

thm_2Etransc_2EDIVISION_EQ (TMVE- hvp8aPV3n7wVLz2wHv4TXc7CnejRDYq)

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Definition 1 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \mathbf{if} (\exists x \in A. p (ap P x)) \mathbf{then} (the (\lambda x. x \in A \wedge p x))$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Ebool_2E_3F` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap V0P (ap (c_2Emin_2E_40 A))))$

Definition 4 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 5 We define `c_2Ebool_2E_T` to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

Definition 6 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (ap (ap (c_2Emin_2E_3D (2^{A-27a}))))$

Definition 7 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. V2t))))$

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Definition 8 We define `c_2Ebool_2E_F` to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2. V0t))$.

Definition 9 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2. (ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_F))$

Definition 10 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2. V2t))))$

Let `c_2Enum_2EREP_num` : ι be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{2}$$

Let `c_2Enum_2ESUC_REP` : ι be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{3}$$

Let `c_2Enum_2EABS_num` : ι be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 11 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Definition 12 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 13 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{5}$$

Definition 14 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 15 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 16 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ (c_2Ebool_2E$

Definition 17 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 18 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{6}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{7}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{8}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})_{ty_2Erealax_2Ereal}) \tag{9}$$

Definition 19 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E_40\ ($

Let $c_2Erealax_2Etreall_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})_{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \tag{10}$$

Definition 20 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 21 We define $c_2Etransc_2Edsize$ to be $\lambda V0D \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(ap\ (c_2$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (11)$$

Definition 22 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2E$

Let $c_2Etransc_2Edivision : \iota$ be given. Assume the following.

$$c_2Etransc_2Edivision \in ((2^{(ty_2Erealax_2Ereal^{ty_2Enum_2Enum})})^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Ereal)}) \quad (12)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. ((V0m = c_2Enum_2E0) \vee (\exists V1n \in ty_2Enum_2Enum. (V0m = (ap\ c_2Enum_2ESUC\ V1n)))))) \quad (13)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V0m)\ V0m))) \quad (14)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (p\ (ap\ (ap\ c_2Earithmetic_2E_3E_3D\ V0n)\ V1m)) \Leftrightarrow (p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V1m)\ V0n)))))) \quad (15)$$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (20)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\
& p V0t))))))
\end{aligned} \tag{21}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(\neg((ap\ c_2Enum_2ESUC\ V0n) = c_2Enum_2E0))) \tag{22}$$

Assume the following.

$$\begin{aligned}
& (((ap\ c_2Eprim_rec_2EPRE\ c_2Enum_2E0) = c_2Enum_2E0) \wedge (\forall V0m \in \\
& ty_2Enum_2Enum.((ap\ c_2Eprim_rec_2EPRE\ (ap\ c_2Enum_2ESUC\ V0m)) = \\
& V0m)))
\end{aligned} \tag{23}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0n) \\ (ap\ c_2Enum_2ESUC\ V0n)))) \tag{24}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealx_2Ereal.(\forall V1y \in ty_2Erealx_2Ereal. \\
& ((p\ (ap\ (ap\ c_2Erealx_2Ereal_lt\ V0x)\ V1y)) \Rightarrow (\neg(V0x = V1y))))
\end{aligned} \tag{25}$$

Assume the following.

$$\begin{aligned}
& (\forall V0D \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}).(\forall V1a \in \\
& ty_2Erealx_2Ereal.(\forall V2b \in ty_2Erealx_2Ereal.((p\ (ap \\
& (ap\ c_2Etransc_2Edivision\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealx_2Ereal \\
& ty_2Erealx_2Ereal)\ V1a)\ V2b))\ V0D)) \Leftrightarrow (((ap\ V0D\ c_2Enum_2E0) = \\
& V1a) \wedge ((\forall V3n \in ty_2Enum_2Enum.((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C \\
& V3n)\ (ap\ c_2Etransc_2Edsize\ V0D)) \Rightarrow (p\ (ap\ (ap\ c_2Erealx_2Ereal_lt \\
& (ap\ V0D\ V3n))\ (ap\ V0D\ (ap\ c_2Enum_2ESUC\ V3n)))))) \wedge (\forall V4n \in \\
& ty_2Enum_2Enum.((p\ (ap\ (ap\ c_2Earithmetic_2E_3E_3D\ V4n)\ (ap\ c_2Etransc_2Edsize \\
& V0D)) \Rightarrow ((ap\ V0D\ V4n) = V2b))))))))))
\end{aligned} \tag{26}$$

Assume the following.

$$\begin{aligned}
& (\forall V0D \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}).(\forall V1a \in \\
& ty_2Erealx_2Ereal.(\forall V2b \in ty_2Erealx_2Ereal.((p\ (ap \\
& (ap\ c_2Etransc_2Edivision\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Erealx_2Ereal \\
& ty_2Erealx_2Ereal)\ V1a)\ V2b))\ V0D)) \Rightarrow (\forall V3n \in ty_2Enum_2Enum. \\
& ((p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V3n)\ (ap\ c_2Etransc_2Edsize\ V0D)) \Rightarrow \\
& (p\ (ap\ (ap\ c_2Erealx_2Ereal_lt\ (ap\ V0D\ c_2Enum_2E0))\ (ap\ V0D\ (\\
& ap\ c_2Enum_2ESUC\ V3n))))))))))
\end{aligned} \tag{27}$$

Theorem 1

$$\begin{aligned} & (\forall V0D \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(\forall V1a \in \\ & ty_2Erealax_2Ereal.(\forall V2b \in ty_2Erealax_2Ereal.((p (ap \\ & (ap c_2Etrasc_2Edivision (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal \\ & ty_2Erealax_2Ereal) V1a) V2b)) V0D)) \Rightarrow ((V1a = V2b) \Leftrightarrow ((ap c_2Etrasc_2Edsize \\ & V0D) = c_2Enum_2E0)))))) \end{aligned}$$