

thm_2Etransc_2ECOS__NEG (TMZq- dUbEHRfKdZnUbwqoSp1A7JYE3388kYt)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 5 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{3}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})\ ty_2Erealax_2Ereal) \tag{4}$$

Definition 6 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 7 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (ty_2Erealax_2Ereal V0a)))$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)))(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal) \quad (5)$$

Let $c_2Erealax_2Etrealeq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealeq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)) \quad (6)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}}) \quad (7)$$

Definition 8 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 9 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (8)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (9)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \quad (10)$$

Definition 10 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal)^{ty_2Enum_2Enum} \quad (11)$$

Let $c_2Ereal_2Epow : \iota$ be given. Assume the following.

$$c_2Ereal_2Epow \in ((ty_2Erealax_2Ereal)^{ty_2Enum_2Enum})^{ty_2Erealax_2Ereal} \quad (12)$$

Let $c_2Earithmetic_2EFACT : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EFACT \in (ty_2Enum_2Enum)^{ty_2Enum_2Enum} \quad (13)$$

Definition 11 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega)^{ty_2Enum_2Enum} \quad (14)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega)^{\omega} \quad (15)$$

Definition 12 We define c_Enum_ESUC to be $\lambda V0m \in ty_Enum_Enum.(ap\ c_Enum_EABS_num)$.
Let $c_Earithmic_E_2B : \iota$ be given. Assume the following.

$$c_Earithmic_E_2B \in ((ty_Enum_Enum^{ty_Enum_Enum})^{ty_Enum_Enum}) \quad (16)$$

Definition 13 We define $c_Earithmic_EBIT2$ to be $\lambda V0n \in ty_Enum_Enum.(ap\ (ap\ c_Earithmic_E_2B))$.

Definition 14 We define $c_Earithmic_ENUMERAL$ to be $\lambda V0x \in ty_Enum_Enum.V0x$.

Let $c_Earithmic_EDIV : \iota$ be given. Assume the following.

$$c_Earithmic_EDIV \in ((ty_Enum_Enum^{ty_Enum_Enum})^{ty_Enum_Enum}) \quad (17)$$

Definition 15 We define $c_Earithmic_EBIT1$ to be $\lambda V0n \in ty_Enum_Enum.(ap\ (ap\ c_Earithmic_E_2B))$.

Let $c_Erealax_Etrealm_neg : \iota$ be given. Assume the following.

$$c_Erealax_Etrealm_neg \in ((ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)}) \quad (18)$$

Definition 16 We define $c_Erealax_Ereal_neg$ to be $\lambda V0T1 \in ty_Erealax_Ereal.(ap\ c_Erealax_Ereal_neg)$.

Let $c_Erealax_Etrealm_inv : \iota$ be given. Assume the following.

$$c_Erealax_Etrealm_inv \in ((ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)}) \quad (19)$$

Definition 17 We define $c_Erealax_Einv$ to be $\lambda V0T1 \in ty_Erealax_Ereal.(ap\ c_Erealax_Ereal_ABS)$.

Definition 18 We define $c_Ereal_E_2F$ to be $\lambda V0x \in ty_Erealax_Ereal.\lambda V1y \in ty_Erealax_Ereal.(c_Erealax_Ereal_E_2F\ x\ y)$.

Let $c_Earithmic_EEVEN : \iota$ be given. Assume the following.

$$c_Earithmic_EEVEN \in (2^{ty_Enum_Enum}) \quad (20)$$

Definition 19 We define c_Ebool_EF to be $(ap\ (c_Ebool_E_21\ 2))\ (\lambda V0t \in 2.V0t)$.

Definition 20 We define c_Ebool_ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(c_Ebool_ECOND\ t1\ t2\ t3))))$.

Let $c_Epair_EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_Epair_EABS_prod\ A_27a\ A_27b \in ((ty_Epair_Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (21)$$

Definition 21 We define $c_Epair_E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_Epair_EABS_prod\ A_27a\ A_27b))\ x\ y)$.

Let $c_2Ereal_2Esum : \iota$ be given. Assume the following.

$$c_2Ereal_2Esum \in ((ty_2Erealax_2Ereal^{(ty_2Erealax_2Ereal^{ty_2Enum_2Enum})})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)}) (22)$$

Definition 22 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E))$

Definition 23 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ P)\ c_2Ebool_2E_3F)))$

Definition 24 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(c_2Eprim_rec_2E_3C\ m\ n)$

Definition 25 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(c_2Earithmetic_2E_3E\ m\ n)$

Definition 26 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ t1\ t2)\ c_2Ebool_2E_5C_2F))$

Definition 27 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.(c_2Earithmetic_2E_3E_3D\ m\ n)$

Let $c_2Erealax_2Etrealm_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) (23)$$

Definition 28 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.(c_2Erealax_2Ereal_add\ T1\ T2)$

Definition 29 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.(c_2Ereal_2Ereal_sub\ x\ y)$

Let $c_2Erealax_2Etrealm_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) (24)$$

Definition 30 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.(c_2Erealax_2Ereal_lt\ T1\ T2)$

Definition 31 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.(c_2Ereal_2Ereal_lte\ x\ y)$

Definition 32 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ (ap\ (ap\ (c_2Ebool_2ECONV\ x)\ c_2Ereal_2Eabs)))$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) (25)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) (26)$$

Definition 33 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a})^{A_27b})$

Assume the following.

$$True \quad (34)$$

Assume the following.

$$\forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. ((V0x = V0x) \Leftrightarrow True)) \quad (35)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty.2Erealax.2Ereal. (\forall V1y \in ty.2Erealax.2Ereal. \\ & (((ap (ap c.2Ereal.2Ereal._sub\ V0x)\ V1y) = (ap c.2Ereal.2Ereal._of_num \\ & \quad c.2Enum.2E0)) \Leftrightarrow (V0x = V1y)))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty.2Erealax.2Ereal. (\forall V1y \in ty.2Erealax.2Ereal. \\ & (((ap (ap c.2Erealax.2Ereal._add\ (ap (ap c.2Erealax.2Ereal._mul \\ & V0x)\ V0x))\ (ap (ap c.2Erealax.2Ereal._mul\ V1y)\ V1y)) = (ap c.2Ereal.2Ereal._of_num \\ & \quad c.2Enum.2E0)) \Leftrightarrow ((V0x = (ap c.2Ereal.2Ereal._of_num\ c.2Enum.2E0)) \wedge \\ & \quad (V1y = (ap c.2Ereal.2Ereal._of_num\ c.2Enum.2E0)))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty.2Erealax.2Ereal. ((ap (ap c.2Ereal.2Epow\ V0x) \\ & (ap c.2Earithmetic.2ENUMERAL\ (ap c.2Earithmetic.2EBIT2\ c.2Earithmetic.2EZERO))) = \\ & \quad (ap (ap c.2Erealax.2Ereal._mul\ V0x)\ V0x))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty.2Erealax.2Ereal. ((ap (ap c.2Erealax.2Ereal._add \\ & (ap (ap c.2Ereal.2Epow\ (ap (ap c.2Erealax.2Ereal._add\ (ap c.2Etransc.2Esin \\ & \quad (ap c.2Erealax.2Ereal._neg\ V0x)))\ (ap c.2Etransc.2Esin\ V0x))) \\ & (ap c.2Earithmetic.2ENUMERAL\ (ap c.2Earithmetic.2EBIT2\ c.2Earithmetic.2EZERO)))) \\ & (ap (ap c.2Ereal.2Epow\ (ap (ap c.2Ereal.2Ereal._sub\ (ap c.2Etransc.2Ecos \\ & \quad (ap c.2Erealax.2Ereal._neg\ V0x)))\ (ap c.2Etransc.2Ecos\ V0x))) \\ & (ap c.2Earithmetic.2ENUMERAL\ (ap c.2Earithmetic.2EBIT2\ c.2Earithmetic.2EZERO)))) = \\ & \quad (ap c.2Ereal.2Ereal._of_num\ c.2Enum.2E0))) \end{aligned} \quad (39)$$

Theorem 1

$$(\forall V0x \in ty.2Erealax.2Ereal. ((ap c.2Etransc.2Ecos\ (ap c.2Erealax.2Ereal._neg\ V0x)) = (ap c.2Etransc.2Ecos\ V0x)))$$