

thm_2Etransc_2EDIFF__ASN__LEMMA (TMYX3VJw6Rq92wfcsPchaRvF5wiz3y7cmph)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{3}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})_{ty_2Erealax_2Ereal}) \tag{4}$$

Definition 5 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap\ P\ x))$ **then** (the $(\lambda x.x \in A \wedge p (ap\ P\ x))$) of type $\iota \Rightarrow \iota$.

Definition 6 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (ty_2Erealax_2Ereal_REP_CLASS\ a)))$

Let $c_2Erealax_2Etrealm_inv : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_inv \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)_{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \tag{5}$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (6)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (7)$$

Definition 7 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 8 We define $c_2Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal_ABS)$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (8)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (9)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \quad (10)$$

Definition 9 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal)^{ty_2Enum_2Enum} \quad (11)$$

Let $c_2Erealax_2Etrealm_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (12)$$

Definition 10 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (13)$$

Definition 11 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal_neg)$

Definition 12 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Erealax_2Etrealm_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (14)$$

Definition 13 We define $c_2Erealax_2Ereal_lte$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$.

Definition 14 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 15 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_7E))$

Definition 16 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$.

Definition 17 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21) 2) (\lambda V2t \in 2)))$

Definition 18 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(\lambda V3t3 \in 2))))$

Definition 19 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap (ap (ap (c_2Ebool_2ECOND) c_2Ereal_2Eabs) c_2Ereal_2Eabs) c_2Ereal_2Eabs))$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2ESND \\ A_27a A_27b \in (A_27b^{(ty_2Epair_2Eprod A_27a A_27b)}) \end{aligned} \quad (15)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EFST \\ A_27a A_27b \in (A_27a^{(ty_2Epair_2Eprod A_27a A_27b)}) \end{aligned} \quad (16)$$

Definition 20 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a})^{A_27b})$

Let $ty_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_2Emetric_2Emetric A0) \quad (17)$$

Let $c_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow c_2Emetric_2Emetric A_27a \in ((ty_2Emetric_2Emetric \\ A_27a)^{(ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod A_27a A_27a)})}) \end{aligned} \quad (18)$$

Definition 21 We define $c_2Emetric_2Emr1$ to be $(ap (c_2Emetric_2Emetric ty_2Erealax_2Ereal) (ap (c_2Emetric_2Emetric) c_2Emetric_2Emetric))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a A_27b \in ((ty_2Epair_2Eprod A_27a A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (19)$$

Definition 22 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epair_2EABS_prod) c_2Epair_2E_2C))$

Let $c_2Enets_2Etendsto : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Enets_2Etendsto\ A_27a \in (((2^{A_27a})^{A_27a})^{(ty_2Epair_2Eprod\ (ty_2Emetric_2E...))}) \quad (20)$$

Let $c_2Emetric_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Edist\ A_27a \in ((ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})^{(ty_2Erealax_2Ereal)}) \quad (21)$$

Definition 23 We define c_2Ebool_2E3F to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap\ V0P\ (ap\ (c_2Emin_2E40$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Etopology_2Etopology\ A0) \quad (22)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Etopology\ A_27a \in ((ty_2Etopology_2Etopology\ A_27a)^{(2^{(2^{A_27a})})}) \quad (23)$$

Definition 24 We define $c_2Emetric_2Emtop$ to be $\lambda A_27a : \iota. \lambda V0m \in (ty_2Emetric_2Emetric\ A_27a). (ap$

Let $c_2Enets_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Enets_2Etends\ A_27a\ A_27b \in (((2^{(ty_2Epair_2Eprod\ (ty_2Etopology_2Etopology\ A_27a)\ ((2^{A_27b})^{A_27b}))})^{A_27a})^{(A_27a^{A_27b})}) \quad (24)$$

Definition 25 We define $c_2Elim_2Etends_real_real$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).$

Definition 26 We define $c_2Elim_2Econtl$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). \lambda V1x \in ty$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ (ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (25)$$

Definition 27 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal. \lambda V1T2 \in ty_2Erealax$

Definition 28 We define c_2Ereal_2E2F to be $\lambda V0x \in ty_2Erealax_2Ereal. \lambda V1y \in ty_2Erealax_2Ereal.$

Definition 29 We define $c_2Elim_2Ediff1$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). \lambda V1l \in ty$

Let $c_2Ereal_2Epow : \iota$ be given. Assume the following.

$$c_2Ereal_2Epow \in ((ty_2Erealax_2Ereal^{ty_2Eenum_2Eenum})^{ty_2Erealax_2Ereal}) \quad (26)$$

Let $c_2Earithmetic_2EFACT : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EFACT \in (ty_2Eenum_2Eenum^{ty_2Eenum_2Eenum}) \quad (27)$$

Definition 30 We define $c_2\text{Earithmetic_2EZERO}$ to be $c_2\text{Enum_2E0}$.

Let $c_2\text{Enum_2EREP_num} : \iota$ be given. Assume the following.

$$c_2\text{Enum_2EREP_num} \in (\omega^{ty_2\text{Enum_2Enum}}) \quad (28)$$

Let $c_2\text{Enum_2ESUC_REP} : \iota$ be given. Assume the following.

$$c_2\text{Enum_2ESUC_REP} \in (\omega^{\omega}) \quad (29)$$

Definition 31 We define $c_2\text{Enum_2ESUC}$ to be $\lambda V0m \in ty_2\text{Enum_2Enum}.$ (ap $c_2\text{Enum_2EABS_num}$

Let $c_2\text{Earithmetic_2E_2B} : \iota$ be given. Assume the following.

$$c_2\text{Earithmetic_2E_2B} \in ((ty_2\text{Enum_2Enum}^{ty_2\text{Enum_2Enum}})^{ty_2\text{Enum_2Enum}}) \quad (30)$$

Definition 32 We define $c_2\text{Earithmetic_2EBIT2}$ to be $\lambda V0n \in ty_2\text{Enum_2Enum}.$ (ap (ap $c_2\text{Earithmetic}$

Definition 33 We define $c_2\text{Earithmetic_2ENUMERAL}$ to be $\lambda V0x \in ty_2\text{Enum_2Enum}.$ $V0x$.

Definition 34 We define $c_2\text{Earithmetic_2EBIT1}$ to be $\lambda V0n \in ty_2\text{Enum_2Enum}.$ (ap (ap $c_2\text{Earithmetic}$

Let $c_2\text{Earithmetic_2E_2D} : \iota$ be given. Assume the following.

$$c_2\text{Earithmetic_2E_2D} \in ((ty_2\text{Enum_2Enum}^{ty_2\text{Enum_2Enum}})^{ty_2\text{Enum_2Enum}}) \quad (31)$$

Let $c_2\text{Earithmetic_2EDIV} : \iota$ be given. Assume the following.

$$c_2\text{Earithmetic_2EDIV} \in ((ty_2\text{Enum_2Enum}^{ty_2\text{Enum_2Enum}})^{ty_2\text{Enum_2Enum}}) \quad (32)$$

Let $c_2\text{Earithmetic_2EEVEN} : \iota$ be given. Assume the following.

$$c_2\text{Earithmetic_2EEVEN} \in (2^{ty_2\text{Enum_2Enum}}) \quad (33)$$

Let $c_2\text{Ereal_2Esum} : \iota$ be given. Assume the following.

$$c_2\text{Ereal_2Esum} \in ((ty_2\text{Erealax_2Ereal}^{(ty_2\text{Erealax_2Ereal}^{ty_2\text{Enum_2Enum}})})^{(ty_2\text{Epair_2Eprod } ty_2\text{Enum_2Enum})}) \quad (34)$$

Definition 35 We define $c_2\text{Eprim_rec_2E_3C}$ to be $\lambda V0m \in ty_2\text{Enum_2Enum}.$ $\lambda V1n \in ty_2\text{Enum_2Enum}$

Definition 36 We define $c_2\text{Earithmetic_2E_3E}$ to be $\lambda V0m \in ty_2\text{Enum_2Enum}.$ $\lambda V1n \in ty_2\text{Enum_2Enum}$

Definition 37 We define $c_2\text{Ebool_2E_5C_2F}$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2\text{Ebool_2E_21 } 2) (\lambda V2t \in 2.$

Definition 38 We define $c_2\text{Earithmetic_2E_3E_3D}$ to be $\lambda V0m \in ty_2\text{Enum_2Enum}.$ $\lambda V1n \in ty_2\text{Enum_2Enum}$

Definition 39 We define $c_2\text{Eseq_2E_2D_2D_3E}$ to be $\lambda V0x \in (ty_2\text{Erealax_2Ereal}^{ty_2\text{Enum_2Enum}}).$ $\lambda V1x \in ty_2\text{Enum_2Enum}$

Definition 40 We define $c_2\text{Eseq_2Esums}$ to be $\lambda V0f \in (ty_2\text{Erealax_2Ereal}^{ty_2\text{Enum_2Enum}}).$ $\lambda V1s \in ty_2\text{Enum_2Enum}$

Definition 41 We define $c_2Eseq_2Esuminf$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(ap (c_2E$

Definition 42 We define $c_2Etransc_2Esin$ to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap c_2Eseq_2Esuminf (\lambda V1n$

Definition 43 We define $c_2Etransc_2Ecos$ to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap c_2Eseq_2Esuminf (\lambda V1n$

Definition 44 We define $c_2Etransc_2Epi$ to be $(ap (ap c_2Erealax_2Ereal_mul (ap c_2Ereal_2Ereal_of_num$

Definition 45 We define $c_2Etransc_2Easn$ to be $\lambda V0y \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E40 ty_2Ere$

Assume the following.

$$True \quad (35)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\\ & p V0t)))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} & (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1l \in \\ & ty_2Erealax_2Ereal.(\forall V2x \in ty_2Erealax_2Ereal.((p (ap \\ & (ap (ap c_2Elim_2Ediff1 V0f) V1l) V2x)) \Rightarrow (p (ap (ap c_2Elim_2Econt1 \\ & V0f) V2x)))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} & (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1g \in \\ & (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V2l \in ty_2Erealax_2Ereal. \\ & (\forall V3a \in ty_2Erealax_2Ereal.(\forall V4x \in ty_2Erealax_2Ereal. \\ & (\forall V5b \in ty_2Erealax_2Ereal.(((p (ap (ap c_2Erealax_2Ereal_lt \\ & V3a) V4x)) \wedge ((p (ap (ap c_2Erealax_2Ereal_lt V4x) V5b)) \wedge ((\forall V6z \in \\ & ty_2Erealax_2Ereal.(((p (ap (ap c_2Erealax_2Ereal_lt V3a) V6z)) \wedge \\ & (p (ap (ap c_2Erealax_2Ereal_lt V6z) V5b))) \Rightarrow (((ap V1g (ap V0f V6z)) = \\ & V6z) \wedge (p (ap (ap c_2Elim_2Econt1 V0f) V6z)))))) \wedge ((p (ap (ap c_2Elim_2Ediff1 \\ & V0f) V2l) V4x)) \wedge (\neg (V2l = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)))))) \Rightarrow \\ & (p (ap (ap (ap c_2Elim_2Ediff1 V1g) (ap c_2Erealax_2Einv V2l)) (\\ & ap V0f V4x)))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap (ap c_2Erealax_2Ereal_lt V0x) V1y)) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte \\
& V0x) V1y))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (p (ap (ap (ap c_2Elim_2Ediff \\
& c_2Etransc_2Esin) (ap c_2Etransc_2Ecos V0x)) V0x)))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0y \in ty_2Erealax_2Ereal. (((p (ap (ap c_2Ereal_2Ereal_lte \\
& (ap c_2Erealax_2Ereal_neg (ap c_2Ereal_2Ereal_of_num (ap \\
& c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \\
& V0y)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte V0y) (ap c_2Ereal_2Ereal_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \Rightarrow \\
& ((ap c_2Etransc_2Esin (ap c_2Etransc_2Easn V0y)) = V0y)))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0y \in ty_2Erealax_2Ereal. (((p (ap (ap c_2Erealax_2Ereal_lt \\
& (ap c_2Erealax_2Ereal_neg (ap c_2Ereal_2Ereal_of_num (ap \\
& c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \\
& V0y)) \wedge (p (ap (ap c_2Erealax_2Ereal_lt V0y) (ap c_2Ereal_2Ereal_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \Rightarrow \\
& ((p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Erealax_2Ereal_neg \\
& (ap (ap c_2Ereal_2E_2F c_2Etransc_2Epi) (ap c_2Ereal_2Ereal_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO)))))) \\
& (ap c_2Etransc_2Easn V0y))) \wedge (p (ap (ap c_2Erealax_2Ereal_lt \\
& (ap c_2Etransc_2Easn V0y)) (ap (ap c_2Ereal_2E_2F c_2Etransc_2Epi) \\
& (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL (\\
& ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))))))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (((p (ap (ap c_2Ereal_2Ereal_lte \\
& (ap c_2Erealax_2Ereal_neg (ap (ap c_2Ereal_2E_2F c_2Etransc_2Epi) \\
& (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL (\\
& ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO)))))) V0x)) \wedge \\
& (p (ap (ap c_2Ereal_2Ereal_lte V0x) (ap (ap c_2Ereal_2E_2F c_2Etransc_2Epi) \\
& (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL (\\
& ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO)))))) \Rightarrow ((ap \\
& c_2Etransc_2Easn (ap c_2Etransc_2Esin V0x)) = V0x)))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.(((p (ap (ap c_2Erealax_2Ereal_lt \\
& (ap c_2Erealax_2Ereal_neg (ap c_2Ereal_2Ereal_of_num (ap \\
& c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \\
& V0x)) \wedge (p (ap (ap c_2Erealax_2Ereal_lt V0x) (ap c_2Ereal_2Ereal_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \Rightarrow \\
& (\neg((ap c_2Etransc_2Ecos (ap c_2Etransc_2Easn V0x)) = (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0))))))
\end{aligned} \tag{46}$$

Theorem 1

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.(((p (ap (ap c_2Erealax_2Ereal_lt \\
& (ap c_2Erealax_2Ereal_neg (ap c_2Ereal_2Ereal_of_num (ap \\
& c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \\
& V0x)) \wedge (p (ap (ap c_2Erealax_2Ereal_lt V0x) (ap c_2Ereal_2Ereal_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \Rightarrow \\
& (p (ap (ap (ap c_2Elim_2Ediff1 c_2Etransc_2Easn) (ap c_2Erealax_2Einv \\
& (ap c_2Etransc_2Ecos (ap c_2Etransc_2Easn V0x)))) V0x))))
\end{aligned}$$