thm_2Etransc_2EDIFF__ACS__LEMMA (TMSe1k1kJKkJ7SxPh6pTT2kxFQYvwz5jHxL)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{1}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{3}$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{4}$$

Definition 5 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 6 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (\mathsf{c\_2Emin\_2E\_40}\ (ty$

Let $c\_2Erealax\_2Etreal\_\_inv : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_inv \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \tag{5}$$

Let $c\_2Erealax\_2Etreal\_\_eq : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{6}$$

Let $c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehr} \tag{7}$$

Definition 7 *We define* c_2Erealax_2Ereal__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_$

Definition 8 *We define* c_2Erealax_2Einv *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal__ABS (

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{8}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{9}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{10}$$

Definition 9 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \tag{11}$$

Let $c\_2Erealax\_2Etreal\_\_add : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \tag{12}$$

Definition 10 *We define* c_2Erealax_2Ereal__add *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Let $c\_2Erealax\_2Etreal\_\_neg : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \tag{13}$$

Definition 11 *We define* c_2Erealax_2Ereal__neg *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal.

Definition 12 *We define* c_2Ereal_2Ereal__sub *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehr} \tag{14}$$

Definition 13 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_$

Definition 14 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 15 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 16 *We define* c_2Ereal_2Ereal__lte *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Definition 17 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 18 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 19 *We define* c_2Ereal_2Eabs *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (ap$ (c_2Ebool_2ECONI

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{15}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{16}$$

Definition 20 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27}$

Let $ty\_2Emetric\_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Emetric\_2Emetric\ A0) \tag{17}$$

Let $c\_2Emetric\_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Emetric\ A\_27a \in ((ty\_2Emetric\_2Emetric\ A\_27a)^{(ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})}) \tag{18}$$

Definition 21 *We define* c_2Emetric_2Emr1 *to be* $(ap\ (c\_2Emetric\_2Emetric\ ty\_2Erealax\_2Ereal)\ (ap$ (c_

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{19}$$

Definition 22 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Enets\_2Etendsto : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Enets\_2Etendsto\ A\_27a \in (((2^{A\_27a})^{A\_27a})^{(ty\_2Epair\_2Eprod\ (ty\_2Emetric\_2En} \quad (20)$$

Let $c\_2Emetric\_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Edist\ A\_27a \in ((ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a} \quad (21)$$

Definition 23 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Let $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \quad (22)$$

Let $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^{A\_27a})})}) \quad (23)$$

Definition 24 *We define* c_2Emetric_2Emtop *to be* $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Emetric\_2Emetric\ A\_27a).(ap$

Let $c\_2Enets\_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Enets\_2Etends\ A\_27a\ A\_27b \in (((2^{(ty\_2Epair\_2Eprod\ (ty\_2Etopology\_2Etopology\ A\_27a)\ ((2^{A\_27b})^{A\_27b}))})^{A\_27a})^{(A\_27a^{A\_27b})}) \quad (24)$$

Definition 25 *We define* c_2Elim_2Etends__real__real *to be* $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).$

Definition 26 *We define* c_2Elim_2Econtl *to be* $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).\lambda V1x \in ty$

Let $c\_2Erealax\_2Etreal\_\_mul : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \quad (25)$$

Definition 27 *We define* c_2Erealax_2Ereal__mul *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Definition 28 *We define* c_2Ereal_2E_2F *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.($

Definition 29 *We define* c_2Elim_2Ediffl *to be* $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).\lambda V1l \in ty\_2$

Let $c\_2Ereal\_2Epow : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})^{ty\_2Erealax\_2Ereal}) \quad (26)$$

Let $c\_2Earithmetic\_2EFACT : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EFACT \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}) \quad (27)$$

Definition 30 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \quad (28)$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \quad (29)$$

Definition 31 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (30)$$

Definition 32 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 33 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $c\_2Earithmetic\_2EDIV : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (31)$$

Definition 34 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (32)$$

Let $c\_2Ereal\_2Esum : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Esum \in ((ty\_2Erealax\_2Ereal^{(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2En} \quad (33)$$

Definition 35 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 36 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 37 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 38 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 39 *We define* c_2Eseq_2E_2D_2D_3E *to be* $\lambda V0x \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1x$

Definition 40 *We define* c_2Eseq_2Esums *to be* $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1s \in ty\_$

Definition 41 *We define* c_2Eseq_2Esuminf *to be* $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(ap$ (c_2E

Definition 42 *We define* c_2Etransc_2Ecos *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap$ c_2Eseq_2Esuminf $(\lambda V1n$

Definition 43 *We define* c_2Etransc_2Epi *to be* $(ap\ (ap$ c_2Erealax_2Ereal__mul $(ap\ c\_2Ereal\_2Ereal\_\_of\_\_n$

Definition 44 *We define* c_2Etransc_2Eacs *to be* $\lambda V0y \in ty\_2Erealax\_2Ereal.(ap$ (c_2Emin_2E_40 ty_2Ere

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{34}$$

Definition 45 *We define* c_2Etransc_2Esin *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap$ c_2Eseq_2Esuminf $(\lambda V1n$

Assume the following.

$$True \tag{35}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{36}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{37}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{38}$$

Assume the following.

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1l \in ty\_2Erealax\_2Ereal.(\forall V2x \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V0f)\ V1l)\ V2x)) \Rightarrow (p\ (ap\ (ap\ \mathsf{c\_2Elim\_2Econtl}\ V0f)\ V2x)))))) \tag{39}$$

Assume the following.

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1g \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V2l \in ty\_2Erealax\_2Ereal.(\forall V3a \in ty\_2Erealax\_2Ereal.(\forall V4x \in ty\_2Erealax\_2Ereal.(\forall V5b \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V3a)\ V4x)) \wedge ((p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V4x)\ V5b)) \wedge ((\forall V6z \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V3a)\ V6z)) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V6z)\ V5b))) \Rightarrow (((ap\ V1g\ (ap\ V0f\ V6z)) = V6z) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Elim\_2Econtl}\ V0f)\ V6z)))))) \wedge ((p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V0f)\ V2l)\ V4x)) \wedge (\neg (V2l = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))))))) \Rightarrow (p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V1g)\ (ap\ \mathsf{c\_2Erealax\_2Einv}\ V2l))\ (ap\ V0f\ V4x)))))))))) \tag{40}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V0x)\ V1y)) \Rightarrow (p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V0x)\ V1y))))) \tag{41}$$

Assume the following.

$$((ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})) \tag{42}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.(((ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V0x) = V1y) \Leftrightarrow (V0x = (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V1y))))) \tag{43}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ \mathsf{c\_2Etransc\_2Ecos})\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ (ap\ \mathsf{c\_2Etransc\_2Esin}\ V0x)))\ V0x))) \tag{44}$$

Assume the following.

$$(\forall V0y \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))\ V0y)) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V0y)\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))) \Rightarrow ((ap\ \mathsf{c\_2Etransc\_2Ecos}\ (ap\ \mathsf{c\_2Etransc\_2Eacs}\ V0y)) = V0y))) \tag{45}$$

Assume the following.

$$(\forall V0y \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))\ V0y)) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V0y)\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))) \Rightarrow ((p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ \mathsf{c\_2Etransc\_2Eacs}\ V0y))) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ (ap\ \mathsf{c\_2Etransc\_2Eacs}\ V0y))\ \mathsf{c\_2Etransc\_2Epi}))))) \tag{46}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V0x)) \wedge (p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V0x)\ \mathsf{c\_2Etransc\_2Epi}))) \Rightarrow ((ap\ \mathsf{c\_2Etransc\_2Eacs}\ (ap\ \mathsf{c\_2Etransc\_2Ecos}\ V0x)) = V0x))) \tag{47}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))\ V0x))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V0x)\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))))\Rightarrow(\neg((ap\ \mathsf{c\_2Etransc\_2Esin}\ (ap\ \mathsf{c\_2Etransc\_2Eacs}\ V0x)) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))))\tag{48}$$

Theorem 1

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))\ V0x))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V0x)\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))))\Rightarrow(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ \mathsf{c\_2Etransc\_2Eacs})\ (ap\ \mathsf{c\_2Erealax\_2Einv}\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ (ap\ \mathsf{c\_2Etransc\_2Esin}\ (ap\ \mathsf{c\_2Etransc\_2Eacs}\ V0x)))))\ V0x))))$$