thm_2Etransc_2EDIFF__COMPOSITE (TMZrQ5bjwnRuNSS33qhVXYKmQHkjsdEWMpX)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\text{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\text{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{1}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{3}$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{4}$$

Definition 5 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\textbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \textbf{then}\ (the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 6 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ (\text{c\_2Emin\_2E\_40}\ (ty$

Let $c\_2Erealax\_2Etreal\_\_add : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \tag{5}$$

Let $c\_2Erealax\_2Etreal\_\_eq : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{6}$$

Let $c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Eh} \tag{7}$$

Definition 7 *We define* c_2Erealax_2Ereal__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_$

Definition 8 *We define* c_2Erealax_2Ereal__add *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{8}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{9}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{10}$$

Definition 9 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{11}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{12}$$

Definition 10 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{13}$$

Definition 11 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{14}$$

Let $c\_2Erealax\_2Etreal\_\_inv : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_inv \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \tag{15}$$

Definition 12 *We define* c_2Erealax_2Einv *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal__ABS

Definition 13 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Definition 14 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 15 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Let $c\_2Ereal\_2Epow : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})^{ty\_2Erealax\_2Ereal}) \tag{16}$$

Let $c\_2Erealax\_2Etreal\_\_mul : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Eh} \tag{17}$$

Definition 16 *We define* c_2Erealax_2Ereal__mul *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Let $c\_2Erealax\_2Etreal\_\_neg : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \tag{18}$$

Definition 17 *We define* c_2Erealax_2Ereal__neg *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal

Definition 18 *We define* c_2Ereal_2Ereal__sub *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2E$

Definition 19 *We define* c_2Ereal_2E_2F *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.($

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \tag{19}$$

Definition 20 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 21 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Definition 22 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Let $c\_2Earithmetic\_2EFACT : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EFACT \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}) \tag{20}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{21}$$

Definition 23 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Ereal\_2Esum : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Esum \in ((ty\_2Erealax\_2Ereal^{(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2En} \tag{22}$$

Definition 24 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap$ (c_2Emin_2E_40

Definition 25 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 26 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 27 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 28 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{23}$$

Definition 29 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_$

Definition 30 *We define* c_2Ereal_2Ereal__lte *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Definition 31 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 32 *We define* c_2Ereal_2Eabs *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (ap$ (c_2Ebool_2ECONI

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{24}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{25}$$

Definition 33 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27}$

Let $ty\_2Emetric\_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Emetric\_2Emetric\ A0) \tag{26}$$

Let $c\_2Emetric\_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Emetric\ A\_27a \in ((ty\_2Emetric\_2Emetric\ A\_27a)^{(ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})}) \tag{27}$$

Definition 34 *We define* c_2Emetric_2Emr1 *to be* $(ap\ (c_2Emetric_2Emetric\ ty_2Erealax_2Ereal)\ (ap\ ($c

Let $c\_2Emetric\_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Edist\ A\_27a \in ((ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a}\tag{28}$$

Let $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0)\tag{29}$$

Let $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^{A\_27a})})})\tag{30}$$

Definition 35 *We define* c_2Emetric_2Emtop *to be* $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Emetric\_2Emetric\ A\_27a).(ap$

Let $c\_2Enets\_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Enets\_2Etends\ A\_27a\ A\_27b \in (((2^{(ty\_2Epair\_2Eprod\ (ty\_2Etopology\_2Etopology\ A\_27a)\ ((2^{A\_27b})^{A\_27b}))})^{A\_27a})^{(A\_27a^{A\_27b})})\tag{31}$$

Definition 36 *We define* c_2Eseq_2E_2D_2D_3E *to be* $\lambda V0x \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1x$

Definition 37 *We define* c_2Eseq_2Esums *to be* $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1s \in ty\_$

Definition 38 *We define* c_2Eseq_2Esuminf *to be* $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(ap$ (c_2E

Definition 39 *We define* c_2Etransc_2Eexp *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap$ c_2Eseq_2Esuminf $(\lambda V1$

Let $c\_2Earithmetic\_2EDIV : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})\tag{32}$$

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum})\tag{33}$$

Definition 40 *We define* c_2Etransc_2Esin *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap$ c_2Eseq_2Esuminf $(\lambda V1n$

Definition 41 *We define* c_2Etransc_2Ecos *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap$ c_2Eseq_2Esuminf $(\lambda V1n$

Let $c\_2Enets\_2Etendsto : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Enets\_2Etendsto\ A\_27a \in (((2^{A\_27a})^{A\_27a})^{(ty\_2Epair\_2Eprod\ (ty\_2Emetric\_2Em}\tag{34}$$

Definition 42 *We define* c_2Elim_2Etends__real__real *to be* $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).$

Definition 43 *We define* c_2Elim_2Ediffl *to be* $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).\lambda V1l \in ty\_2$

Assume the following.

$$True \tag{35}$$

Assume the following.

$$(\forall V0t \in 2.(((True\wedge(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\wedge True)\Leftrightarrow (p\ V0t))\wedge(((False\wedge(p\ V0t))\Leftrightarrow False)\wedge((((p\ V0t)\wedge False)\Leftrightarrow False)\wedge (((p\ V0t)\wedge(p\ V0t))\Leftrightarrow(p\ V0t))))))) \tag{36}$$

Assume the following.

$$(\forall V0t \in 2.(((True\Leftrightarrow(p\ V0t))\Leftrightarrow(p\ V0t))\wedge((((p\ V0t)\Leftrightarrow True)\Leftrightarrow (p\ V0t))\wedge(((False\Leftrightarrow(p\ V0t))\Leftrightarrow(\neg(p\ V0t)))\wedge(((p\ V0t)\Leftrightarrow False)\Leftrightarrow(\neg(p\ V0t)))))) \tag{37}$$

Assume the following.

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1g \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V2l \in ty\_2Erealax\_2Ereal. (\forall V3m \in ty\_2Erealax\_2Ereal.(\forall V4x \in ty\_2Erealax\_2Ereal. (((p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V0f)\ V2l)\ V4x))\wedge(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V1g)\ V3m)\ V4x)))\Rightarrow(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ (\lambda V5x \in ty\_2Erealax\_2Ereal. (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ V0f\ V5x))\ (ap\ V1g\ V5x))))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V2l)\ V3m))\ V4x)))))))) \tag{38}$$

Assume the following.

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1g \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V2l \in ty\_2Erealax\_2Ereal. (\forall V3m \in ty\_2Erealax\_2Ereal.(\forall V4x \in ty\_2Erealax\_2Ereal. (((p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V0f)\ V2l)\ V4x))\wedge(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V1g)\ V3m)\ V4x)))\Rightarrow(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ (\lambda V5x \in ty\_2Erealax\_2Ereal. (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ V0f\ V5x))\ (ap\ V1g\ V5x))))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V2l)\ (ap\ V1g\ V4x)))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V3m)\ (ap\ V0f\ V4x))))\ V4x)))))))) \tag{39}$$

Assume the following.

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1l \in ty\_2Erealax\_2Ereal.(\forall V2x \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V0f)\ V1l)\ V2x))\Rightarrow(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ (\lambda V3x \in ty\_2Erealax\_2Ereal.(ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ (ap\ V0f\ V3x))))\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V1l))\ V2x)))))) \tag{40}$$

Assume the following.

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1g \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V2l \in ty\_2Erealax\_2Ereal.(\forall V3m \in ty\_2Erealax\_2Ereal.(\forall V4x \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V0f)\ V2l)\ V4x))\wedge(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V1g)\ V3m)\ V4x)))\Rightarrow(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ (\lambda V5x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ (ap\ V0f\ V5x))\ (ap\ V1g\ V5x))))\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ V2l)\ V3m))\ V4x))))))) \tag{41}$$

Assume the following.

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1g \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V2l \in ty\_2Erealax\_2Ereal.(\forall V3m \in ty\_2Erealax\_2Ereal.(\forall V4x \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V0f)\ V2l)\ (ap\ V1g\ V4x)))\wedge(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V1g)\ V3m)\ V4x)))\Rightarrow(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ (\lambda V5x \in ty\_2Erealax\_2Ereal.(ap\ V0f\ (ap\ V1g\ V5x))))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V2l)\ V3m))\ V4x))))))) \tag{42}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1x \in ty\_2Erealax\_2Ereal.(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ (\lambda V2x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ c\_2Ereal\_2Epow\ V2x)\ V0n)))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ V0n))\ (ap\ (ap\ c\_2Ereal\_2Epow\ V1x)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0n)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))))\ V1x)))) \tag{43}$$

Assume the following.

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1l \in ty\_2Erealax\_2Ereal.(\forall V2x \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V0f)\ V1l)\ V2x))\wedge(\neg((ap\ V0f\ V2x) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))))\Rightarrow(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ (\lambda V3x \in ty\_2Erealax\_2Ereal.(ap\ \mathsf{c\_2Erealax\_2Einv}\ (ap\ V0f\ V3x))))\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ (ap\ (ap\ \mathsf{c\_2Ereal\_2E\_2F}\ V1l)\ (ap\ (ap\ c\_2Ereal\_2Epow\ (ap\ V0f\ V2x))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO}))))))\ V2x)))))) \tag{44}$$

Assume the following.

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1g \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V2l \in ty\_2Erealax\_2Ereal.(\forall V3m \in ty\_2Erealax\_2Ereal.(\forall V4x \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V0f)\ V2l)\ V4x))\wedge((p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V1g)\ V3m)\ V4x))\wedge(\neg((ap\ V1g\ V4x) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))))\Rightarrow(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ (\lambda V5x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ \mathsf{c\_2Ereal\_2E\_2F}\ (ap\ V0f\ V5x))\ (ap\ V1g\ V5x))))\ (ap\ (ap\ \mathsf{c\_2Ereal\_2E\_2F}\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V2l)\ (ap\ V1g\ V4x)))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V3m)\ (ap\ V0f\ V4x))))\ (ap\ (ap\ c\_2Ereal\_2Epow\ (ap\ V1g\ V4x))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO})))))\ V4x))))))) \tag{45}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ \mathsf{c\_2Etransc\_2Eexp})\ (ap\ \mathsf{c\_2Etransc\_2Eexp}\ V0x))\ V0x))) \tag{46}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ \mathsf{c\_2Etransc\_2Esin})\ (ap\ \mathsf{c\_2Etransc\_2Ecos}\ V0x))\ V0x))) \tag{47}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ \mathsf{c\_2Etransc\_2Ecos})\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ (ap\ \mathsf{c\_2Etransc\_2Esin}\ V0x)))\ V0x))) \tag{48}$$

Theorem 1

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1l \in ty\_2Erealax\_2Ereal.(\forall V2x \in ty\_2Erealax\_2Ereal.(\forall V3g \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V4m \in ty\_2Erealax\_2Ereal.(\forall V5n \in ty\_2Enum\_2Enum.((((p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V0f)\ V1l)\ V2x))\wedge(\neg((ap\ V0f\ V2x) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))))\Rightarrow(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ (\lambda V6x \in ty\_2Erealax\_2Ereal.(ap\ \mathsf{c\_2Erealax\_2Einv}\ (ap\ V0f\ V6x))))\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ (ap\ (ap\ \mathsf{c\_2Ereal\_2E\_2F}\ V1l)\ (ap\ (ap\ c\_2Ereal\_2Epow\ (ap\ V0f\ V2x))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO}))))))\ V2x)))\wedge(((((p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V0f)\ V1l)\ V2x))\wedge((p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V3g)\ V4m)\ V2x))\wedge(\neg((ap\ V3g\ V2x) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))))\Rightarrow(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ (\lambda V7x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ \mathsf{c\_2Ereal\_2E\_2F}\ (ap\ V0f\ V7x))\ (ap\ V3g\ V7x))))\ (ap\ (ap\ \mathsf{c\_2Ereal\_2E\_2F}\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V1l)\ (ap\ V3g\ V2x)))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V4m)\ (ap\ V0f\ V2x))))\ (ap\ (ap\ c\_2Ereal\_2Epow\ (ap\ V3g\ V2x))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO})))))\ V2x)))\wedge((((p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V0f)\ V1l)\ V2x))\wedge(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V3g)\ V4m)\ V2x)))\Rightarrow(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ (\lambda V8x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ V0f\ V8x))\ (ap\ V3g\ V8x))))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V1l)\ V4m))\ V2x)))\wedge((((p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V0f)\ V1l)\ V2x))\wedge(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V3g)\ V4m)\ V2x)))\Rightarrow(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ (\lambda V9x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ V0f\ V9x))\ (ap\ V3g\ V9x))))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V1l)\ (ap\ V3g\ V2x)))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V4m)\ (ap\ V0f\ V2x))))\ V2x)))\wedge((((p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V0f)\ V1l)\ V2x))\wedge(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V3g)\ V4m)\ V2x)))\Rightarrow(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ (\lambda V10x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ (ap\ V0f\ V10x))\ (ap\ V3g\ V10x))))\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ V1l)\ V4m))\ V2x)))\wedge(((p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V0f)\ V1l)\ V2x))\Rightarrow(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ (\lambda V11x \in ty\_2Erealax\_2Ereal.(ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ (ap\ V0f\ V11x))))\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V1l))\ V2x)))\wedge(((p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V3g)\ V4m)\ V2x))\Rightarrow(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ (\lambda V12x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ c\_2Ereal\_2Epow\ (ap\ V3g\ V12x))\ V5n)))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ V5n))\ (ap\ (ap\ c\_2Ereal\_2Epow\ (ap\ V3g\ V2x))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V5n)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))))\ V4m))\ V2x)))\wedge(((p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V3g)\ V4m)\ V2x))\Rightarrow(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ (\lambda V13x \in ty\_2Erealax\_2Ereal.(ap\ \mathsf{c\_2Etransc\_2Eexp}\ (ap\ V3g\ V13x))))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ \mathsf{c\_2Etransc\_2Eexp}\ (ap\ V3g\ V2x)))\ V4m))\ V2x)))\wedge(((p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V3g)\ V4m)\ V2x))\Rightarrow(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ (\lambda V14x \in ty\_2Erealax\_2Ereal.(ap\ \mathsf{c\_2Etransc\_2Esin}\ (ap\ V3g\ V14x))))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ \mathsf{c\_2Etransc\_2Ecos}\ (ap\ V3g\ V2x)))\ V4m))\ V2x)))\wedge((p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V3g)\ V4m)\ V2x))\Rightarrow(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ (\lambda V15x \in ty\_2Erealax\_2Ereal.(ap\ \mathsf{c\_2Etransc\_2Ecos}\ (ap\ V3g\ V15x))))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ (ap\ \mathsf{c\_2Etransc\_2Esin}\ (ap\ V3g\ V2x))))\ V4m))\ V2x))))))))))))))))))$$