

thm_2Etransc_2EDIFF__TAN (TMUXoz5nrEQnL4W8KY3dtEDUvag2QPcsjBG)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Ebool_2ET to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \tag{3}$$

Definition 3 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{4}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{5}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{6}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \tag{7}$$

Definition 4 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \text{ of type } \iota \Rightarrow \iota).$

Definition 5 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a} \text{ of type } \iota \Rightarrow \iota).$

Definition 6 We define `c_2Erealax_2Ereal__REP` to be $\lambda V0a \in \text{ty_2Erealax_2Ereal}. (\text{ap } (\text{c_2Emin_2E_40 } (2^{A-27a} \text{ of type } \iota \Rightarrow \iota).$

Let `c_2Erealax_2Etreall__neg` : ι be given. Assume the following.

$$c_2Erealax_2Etreall_neg \in ((\text{ty_2Epair_2Eprod } \text{ty_2Ehreal_2Ehreal} \text{ ty_2Ehreal_2Ehreal})^{(\text{ty_2Epair_2Eprod } \text{ty_2Ehreal_2Ehreal } \text{ty_2Ehreal_2Ehreal})}) \quad (8)$$

Let `c_2Erealax_2Etreall__eq` : ι be given. Assume the following.

$$c_2Erealax_2Etreall_eq \in ((2^{(\text{ty_2Epair_2Eprod } \text{ty_2Ehreal_2Ehreal } \text{ty_2Ehreal_2Ehreal})})^{(\text{ty_2Epair_2Eprod } \text{ty_2Ehreal_2Ehreal } \text{ty_2Ehreal_2Ehreal})}) \quad (9)$$

Let `c_2Erealax_2Ereal__ABS__CLASS` : ι be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (\text{ty_2Erealax_2Ereal})^{(2^{(\text{ty_2Epair_2Eprod } \text{ty_2Ehreal_2Ehreal } \text{ty_2Ehreal_2Ehreal})})} \quad (10)$$

Definition 7 We define `c_2Erealax_2Ereal__ABS` to be $\lambda V0r \in (\text{ty_2Epair_2Eprod } \text{ty_2Ehreal_2Ehreal } \text{ty_2Ehreal_2Ehreal})$

Definition 8 We define `c_2Erealax_2Ereal__neg` to be $\lambda V0T1 \in \text{ty_2Erealax_2Ereal}. (\text{ap } \text{c_2Erealax_2Ereal_neg} \text{ of type } \iota \Rightarrow \iota).$

Let `c_2Erealax_2Etreall__add` : ι be given. Assume the following.

$$c_2Erealax_2Etreall_add \in (((\text{ty_2Epair_2Eprod } \text{ty_2Ehreal_2Ehreal} \text{ ty_2Ehreal_2Ehreal})^{(\text{ty_2Epair_2Eprod } \text{ty_2Ehreal_2Ehreal } \text{ty_2Ehreal_2Ehreal})})^{(\text{ty_2Epair_2Eprod } \text{ty_2Ehreal_2Ehreal } \text{ty_2Ehreal_2Ehreal})}) \quad (11)$$

Definition 9 We define `c_2Erealax_2Ereal__add` to be $\lambda V0T1 \in \text{ty_2Erealax_2Ereal}. \lambda V1T2 \in \text{ty_2Erealax_2Ereal}. (\text{ap } \text{c_2Erealax_2Ereal_add} \text{ of type } \iota \Rightarrow \iota).$

Definition 10 We define `c_2Ereal_2Ereal__sub` to be $\lambda V0x \in \text{ty_2Erealax_2Ereal}. \lambda V1y \in \text{ty_2Erealax_2Ereal}. (\text{ap } \text{c_2Ereal_2Ereal_sub} \text{ of type } \iota \Rightarrow \iota).$

Let `c_2Erealax_2Etreall__inv` : ι be given. Assume the following.

$$c_2Erealax_2Etreall_inv \in ((\text{ty_2Epair_2Eprod } \text{ty_2Ehreal_2Ehreal} \text{ ty_2Ehreal_2Ehreal})^{(\text{ty_2Epair_2Eprod } \text{ty_2Ehreal_2Ehreal } \text{ty_2Ehreal_2Ehreal})}) \quad (12)$$

Definition 11 We define `c_2Erealax_2Einv` to be $\lambda V0T1 \in \text{ty_2Erealax_2Ereal}. (\text{ap } \text{c_2Erealax_2Ereal_inv} \text{ of type } \iota \Rightarrow \iota).$

Let `c_2Erealax_2Etreall__mul` : ι be given. Assume the following.

$$c_2Erealax_2Etreall_mul \in (((\text{ty_2Epair_2Eprod } \text{ty_2Ehreal_2Ehreal} \text{ ty_2Ehreal_2Ehreal})^{(\text{ty_2Epair_2Eprod } \text{ty_2Ehreal_2Ehreal } \text{ty_2Ehreal_2Ehreal})})^{(\text{ty_2Epair_2Eprod } \text{ty_2Ehreal_2Ehreal } \text{ty_2Ehreal_2Ehreal})}) \quad (13)$$

Definition 12 We define `c_2Erealax_2Ereal__mul` to be $\lambda V0T1 \in \text{ty_2Erealax_2Ereal}. \lambda V1T2 \in \text{ty_2Erealax_2Ereal}. (\text{ap } \text{c_2Erealax_2Ereal_mul} \text{ of type } \iota \Rightarrow \iota).$

Definition 13 We define c_Ebool_2EF to be $(ap (c_Ebool_2E_21\ 2) (\lambda V0t \in 2.V0t))$.

Definition 14 We define $c_Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 15 We define $c_Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21\ 2) (\lambda V2t \in 2.V2t))))$

Definition 16 We define $c_Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_2E_21\ 2) (\lambda V2t \in 2.V2t))))$

Definition 17 We define $c_Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap\ c_Emin_2E_3D_3D_3E\ V0t)\ c_Ebool_2E_21\ 2))$

Let $c_Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (14)$$

Definition 18 We define $c_Ereal_2E_2F$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.(c_Ereal_2Ereal_of_num\ x\ y)$

Let $c_2Erealax_2Etreallt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreallt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (15)$$

Definition 19 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.(c_2Erealax_2Etreallt\ T1\ T2)$

Definition 20 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.(c_2Ereal_2Ereal_lt\ x\ y)$

Definition 21 We define c_Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(c_Ebool_2E_21\ 2\ t1\ t2))))$

Definition 22 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap (ap (ap (c_Ebool_2ECOND\ x)\ y)\ z))$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND \\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (16)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (17)$$

Definition 23 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a\ A_27b}))$

Let $ty_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Emetric_2Emetric\ A0) \quad (18)$$

Let $c_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Emetric\ A_27a \in ((ty_2Emetric_2Emetric \\ A_27a)^{(ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})}) \end{aligned} \quad (19)$$

Let $c_2Earithmic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (28)$$

Definition 31 We define $c_2Earithmic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap (ap c_2Earithmic_2EBIT1))$

Definition 32 We define $c_2Earithmic_2EZERO$ to be c_2Enum_2E0 .

Definition 33 We define $c_2Earithmic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum. (ap (ap c_2Earithmic_2EBIT2))$

Definition 34 We define $c_2Earithmic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Let $c_2Ereal_2Epow : \iota$ be given. Assume the following.

$$c_2Ereal_2Epow \in ((ty_2Erealax_2Ereal^{ty_2Enum_2Enum})^{ty_2Erealax_2Ereal}) \quad (29)$$

Let $c_2Earithmic_2EFACT : \iota$ be given. Assume the following.

$$c_2Earithmic_2EFACT \in (ty_2Enum_2Enum^{ty_2Enum_2Enum}) \quad (30)$$

Let $c_2Earithmic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (31)$$

Let $c_2Earithmic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (32)$$

Let $c_2Ereal_2Esum : \iota$ be given. Assume the following.

$$c_2Ereal_2Esum \in ((ty_2Erealax_2Ereal^{(ty_2Erealax_2Ereal^{ty_2Enum_2Enum})})^{(ty_2Epair_2Eprod ty_2Enum_2Enum)}) \quad (33)$$

Definition 35 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum. (c_2Eprim_rec_2E_3C m n)$

Definition 36 We define $c_2Earithmic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum. (c_2Earithmic_2E_3E m n)$

Definition 37 We define $c_2Earithmic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum. (c_2Earithmic_2E_3E_3D m n)$

Definition 38 We define $c_2Eseq_2E_2D_2D_3E$ to be $\lambda V0x \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}). \lambda V1x \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}). (c_2Eseq_2E_2D_2D_3E x y)$

Definition 39 We define c_2Eseq_2Esums to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}). \lambda V1s \in ty_2Enum_2Enum. (c_2Eseq_2Esums f s)$

Definition 40 We define $c_2Eseq_2Esuminf$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}). (ap (c_2Eseq_2Esuminf f))$

Definition 41 We define $c_2Etransc_2Ecos$ to be $\lambda V0x \in ty_2Erealax_2Ereal. (ap c_2Eseq_2Esuminf (\lambda V1n. c_2Etransc_2Ecos x n))$

Let $c_2Earithmic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (34)$$

Definition 42 We define $c_2Etransc_2Esin$ to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ c_2Eseq_2Esuminf\ (\lambda V1m$

Definition 43 We define $c_2Etransc_2Etan$ to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ (ap\ c_2Ereal_2E2F\ (ap\ c$

Assume the following.

$$True \quad (35)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (36)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (37)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \wedge ((p\ V1t2) \wedge (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \wedge (p\ V2t3)))))) \quad (38)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (39)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (40)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (41)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (42)$$

Assume the following.

$$(\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1g \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V2l \in ty_2Erealax_2Ereal.(\forall V3m \in ty_2Erealax_2Ereal.(\forall V4x \in ty_2Erealax_2Ereal.(((p\ (ap\ (ap\ (ap\ c_2Elim_2Ediff\ V0f)\ V2l)\ (ap\ V1g\ V4x))) \wedge (p\ (ap\ (ap\ (ap\ c_2Elim_2Ediff\ V1g)\ V3m)\ V4x))) \Rightarrow (p\ (ap\ (ap\ (ap\ c_2Elim_2Ediff\ (\lambda V5x \in ty_2Erealax_2Ereal.(ap\ V0f\ (ap\ V1g\ V5x))))\ (ap\ (ap\ c_2Erealax_2Ereal_mul\ V2l)\ V3m))\ V4x)))))))))) \quad (43)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.(p (ap (ap (ap c_2Elim_2Ediff1 \\
& (\lambda V1x \in ty_2Erealax_2Ereal.V1x)) (ap c_2Ereal_2Ereal_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \\
& V0x)))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1g \in \\
& (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V2l \in ty_2Erealax_2Ereal. \\
& (\forall V3m \in ty_2Erealax_2Ereal.(\forall V4x \in ty_2Erealax_2Ereal. \\
& (((p (ap (ap (ap c_2Elim_2Ediff1 V0f) V2l) V4x)) \wedge ((p (ap (ap (ap c_2Elim_2Ediff1 \\
& V1g) V3m) V4x)) \wedge (\neg((ap V1g V4x) = (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)))))) \Rightarrow (p (ap (ap (ap c_2Elim_2Ediff1 (\lambda V5x \in ty_2Erealax_2Ereal. \\
& (ap (ap c_2Ereal_2E_2F (ap V0f V5x)) (ap V1g V5x)))) (ap (ap c_2Ereal_2E_2F \\
& (ap (ap c_2Ereal_2Ereal_sub (ap (ap c_2Erealax_2Ereal_mul V2l) \\
& (ap V1g V4x))) (ap (ap c_2Erealax_2Ereal_mul V3m) (ap V0f V4x)))) \\
& (ap (ap c_2Ereal_2Epow (ap V1g V4x)) (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO)))))) V4x))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap (ap c_2Erealax_2Ereal_add V0x) V1y) = (ap (ap c_2Erealax_2Ereal_add \\
& V1y) V0x))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_mul \\
& V0x) (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) = V0x))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap c_2Erealax_2Ereal_neg (ap (ap c_2Erealax_2Ereal_mul V0x) \\
& V1y)) = (ap (ap c_2Erealax_2Ereal_mul (ap c_2Erealax_2Ereal_neg \\
& V0x)) V1y))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.((ap c_2Erealax_2Ereal_neg \\
& (ap c_2Erealax_2Ereal_neg V0x)) = V0x))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.((ap c_2Erealax_2Einv V0x) = \\
& (ap (ap c_2Ereal_2E_2F (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) V0x)))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Ereal_2Epow V0x) \\ & (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) = \\ & (ap (ap c_2Erealax_2Ereal_mul V0x) V0x))) \end{aligned} \quad (51)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (52)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (53)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\ & (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \end{aligned} \quad (54)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\ & ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \end{aligned} \quad (55)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow ((p V0A) \Rightarrow False) \Rightarrow False))) \quad (56)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(\\ & p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\ & ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (57)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\ & (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (58)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (\\ & (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\\ & \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \end{aligned} \quad (59)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\ & (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \end{aligned} \quad (60)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p)))) \quad (61)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (\neg(p \ V1q))))) \quad (62)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(p \ (ap \ (ap \ (ap \ c_2Elim_2Ediff \ c_2Etransc_2Esin) \ (ap \ c_2Etransc_2Ecos \ V0x)) \ V0x))) \quad (63)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(p \ (ap \ (ap \ (ap \ c_2Elim_2Ediff \ c_2Etransc_2Ecos) \ (ap \ c_2Erealax_2Ereal_neg \ (ap \ c_2Etransc_2Esin \ V0x))) \ V0x))) \quad (64)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap \ (ap \ c_2Erealax_2Ereal_add \ (ap \ (ap \ c_2Ereal_2Epow \ (ap \ c_2Etransc_2Esin \ V0x)) \ (ap \ c_2Earithmetic_2ENUMERAL \ (ap \ c_2Earithmetic_2EBIT2 \ c_2Earithmetic_2EZERO)))) \ (ap \ (ap \ c_2Ereal_2Epow \ (ap \ c_2Etransc_2Ecos \ V0x)) \ (ap \ c_2Earithmetic_2ENUMERAL \ (ap \ c_2Earithmetic_2EBIT2 \ c_2Earithmetic_2EZERO)))))) = (ap \ c_2Ereal_2Ereal_of_num \ (ap \ c_2Earithmetic_2ENUMERAL \ (ap \ c_2Earithmetic_2EBIT1 \ c_2Earithmetic_2EZERO)))))) \quad (65)$$

Theorem 1

$$(\forall V0x \in ty_2Erealax_2Ereal.((\neg((ap \ c_2Etransc_2Ecos \ V0x) = (ap \ c_2Ereal_2Ereal_of_num \ c_2Enum_2E0))) \Rightarrow (p \ (ap \ (ap \ (ap \ c_2Elim_2Ediff \ c_2Etransc_2Etan) \ (ap \ c_2Erealax_2Einv \ (ap \ (ap \ c_2Ereal_2Epow \ (ap \ c_2Etransc_2Ecos \ V0x)) \ (ap \ c_2Earithmetic_2ENUMERAL \ (ap \ c_2Earithmetic_2EBIT2 \ c_2Earithmetic_2EZERO)))))) \ V0x))))$$