

thm\_2Etransc\_2ECOS\_\_TOTAL  
(TMJyuykc1jg9uX6c2qoTZGB98S4PL7FwBVi)

October 26, 2020

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 3** We define  $c\_2Ebool\_2ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 5** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2$

**Definition 6** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$  of type  $\iota \Rightarrow \iota$ .

**Definition 7** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40 A$

**Definition 8** We define  $c\_2Ebool\_2E\_3F\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap c\_2Ebool\_2E\_2F\_5C$

**Definition 9** We define  $c\_2Ebool\_2EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 10** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (4)$$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (5)$$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (6)$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (7)$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal}) \quad (8)$$

**Definition 11** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (c\_2Emin\_2E\_40\ ty\_2Erealax\_2Ereal\_REP\_CLASS)\ a)$

Let  $c\_2Erealax\_2Etrealm\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (9)$$

Let  $c\_2Erealax\_2Etrealm\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (10)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})}) \quad (11)$$

**Definition 12** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal).c\_2Erealax\_2Ereal\_ABS\_CLASS\ r$

**Definition 13** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal.c\_2Erealax\_2Etrealm\_add\ T1\ T2$

Let  $c\_2Erealax\_2Etrealm\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal}) \quad (12)$$

**Definition 14** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal\_neg\ T1)$

**Definition 15** We define  $c\_2Ereal\_2Ereal\_sub$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.c\_2Erealax\_2Ereal\_neg\ (c\_2Erealax\_2Ereal\_add\ x\ y)$

Let  $c\_2Erealax\_2Etrealm\_inv : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_inv \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)) \quad (13)$$

**Definition 16** We define  $c\_2Erealax\_2Einv$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal\_ABS$

Let  $c\_2Erealax\_2Etrealm\_mul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)) \quad (14)$$

**Definition 17** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal.$

**Definition 18** We define  $c\_2Ereal\_2E\_2F$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.$

Let  $c\_2Erealax\_2Etrealm\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)) \quad (15)$$

**Definition 19** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal.$

**Definition 20** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E\_7E$

**Definition 21** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.$

**Definition 22** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.$

**Definition 23** We define  $c\_2Ereal\_2Eabs$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b)^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)} \quad (16)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a)^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)} \quad (17)$$

**Definition 24** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c)^{A\_27a}$

Let  $ty\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Emetric\_2Emetric\ A0) \quad (18)$$

Let  $c\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Emetric\ A\_27a \in ((ty\_2Emetric\_2Emetric\ A\_27a)^{(ty\_2Erealax\_2Ereal)^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}})) \quad (19)$$



**Definition 34** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num$   
Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})ty\_2Enum\_2Enum) \quad (28)$$

**Definition 35** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

**Definition 36** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Ereal\_2Epow : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})ty\_2Erealax\_2Ereal) \quad (29)$$

Let  $c\_2Earithmetic\_2EFACT : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EFACT \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}) \quad (30)$$

**Definition 37** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Let  $c\_2Earithmetic\_2EDIV : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})ty\_2Enum\_2Enum) \quad (31)$$

Let  $c\_2Earithmetic\_2EEVEN : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (32)$$

Let  $c\_2Ereal\_2Esum : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Esum \in ((ty\_2Erealax\_2Ereal^{(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum)}) \quad (33)$$

**Definition 38** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 39** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 40** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 41** We define  $c\_2Eseq\_2E\_2D\_2D\_3E$  to be  $\lambda V0x \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1x$

**Definition 42** We define  $c\_2Eseq\_2Esums$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1s \in ty\_2Enum$

**Definition 43** We define  $c\_2Eseq\_2Esuminf$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(ap\ (c\_2E$

**Definition 44** We define  $c\_2Etransc\_2Ecos$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ c\_2Eseq\_2Esuminf\ (\lambda V1n$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})ty\_2Enum\_2Enum) \quad (34)$$

**Definition 45** We define  $c\_2Etransc\_2Esin$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ c\_2Eseq\_2Esuminf\ (\lambda V1n$

**Definition 46** We define  $c\_2Etransc\_2Epi$  to be  $(ap\ (ap\ c\_2Erealax\_2Ereal\_mul\ (ap\ c\_2Ereal\_2Ereal\_of\_m$

Assume the following.

$$True \quad (35)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (36)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (37)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (38)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (39)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (40)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (41)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (42)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (43)$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1l \in \\
& \quad ty\_2Erealax\_2Ereal.(\forall V2m \in ty\_2Erealax\_2Ereal.(\forall V3x \in \\
& \quad ty\_2Erealax\_2Ereal.(((p (ap (ap (ap (ap c\_2Elim\_2Ediff1 V0f) V1l) \\
& \quad V3x)) \wedge (p (ap (ap (ap c\_2Elim\_2Ediff1 V0f) V2m) V3x))) \Rightarrow (V1l = V2m)))))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1l \in \\
& \quad ty\_2Erealax\_2Ereal.(\forall V2x \in ty\_2Erealax\_2Ereal.((p (ap \\
& \quad (ap (ap c\_2Elim\_2Ediff1 V0f) V1l) V2x)) \Rightarrow (p (ap (ap c\_2Elim\_2Econt1 \\
& \quad V0f) V2x))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1a \in \\
& \quad ty\_2Erealax\_2Ereal.(\forall V2b \in ty\_2Erealax\_2Ereal.(\forall V3y \in \\
& \quad ty\_2Erealax\_2Ereal.(((p (ap (ap c\_2Ereal\_2Ereal\_lte V1a) V2b)) \wedge \\
& \quad (((p (ap (ap c\_2Ereal\_2Ereal\_lte (ap V0f V2b) V3y)) \wedge (p (ap (ap \\
& \quad c\_2Ereal\_2Ereal\_lte V3y) (ap V0f V1a)))) \wedge (\forall V4x \in ty\_2Erealax\_2Ereal. \\
& \quad (((p (ap (ap c\_2Ereal\_2Ereal\_lte V1a) V4x)) \wedge (p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& \quad V4x) V2b))) \Rightarrow (p (ap (ap c\_2Elim\_2Econt1 V0f) V4x)))))) \Rightarrow (\exists V5x \in \\
& \quad ty\_2Erealax\_2Ereal.((p (ap (ap c\_2Ereal\_2Ereal\_lte V1a) V5x)) \wedge \\
& \quad ((p (ap (ap c\_2Ereal\_2Ereal\_lte V5x) V2b)) \wedge ((ap V0f V5x) = V3y)))))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1a \in \\
& \quad ty\_2Erealax\_2Ereal.(\forall V2b \in ty\_2Erealax\_2Ereal.(((p ( \\
& \quad ap (ap c\_2Erealax\_2Ereal\_lt V1a) V2b)) \wedge (((ap V0f V1a) = (ap V0f \\
& \quad V2b)) \wedge (\forall V3x \in ty\_2Erealax\_2Ereal.(((p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& \quad V1a) V3x)) \wedge (p (ap (ap c\_2Ereal\_2Ereal\_lte V3x) V2b))) \Rightarrow (p (ap ( \\
& \quad ap c\_2Elim\_2Econt1 V0f) V3x)))) \wedge (\forall V4x \in ty\_2Erealax\_2Ereal. \\
& \quad (((p (ap (ap c\_2Erealax\_2Ereal\_lt V1a) V4x)) \wedge (p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& \quad V4x) V2b))) \Rightarrow (p (ap (ap c\_2Elim\_2Edifferentiable V0f) V4x)))))) \Rightarrow \\
& \quad (\exists V5z \in ty\_2Erealax\_2Ereal.((p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& \quad V1a) V5z)) \wedge ((p (ap (ap c\_2Erealax\_2Ereal\_lt V5z) V2b)) \wedge (p (ap \\
& \quad (ap (ap c\_2Elim\_2Ediff1 V0f) (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0) \\
& \quad V5z)))))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. \\
& \quad ((V0x = V1y) \vee ((p (ap (ap c\_2Erealax\_2Ereal\_lt V0x) V1y)) \vee (p (ap \\
& \quad (ap c\_2Erealax\_2Ereal\_lt V1y) V0x))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal.(\neg (p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& \quad V0x) V0x))))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Erealax\_2Ereal\_lt V0x) V1y)) \Rightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& V0x) V1y))))))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2z \in ty\_2Erealax\_2Ereal. (((p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& V0x) V1y)) \wedge (p (ap (ap c\_2Ereal\_2Ereal\_lte V1y) V2z))) \Rightarrow (p (ap ( \\
& ap c\_2Erealax\_2Ereal\_lt V0x) V2z))))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2z \in ty\_2Erealax\_2Ereal. (((p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& V0x) V1y)) \wedge (p (ap (ap c\_2Erealax\_2Ereal\_lt V1y) V2z))) \Rightarrow (p (ap ( \\
& ap c\_2Erealax\_2Ereal\_lt V0x) V2z))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (((ap c\_2Erealax\_2Ereal\_neg \\
& V0x) = (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) \Leftrightarrow (V0x = (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (p (ap (ap (ap c\_2Elim\_2Ediff \\
& c\_2Etransc\_2Ecos) (ap c\_2Erealax\_2Ereal\_neg (ap c\_2Etransc\_2Esin \\
& V0x))) V0x)))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& ((ap c\_2Etransc\_2Ecos (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) = \\
& (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL ( \\
& ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& (p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)) c\_2Etransc\_2Epi))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& ((ap c\_2Etransc\_2Ecos c\_2Etransc\_2Epi) = (ap c\_2Erealax\_2Ereal\_neg \\
& (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL ( \\
& ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))))))
\end{aligned} \tag{57}$$



Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal.(((p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) V0x)) \wedge (p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& V0x) c\_2Etransc\_2Epi)))) \Rightarrow (p (ap (ap c\_2Erealax\_2Ereal\_lt (ap \\
& c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) (ap c\_2Etransc\_2Esin \\
& V0x))))))
\end{aligned} \tag{58}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0y \in ty\_2Erealax\_2Ereal.(((p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& (ap c\_2Erealax\_2Ereal\_neg (ap c\_2Ereal\_2Ereal\_of\_num (ap \\
& c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \\
& V0y)) \wedge (p (ap (ap c\_2Ereal\_2Ereal\_lte V0y) (ap c\_2Ereal\_2Ereal\_of\_num \\
& (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO)))))) \Rightarrow \\
& (p (ap (c\_2Ebool\_2E\_3F\_21 ty\_2Erealax\_2Ereal) (\lambda V1x \in ty\_2Erealax\_2Ereal. \\
& (ap (ap c\_2Ebool\_2E\_2F\_5C (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)) V1x)) (ap (ap c\_2Ebool\_2E\_2F\_5C (ap (ap c\_2Ereal\_2Ereal\_lte \\
& V1x) c\_2Etransc\_2Epi)) (ap (ap (c\_2Emin\_2E\_3D ty\_2Erealax\_2Ereal) \\
& (ap c\_2Etransc\_2Ecos V1x)) V0y)))))))))
\end{aligned}$$