thm_2Etransc_2EDIVISION__APPEND (TMaRHcD-nDDW4QtGZLcV7dnHc4U4rYiqAgyd)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40 A

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{2}$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{3}$$

Definition 4 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D $(2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ ($c_2Emin_2E_3D $(2^{A\_27}$

Definition 6 *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V0t \in 2.V0t))$.

Definition 7 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 8 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in 2$

Definition 9 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(a$

Definition 10 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 11 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{4}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{5}$$

Let $c\_2Etransc\_2Etdiv : \iota$ be given. Assume the following.

$$c\_2Etransc\_2Etdiv \in ((2^{(ty\_2Epair\_2Eprod\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum} \tag{6}$$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{7}$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{8}$$

Definition 12 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap$ (c_2Emin_2E_40 $(t$

Let $c\_2Erealax\_2Etreal\_\_neg : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \tag{9}$$

Let $c\_2Erealax\_2Etreal\_\_eq : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \tag{10}$$

Let $c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehr} \tag{11}$$

Definition 13 *We define* c_2Erealax_2Ereal__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty$

Definition 14 *We define* c_2Erealax_2Ereal__neg *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal

Let $c\_2Erealax\_2Etreal\_\_add : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \tag{12}$$

Definition 15 *We define* c_2Erealax_2Ereal__add *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Definition 16 *We define* c_2Ereal_2Ereal__sub *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2E$

Let $c\_2Etransc\_2Efine : \iota$ be given. Assume the following.

$$c\_2Etransc\_2Efine \in ((2^{(ty\_2Epair\_2Eprod\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enu} \tag{13}$$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{14}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{15}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{16}$$

Definition 17 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Definition 18 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 19 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 20 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 21 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{17}$$

Definition 22 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehr} \tag{18}$$

Definition 23 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_$

Definition 24 *We define* c_2Ereal_2Ereal__lte *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Definition 25 *We define* c_2Etransc_2Edsize *to be* $\lambda V0D \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(ap$ (c_2

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{19}$$

Definition 26 *We define* $\mathsf{c\_2Epair\_2E\_2C}$ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Etransc\_2Edivision : \iota$ be given. Assume the following.

$$c\_2Etransc\_2Edivision \in ((2^{(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})})^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Ereal} \tag{20}$$

Assume the following.

$$((\forall V0m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ \mathsf{c\_2Enum\_2E0})\ V0m) = \mathsf{c\_2Enum\_2E0}))\wedge(\forall V1m \in ty\_2Enum\_2Enum.(\forall V2n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V1m))\ V2n) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ ty\_2Enum\_2Enum)\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V1m)\ V2n))\ \mathsf{c\_2Enum\_2E0})\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V1m)\ V2n))))))) \tag{21}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V0m)))) \tag{22}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V2p)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))\ V2p))))) \tag{23}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ V1n))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V1n)\ V2p)))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ V2p)))))) \tag{24}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ V1n))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ V1n))))) \tag{25}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ V1n))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))\ V1n))))) \tag{26}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ \mathsf{c\_2Enum\_2E0})\ V0n))) \tag{27}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((\mathsf{c\_2Enum\_2E0} = V0m) \lor (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})\ V0m)))) \tag{28}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n))))) \tag{29}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m)))) \tag{30}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((\neg(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V0m))))) \tag{31}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((\neg(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V1n)\ V0m))))) \tag{32}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ \mathsf{c\_2Enum\_2E0})\ V0m) = \mathsf{c\_2Enum\_2E0}) \land ((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0m)\ \mathsf{c\_2Enum\_2E0}) = V0m))) \tag{33}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V2p))\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1n)\ V2p))) \Leftrightarrow (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ V1n)))))) \tag{34}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n)) \land (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2p))) \Rightarrow (p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V2p)))))) \tag{35}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V1n)\ V2p)))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ V2p)))))) \quad (36)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V0m))) \quad (37)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ V1n))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n))))) \quad (38)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0n)\ V1m))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1m)\ V0n)))\Rightarrow(V0n = V1m)))) \quad (39)$$

Assume the following.

$$(\forall V0a \in ty\_2Enum\_2Enum.(\forall V1c \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0a)\ V1c))\ V1c) = V0a))) \quad (40)$$

Assume the following.

$$(\forall V0c \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0c)\ V0c) = \mathsf{c\_2Enum\_2E0})) \quad (41)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ V1m))\Rightarrow(\forall V2p \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1m)\ V2p)))))))) \quad (42)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E\_3D}\ V0n)\ V1m))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1m)\ V0n))))) \quad (43)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n))\Leftrightarrow(\exists V2p \in ty\_2Enum\_2Enum.(V1n = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V2p)))))) \quad (44)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V1n)\ V2p)))\Leftrightarrow((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V2p))\ V1n))\lor(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ \mathsf{c\_2Enum\_2E0}))))))) \quad (45)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E\_3D}\ V0m)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V1n)\ V2p)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E\_3D}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V2p))\ V1n)))))) \quad (46)$$

Assume the following.

$$True \quad (47)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1)\Rightarrow(p\ V1t2))\Rightarrow(((p\ V1t2)\Rightarrow(p\ V0t1))\Rightarrow((p\ V0t1)\Leftrightarrow(p\ V1t2)))))) \quad (48)$$

Assume the following.

$$(\forall V0t \in 2.(False\Rightarrow(p\ V0t))) \quad (49)$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t)\lor(\neg(p\ V0t)))) \quad (50)$$

Assume the following.

$$(\forall V0t \in 2.(((True\land(p\ V0t))\Leftrightarrow(p\ V0t))\land((((p\ V0t)\land True)\Leftrightarrow(p\ V0t))\land(((False\land(p\ V0t))\Leftrightarrow False)\land((((p\ V0t)\land False)\Leftrightarrow False)\land(((p\ V0t)\land(p\ V0t))\Leftrightarrow(p\ V0t))))))) \quad (51)$$

Assume the following.

$$(\forall V0t \in 2.(((True\lor(p\ V0t))\Leftrightarrow True)\land((((p\ V0t)\lor True)\Leftrightarrow True)\land(((False\lor(p\ V0t))\Leftrightarrow(p\ V0t))\land((((p\ V0t)\lor False)\Leftrightarrow(p\ V0t))\land(((p\ V0t)\lor(p\ V0t))\Leftrightarrow(p\ V0t))))))) \quad (52)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{53}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \tag{54}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{55}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{56}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{57}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27a.(((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1)\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF})\ V0t1)\ V1t2) = V1t2)))) \tag{58}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).(\forall V1x \in A\_27a.((\forall V2y \in A\_27a.((p\ (ap\ V0P\ V2y)) \Leftrightarrow (V2y = V1x))) \Rightarrow ((ap\ (\mathsf{c\_2Emin\_2E\_40}\ A\_27a)\ V0P) = V1x)))) \tag{59}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg ((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg (p\ V0A)) \vee (\neg (p\ V1B)))) \wedge ((\neg ((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg (p\ V0A)) \wedge (\neg (p\ V1B))))))) \tag{60}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\neg (p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ \mathsf{c\_2Enum\_2E0})))) \tag{61}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0n)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n)))) \tag{62}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\neg(p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V0x)\ V0x)))) \tag{63}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.(\forall V2z \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V0x)\ V1y))\land(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V1y)\ V2z)))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V0x)\ V2z)))))) \tag{64}$$

Assume the following.

$$(\forall V0a \in ty\_2Erealax\_2Ereal.(\forall V1b \in ty\_2Erealax\_2Ereal.(\forall V2D \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V3p \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).((p\ (ap\ (ap\ c\_2Etransc\_2Etdiv\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V0a)\ V1b))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}))\ V2D)\ V3p)))\Leftrightarrow((p\ (ap\ (ap\ c\_2Etransc\_2Edivision\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V0a)\ V1b))\ V2D))\land(\forall V4n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ V2D\ V4n))\ (ap\ V3p\ V4n)))\land(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ V3p\ V4n))\ (ap\ V2D\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V4n)))))))))))) \tag{65}$$

Assume the following.

$$(\forall V0g \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1D \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V2p \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).((p\ (ap\ (ap\ c\_2Etransc\_2Efine\ V0g)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}))\ V1D)\ V2p)))\Leftrightarrow(\forall V3n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V3n)\ (ap\ \mathsf{c\_2Etransc\_2Edsize}\ V1D)))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ (ap\ V1D\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V3n)))\ (ap\ V1D\ V3n)))\ (ap\ V0g\ (ap\ V2p\ V3n)))))))))) \tag{66}$$

Assume the following.

$$(\forall V0D \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V1a \in ty\_2Erealax\_2Ereal.(\forall V2b \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ c\_2Etransc\_2Edivision\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V1a)\ V2b))\ V0D))\Rightarrow((ap\ V0D\ \mathsf{c\_2Enum\_2E0}) = V1a))))) \tag{67}$$

Assume the following.

$$(\forall V0D \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V1a \in ty\_2Erealax\_2Ereal.(\forall V2b \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ c\_2Etransc\_2Edivision\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V1a)\ V2b))\ V0D))\Leftrightarrow(((ap\ V0D\ \mathsf{c\_2Enum\_2E0}) = V1a)\wedge((\forall V3n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V3n)\ (ap\ \mathsf{c\_2Etransc\_2Edsize}\ V0D)))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ (ap\ V0D\ V3n))\ (ap\ V0D\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V3n))))))\wedge(\forall V4n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E\_3D}\ V4n)\ (ap\ \mathsf{c\_2Etransc\_2Edsize}\ V0D)))\Rightarrow((ap\ V0D\ V4n) = V2b)))))))) \quad (68)$$

Assume the following.

$$(\forall V0D \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V1a \in ty\_2Erealax\_2Ereal.(\forall V2b \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ c\_2Etransc\_2Edivision\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V1a)\ V2b))\ V0D))\Rightarrow((ap\ V0D\ (ap\ \mathsf{c\_2Etransc\_2Edsize}\ V0D)) = V2b))))) \quad (69)$$

Assume the following.

$$(\forall V0D \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V1a \in ty\_2Erealax\_2Ereal.(\forall V2b \in ty\_2Erealax\_2Ereal.(\forall V3m \in ty\_2Enum\_2Enum.(\forall V4n \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ c\_2Etransc\_2Edivision\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V1a)\ V2b))\ V0D))\wedge((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V3m)\ V4n))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V4n)\ (ap\ \mathsf{c\_2Etransc\_2Edsize}\ V0D)))))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ (ap\ V0D\ V3m))\ (ap\ V0D\ V4n))))))))) \quad (70)$$

Assume the following.

$$(\forall V0D \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V1a \in ty\_2Erealax\_2Ereal.(\forall V2b \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ c\_2Etransc\_2Edivision\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V1a)\ V2b))\ V0D))\Rightarrow((V1a = V2b)\Leftrightarrow((ap\ \mathsf{c\_2Etransc\_2Edsize}\ V0D) = \mathsf{c\_2Enum\_2E0})))))) \quad (71)$$

Assume the following.

$$(\forall V0D \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V1a \in ty\_2Erealax\_2Ereal.(\forall V2b \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ c\_2Etransc\_2Edivision\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V1a)\ V2b))\ V0D))\Rightarrow(\forall V3r \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V1a)\ (ap\ V0D\ V3r)))))))) \quad (72)$$

Assume the following.

$$(\forall V0D \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V1a \in ty\_2Erealax\_2Ereal.(\forall V2b \in ty\_2Erealax\_2Ereal.(\forall V3n \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ c\_2Etransc\_2Edivision\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V1a)\ V2b))\ V0D))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V3n)\ (ap\ \mathsf{c\_2Etransc\_2Edsize}\ V0D))))\Rightarrow (p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ (ap\ V0D\ V3n))\ V2b))))))) \tag{73}$$

Theorem 1

$$(\forall V0g \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1a \in ty\_2Erealax\_2Ereal.(\forall V2b \in ty\_2Erealax\_2Ereal.(\forall V3c \in ty\_2Erealax\_2Ereal.(((\exists V4D1 \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\exists V5p1 \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).((p\ (ap\ (ap\ c\_2Etransc\_2Etdiv\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V1a)\ V2b))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}))\ V4D1)\ V5p1)))\wedge(p\ (ap\ (ap\ c\_2Etransc\_2Efine\ V0g)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}))\ V4D1)\ V5p1))))))\wedge(\exists V6D2 \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\exists V7p2 \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).((p\ (ap\ (ap\ c\_2Etransc\_2Etdiv\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V2b)\ V3c))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}))\ V6D2)\ V7p2)))\wedge(p\ (ap\ (ap\ c\_2Etransc\_2Efine\ V0g)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}))\ V6D2)\ V7p2)))))))\Rightarrow(\exists V8D \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\exists V9p \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).((p\ (ap\ (ap\ c\_2Etransc\_2Etdiv\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V1a)\ V3c))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}))\ V8D)\ V9p)))\wedge(p\ (ap\ (ap\ c\_2Etransc\_2Efine\ V0g)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}))\ V8D)\ V9p))))))))))$$