thm_2Etransc_2EDIFF__COMPOSITE__EXP (TMYDJ7yGcnPzUDAnyc7ZyYiF5mApGoJDLSg)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $c\_2Ereal\_2Epow : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})^{ty\_2Erealax\_2Ereal}) \tag{3}$$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{4}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{5}$$

Definition 5 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \tag{6}$$

Let $c\_2Earithmetic\_2EFACT : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EFACT \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}) \tag{7}$$

Definition 6 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{8}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{9}$$

Definition 7 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{10}$$

Definition 8 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_$

Definition 9 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $c\_2Earithmetic\_2EDIV : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{11}$$

Definition 10 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{12}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{13}$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{14}$$

Definition 11 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^{A}.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 12 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap$ (c_2Emin_2E_40 $(t$

Let $c\_2Erealax\_2Etreal\_\_neg : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \tag{15}$$

Let $c\_2Erealax\_2Etreal\_\_eq : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{16}$$

Let $c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Eh} \tag{17}$$

Definition 13 *We define* c_2Erealax_2Ereal__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ t$

Definition 14 *We define* c_2Erealax_2Ereal__neg *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal

Let $c\_2Erealax\_2Etreal\_\_inv : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_inv \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \tag{18}$$

Definition 15 *We define* c_2Erealax_2Einv *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal__ABS

Let $c\_2Erealax\_2Etreal\_\_mul : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Eh} \tag{19}$$

Definition 16 *We define* c_2Erealax_2Ereal__mul *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealа$

Definition 17 *We define* c_2Ereal_2E_2F *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.($

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \tag{20}$$

Definition 18 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 19 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 20 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{21}$$

Definition 21 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Ereal\_2Esum : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Esum \in ((ty\_2Erealax\_2Ereal^{(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2En} \tag{22}$$

Definition 22 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2E}$

Definition 23 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}$

Definition 24 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 25 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 26 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 27 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Erealax\_2Etreal\_\_add : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \tag{23}$$

Definition 28 *We define* c_2Erealax_2Ereal__add *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Definition 29 *We define* c_2Ereal_2Ereal__sub *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2E$

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehr} \tag{24}$$

Definition 30 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_$

Definition 31 *We define* c_2Ereal_2Ereal__lte *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Definition 32 *We define* c_2Ereal_2Eabs *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECONI}$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{25}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{26}$$

Definition 33 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27}$

Let $ty\_2Emetric\_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Emetric\_2Emetric\ A0) \tag{27}$$

Let $c\_2Emetric\_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Emetric\ A\_27a \in ((ty\_2Emetric\_2Emetric\ A\_27a)^{(ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})}) \tag{28}$$

Definition 34 *We define* c_2Emetric_2Emr1 *to be* $(ap\ (c\_2Emetric\_2Emetric\ ty\_2Erealax\_2Ereal)\ (ap\ (c$

Let $c\_2Emetric\_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Edist\ A\_27a \in ((ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a} \tag{29}$$

Let $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \tag{30}$$

Let $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^{A\_27a})})}) \tag{31}$$

Definition 35 *We define* c_2Emetric_2Emtop *to be* $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Emetric\_2Emetric\ A\_27a).(ap$

Let $c\_2Enets\_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Enets\_2Etends\ A\_27a\ A\_27b \in (((2^{(ty\_2Epair\_2Eprod\ (ty\_2Etopology\_2Etopology\ A\_27a)\ ((2^{A\_27b})^{A\_27b}))})^{A\_27a})^{(A\_27a^{A\_27b})}) \tag{32}$$

Definition 36 *We define* c_2Eseq_2E_2D_2D_3E *to be* $\lambda V0x \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1x$

Definition 37 *We define* c_2Eseq_2Esums *to be* $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1s \in ty\_$

Definition 38 *We define* c_2Eseq_2Esuminf *to be* $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(ap$ (c_2E

Definition 39 *We define* c_2Etransc_2Ecos *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap$ c_2Eseq_2Esuminf $(\lambda V1n$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{33}$$

Definition 40 *We define* c_2Etransc_2Esin *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap$ c_2Eseq_2Esuminf $(\lambda V1n$

Definition 41 *We define* c_2Etransc_2Eexp *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap$ c_2Eseq_2Esuminf $(\lambda V1n$

Let $c\_2Enets\_2Etendsto : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Enets\_2Etendsto\ A\_27a \in (((2^{A\_27a})^{A\_27a})^{(ty\_2Epair\_2Eprod\ (ty\_2Emetric\_2En} \tag{34}$$

Definition 42 *We define* c_2Elim_2Etends__real__real *to be* $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal})$.

Definition 43 *We define* c_2Elim_2Ediffl *to be* $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).\lambda V1l \in ty\_2$

Assume the following.

$$True \tag{35}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{36}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{37}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{38}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{39}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{40}$$

Assume the following.

$$
\begin{array}{c}
(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1l \in \\
ty\_2Erealax\_2Ereal.(\forall V2x \in ty\_2Erealax\_2Ereal.(\forall V3g \in \\
(ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V4m \in ty\_2Erealax\_2Ereal. \\
(\forall V5n \in ty\_2Enum\_2Enum.((((p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl} \\
V0f)\ V1l)\ V2x)) \wedge (\neg((ap\ V0f\ V2x) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num \\
\mathsf{c\_2Enum\_2E0}))))\Rightarrow(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ (\lambda V6x \in ty\_2Erealax\_2Ereal. \\
(ap\ \mathsf{c\_2Erealax\_2Einv}\ (ap\ V0f\ V6x))))\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg} \\
(ap\ (ap\ \mathsf{c\_2Ereal\_2E\_2F}\ V1l)\ (ap\ (ap\ c\_2Ereal\_2Epow\ (ap\ V0f\ V2x)) \\
(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO})))))) \\
V2x))) \wedge ((((p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V0f)\ V1l)\ V2x)) \wedge ((p\ (ap \\
(ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V3g)\ V4m)\ V2x)) \wedge (\neg((ap\ V3g\ V2x) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num \\
\mathsf{c\_2Enum\_2E0})))))\Rightarrow(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ (\lambda V7x \in ty\_2Erealax\_2Ereal. \\
(ap\ (ap\ \mathsf{c\_2Ereal\_2E\_2F}\ (ap\ V0f\ V7x))\ (ap\ V3g\ V7x))))\ (ap\ (ap\ \mathsf{c\_2Ereal\_2E\_2F} \\
(ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V1l) \\
(ap\ V3g\ V2x)))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V4m)\ (ap\ V0f\ V2x)))) \\
(ap\ (ap\ c\_2Ereal\_2Epow\ (ap\ V3g\ V2x))\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL} \\
(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO})))))\ V2x))) \wedge \\
((((p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V0f)\ V1l)\ V2x)) \wedge (p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl} \\
V3g)\ V4m)\ V2x)))\Rightarrow(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ (\lambda V8x \in ty\_2Erealax\_2Ereal. \\
(ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ V0f\ V8x))\ (ap\ V3g\ V8x))))\ (ap \\
(ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V1l)\ V4m))\ V2x))) \wedge ((((p\ (ap\ (ap\ (ap \\
\mathsf{c\_2Elim\_2Ediffl}\ V0f)\ V1l)\ V2x)) \wedge (p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl} \\
V3g)\ V4m)\ V2x)))\Rightarrow(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ (\lambda V9x \in ty\_2Erealax\_2Ereal. \\
(ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ V0f\ V9x))\ (ap\ V3g\ V9x))))\ (ap \\
(ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V1l) \\
(ap\ V3g\ V2x)))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V4m)\ (ap\ V0f\ V2x)))) \\
V2x))) \wedge ((((p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V0f)\ V1l)\ V2x)) \wedge (p\ (ap \\
(ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V3g)\ V4m)\ V2x)))\Rightarrow(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl} \\
(\lambda V10x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub} \\
(ap\ V0f\ V10x))\ (ap\ V3g\ V10x))))\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ V1l) \\
V4m))\ V2x))) \wedge (((p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V0f)\ V1l)\ V2x))\Rightarrow(\\
p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ (\lambda V11x \in ty\_2Erealax\_2Ereal.(\\
ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ (ap\ V0f\ V11x))))\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg} \\
V1l))\ V2x))) \wedge (((p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V3g)\ V4m)\ V2x))\Rightarrow(\\
p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ (\lambda V12x \in ty\_2Erealax\_2Ereal.(\\
ap\ (ap\ c\_2Ereal\_2Epow\ (ap\ V3g\ V12x))\ V5n)))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul} \\
(ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num \\
V5n))\ (ap\ (ap\ c\_2Ereal\_2Epow\ (ap\ V3g\ V2x))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D \\
V5n)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1} \\
\mathsf{c\_2Earithmetic\_2EZERO}))))))\ V4m))\ V2x))) \wedge (((p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl} \\
V3g)\ V4m)\ V2x))\Rightarrow(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ (\lambda V13x \in ty\_2Erealax\_2Ereal. \\
(ap\ \mathsf{c\_2Etransc\_2Eexp}\ (ap\ V3g\ V13x))))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul} \\
(ap\ \mathsf{c\_2Etransc\_2Eexp}\ (ap\ V3g\ V2x)))\ V4m))\ V2x))) \wedge (((p\ (ap\ (ap\ (ap \\
\mathsf{c\_2Elim\_2Ediffl}\ V3g)\ V4m)\ V2x))\Rightarrow(p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl} \\
(\lambda V14x \in ty\_2Erealax\_2Ereal.(ap\ \mathsf{c\_2Etransc\_2Esin}\ (ap\ V3g\ V14x)))) \\
(ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ \mathsf{c\_2Etransc\_2Ecos}\ (ap\ V3g\ V2x))) \\
V4m))\ V2x))) \wedge ((p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V3g)\ V4m)\ V2x))\Rightarrow(p \\
(ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ (\lambda V15x \in ty\_2Erealax\_2Ereal.(ap \\
\mathsf{c\_2Etransc\_2Ecos}\ (ap\ V3g\ V15x))))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul} \\
(ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ (ap\ \mathsf{c\_2Etransc\_2Esin}\ (ap\ V3g\ V2x)))) \\
V4m))\ V2x)))))))))))))))))
\end{array}
\tag{41}
$$

Theorem 1

$$(\forall V0g \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1m \in ty\_2Erealax\_2Ereal.(\forall V2x \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ V0g)\ V1m)\ V2x)) \Rightarrow (p\ (ap\ (ap\ (ap\ \mathsf{c\_2Elim\_2Ediffl}\ (\lambda V3x \in ty\_2Erealax\_2Ereal.(ap\ \mathsf{c\_2Etransc\_2Eexp}\ (ap\ V0g\ V3x))))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ \mathsf{c\_2Etransc\_2Eexp}\ (ap\ V0g\ V2x)))\ V1m))\ V2x))))))$$