ty\_2Erealax\_2Ereal) (ap c\_2Etransc\_2Ecos V0x)) (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0))))))$$