

# thm\_2Etransc\_2ECOS\_\_NPI (TMXmk- sKiu6SQRiCC386AFYerTJumz5B4PhD)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

**Definition 3** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum)^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum} \tag{2}$$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{3}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{4}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum)^{\omega} \tag{5}$$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda \tau a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a})))$

**Definition 5** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num$

Let  $ty\_2Erealx\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealx\_2Ereal \tag{6}$$

Let  $c\_2Ereal\_2Epow : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealx\_2Ereal)^{ty\_2Enum\_2Enum})^{ty\_2Erealx\_2Ereal} \tag{7}$$

**Definition 6** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21) 2) (\lambda V2t \in 2)))$ .  
Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (8)$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (9)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax}) \quad (10)$$

**Definition 7** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p (ap\ P\ x)) \text{ then } (the\ (\lambda x.x \in A \wedge p\ x)) \text{ of type } \iota \Rightarrow \iota$ .

**Definition 8** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_2Emin\_2E\_40\ ty\_2Erealax\_2Ereal\_REP\_CLASS))$ .

Let  $c\_2Erealax\_2Etrealmul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealmul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (11)$$

Let  $c\_2Erealax\_2Etrealeq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealeq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (12)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})} \quad (13)$$

**Definition 9** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$ .

**Definition 10** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$ .

Let  $c\_2Erealax\_2Erealneg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Erealneg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (14)$$

**Definition 11** We define  $c\_2Erealax\_2Erealneg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Erealneg)$ .

Let  $c\_2Erealax\_2Erealadd : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Erealadd \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (15)$$

**Definition 12** We define  $c\_Erealax\_Ereal\_add$  to be  $\lambda V0T1 \in ty\_Erealax\_Ereal.\lambda V1T2 \in ty\_Erealax\_Ereal.$

**Definition 13** We define  $c\_Ereal\_Ereal\_sub$  to be  $\lambda V0x \in ty\_Erealax\_Ereal.\lambda V1y \in ty\_Erealax\_Ereal.$

Let  $c\_Enum\_EZERO\_REP : \iota$  be given. Assume the following.

$$c\_Enum\_EZERO\_REP \in \omega \tag{16}$$

**Definition 14** We define  $c\_Enum\_E0$  to be  $(ap\ c\_Enum\_EABS\_num\ c\_Enum\_EZERO\_REP)$ .

**Definition 15** We define  $c\_Earithmetic\_EZERO$  to be  $c\_Enum\_E0$ .

**Definition 16** We define  $c\_Earithmetic\_EBIT1$  to be  $\lambda V0n \in ty\_Enum\_Enum.(ap\ (ap\ c\_Earithmetic\_E0\ n))$ .

**Definition 17** We define  $c\_Earithmetic\_ENUMERAL$  to be  $\lambda V0x \in ty\_Enum\_Enum.V0x$ .

Let  $c\_Ereal\_Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_Ereal\_Ereal\_of\_num \in (ty\_Erealax\_Ereal^{ty\_Enum\_Enum}) \tag{17}$$

Let  $c\_Earithmetic\_EFACT : \iota$  be given. Assume the following.

$$c\_Earithmetic\_EFACT \in (ty\_Enum\_Enum^{ty\_Enum\_Enum}) \tag{18}$$

**Definition 18** We define  $c\_Earithmetic\_EBIT2$  to be  $\lambda V0n \in ty\_Enum\_Enum.(ap\ (ap\ c\_Earithmetic\_E0\ n))$ .

Let  $c\_Earithmetic\_EDIV : \iota$  be given. Assume the following.

$$c\_Earithmetic\_EDIV \in ((ty\_Enum\_Enum^{ty\_Enum\_Enum})^{ty\_Enum\_Enum}) \tag{19}$$

Let  $c\_Erealax\_Etrealm\_inv : \iota$  be given. Assume the following.

$$c\_Erealax\_Etrealm\_inv \in ((ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)^{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)}) \tag{20}$$

**Definition 19** We define  $c\_Erealax\_Einv$  to be  $\lambda V0T1 \in ty\_Erealax\_Ereal.(ap\ c\_Erealax\_Ereal\_ABS\ T1)$ .

**Definition 20** We define  $c\_Ereal\_E2F$  to be  $\lambda V0x \in ty\_Erealax\_Ereal.\lambda V1y \in ty\_Erealax\_Ereal.$

Let  $c\_Earithmetic\_EEVEN : \iota$  be given. Assume the following.

$$c\_Earithmetic\_EEVEN \in (2^{ty\_Enum\_Enum}) \tag{21}$$

**Definition 21** We define  $c\_Ebool\_EF$  to be  $(ap\ (c\_Ebool\_E21\ 2))\ (\lambda V0t \in 2.V0t)$ .

**Definition 22** We define  $c\_Ebool\_ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.V0t1\ t2)))$ .

Let  $c\_Epair\_EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_Epair\_EABS\_prod\ A\_27a\ A\_27b \in ((ty\_Epair\_Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \tag{22}$$

**Definition 23** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Ereal\_2Esum : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Esum \in ((ty\_2Erealax\_2Ereal^{(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum)}) \quad (23)$$

**Definition 24** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E\ V0t) c\_2Ebool\_2E\_7E$

**Definition 25** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_4O$

**Definition 26** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 27** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 28** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21\ 2) (\lambda V2t \in$

**Definition 29** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_2Erealax\_2Etrealm\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (24)$$

**Definition 30** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 31** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

**Definition 32** We define  $c\_2Ereal\_2Eabs$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap (ap (ap (c\_2Ebool\_2ECONJ$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (25)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \quad (26)$$

**Definition 33** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27a$

Let  $ty\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Emetric\_2Emetric\ A0) \quad (27)$$

Let  $c\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Emetric\ A\_27a \in ((ty\_2Emetric\_2Emetric\ A\_27a)^{(ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})}) \quad (28)$$

**Definition 34** We define  $c\_2Emetric\_2Emr1$  to be  $(ap (c\_2Emetric\_2Emetric ty\_2Erealax\_2Ereal) (ap (c\_2Emetric\_2Edist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emetric\_2Edist A\_27a \in ((ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod A\_27a A\_27a)})) \quad (29)$$

Let  $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Etopology\_2Etopology A0) \quad (30)$$

Let  $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Etopology\_2Etopology A\_27a \in ((ty\_2Etopology\_2Etopology A\_27a)^{(2^{(2^A - 27a)})}) \quad (31)$$

**Definition 35** We define  $c\_2Emetric\_2Emtop$  to be  $\lambda A\_27a : \iota. \lambda V0m \in (ty\_2Emetric\_2Emetric A\_27a). (ap$   
Let  $c\_2Enets\_2Eetends : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Enets\_2Eetends A\_27a A\_27b \in (((2^{(ty\_2Epair\_2Eprod (ty\_2Etopology\_2Etopology A\_27a) ((2^{A-27b})^{A-27b}))})_{A\_27a})_{(A\_27a^{A-27b})}) \quad (32)$$

**Definition 36** We define  $c\_2Eseq\_2E\_2D\_2D\_23E$  to be  $\lambda V0x \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). \lambda V1x$

**Definition 37** We define  $c\_2Eseq\_2Esums$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). \lambda V1s \in ty\_2$

**Definition 38** We define  $c\_2Eseq\_2Esuminf$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). (ap (c\_2E$

**Definition 39** We define  $c\_2Etransc\_2Ecos$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal. (ap c\_2Eseq\_2Esuminf (\lambda V1$

**Definition 40** We define  $c\_2Etransc\_2Epi$  to be  $(ap (ap c\_2Erealax\_2Ereal\_mul (ap c\_2Ereal\_2Ereal\_of\_n$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (33)$$

**Definition 41** We define  $c\_2Etransc\_2Esin$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal. (ap c\_2Eseq\_2Esuminf (\lambda V1$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum. ((ap c\_2Enum\_2ESUC V0m) = (ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \quad (34)$$

Assume the following.

$$True \quad (35)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (36)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (37)$$

Assume the following.

$$\begin{aligned} & (\forall V0P \in (2^{ty\_2Enum\_2Enum}). (((p\ (ap\ V0P\ c\_2Enum\_2E0)) \wedge \\ & (\forall V1n \in ty\_2Enum\_2Enum. ((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ c\_2Enum\_2ESUC\ V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum. (p\ (ap\ V0P\ V2n)))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\ & ((ap\ (ap\ c\_2Erealax\_2Ereal\_mul\ V0x\ V1y) = (ap\ (ap\ c\_2Erealax\_2Ereal\_mul\ V1y\ V0x)))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap\ (ap\ c\_2Erealax\_2Ereal\_mul \\ & (ap\ c\_2Ereal\_2Ereal\_of\_num\ (ap\ c\_2Earithmic\_2ENUMERAL\ ( \\ & ap\ c\_2Earithmic\_2EBIT1\ c\_2Earithmic\_2EZERO))))\ V0x) = V0x)) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\ & (\forall V2z \in ty\_2Erealax\_2Ereal. ((ap\ (ap\ c\_2Erealax\_2Ereal\_mul \\ & (ap\ (ap\ c\_2Erealax\_2Ereal\_add\ V0x\ V1y))\ V2z) = (ap\ (ap\ c\_2Erealax\_2Ereal\_add \\ & (ap\ (ap\ c\_2Erealax\_2Ereal\_mul\ V0x\ V2z))\ (ap\ (ap\ c\_2Erealax\_2Ereal\_mul \\ & V1y\ V2z)))))) \end{aligned} \quad (41)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap\ (ap\ c\_2Erealax\_2Ereal\_mul \\ & (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0))\ V0x) = (ap\ c\_2Ereal\_2Ereal\_of\_num \\ & c\_2Enum\_2E0))) \end{aligned} \quad (42)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap\ (ap\ c\_2Erealax\_2Ereal\_mul \\ & V0x)\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0)) = (ap\ c\_2Ereal\_2Ereal\_of\_num \\ & c\_2Enum\_2E0))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& (ap (ap c\_2Erealax\_2Ereal\_add (ap c\_2Ereal\_2Ereal\_of\_num \\
V0m)) (ap c\_2Ereal\_2Ereal\_of\_num V1n)) = (ap c\_2Ereal\_2Ereal\_of\_num \\
& (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Ereal\_2Ereal\_sub \\
V0x) (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) = V0x))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0x \in ty\_2Erealax\_2Ereal. ((ap (ap c\_2Ereal\_2Epow V0x) \\
c\_2Enum\_2E0) = (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \wedge (\forall V1x \in \\
ty\_2Erealax\_2Ereal. (\forall V2n \in ty\_2Enum\_2Enum. ((ap (ap c\_2Ereal\_2Epow \\
V1x) (ap c\_2Enum\_2ESUC V2n)) = (ap (ap c\_2Erealax\_2Ereal\_mul V1x) \\
& (ap (ap c\_2Ereal\_2Epow V1x) V2n))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& ((ap c\_2Etransc\_2Ecos (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) = \\
& (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL ( \\
& ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((ap c\_2Etransc\_2Ecos (ap (ap c\_2Erealax\_2Ereal\_add V0x) V1y)) = \\
& (ap (ap c\_2Ereal\_2Ereal\_sub (ap (ap c\_2Erealax\_2Ereal\_mul ( \\
ap c\_2Etransc\_2Ecos V0x)) (ap c\_2Etransc\_2Ecos V1y))) (ap (ap c\_2Erealax\_2Ereal\_mul \\
& (ap c\_2Etransc\_2Esin V0x)) (ap c\_2Etransc\_2Esin V1y))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& ((ap c\_2Etransc\_2Ecos c\_2Etransc\_2Epi) = (ap c\_2Erealax\_2Ereal\_neg \\
& (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL ( \\
& ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))))))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& ((ap c\_2Etransc\_2Esin c\_2Etransc\_2Epi) = (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0))
\end{aligned} \tag{50}$$

**Theorem 1**

$$(\forall V0n \in ty\_2Enum\_2Enum. ((ap\ c\_2Etrasc\_2Ecos (ap (ap\ c\_2Erealx\_2Ereal\_mul (ap\ c\_2Ereal\_2Ereal\_of\_num\ V0n))\ c\_2Etrasc\_2Epi)) = (ap (ap\ c\_2Ereal\_2Epow (ap\ c\_2Erealx\_2Ereal\_neg (ap\ c\_2Ereal\_2Ereal\_of\_num (ap\ c\_2Earithmetic\_2ENUMERAL (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO))))))\ V0n)))$$