

thm\_2Etransc\_2EDIVISION\_\_LBOUND\_\_LT  
(TMUoWC14SamWjJk9WVegEfch98Cg2UcbRQL)

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**Definition 1** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow p \Rightarrow Q)$  of type  $\iota$ .

**Definition 2** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 3** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 5** We define  $c\_2Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

**Definition 6** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 7** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F$

**Definition 8** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{2}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{3}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{4}$$

**Definition 9** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num ($

**Definition 10** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.$ if  $(\exists x \in A.p (ap P x))$  then (the  $(\lambda x.x \in A \wedge p x)$  of type  $\iota \Rightarrow \iota$ ).

**Definition 11** We define  $c\_2Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_2Emin\_2E\_40$

**Definition 12** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 13** We define  $c\_2Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 14** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 15** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{5}$$

**Definition 16** We define  $c\_2Enum\_2E0$  to be  $(ap c\_2Enum\_2EABS\_num c\_2Enum\_2EZERO\_REP)$ .

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{6}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{7}$$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{8}$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal}) \tag{9}$$

**Definition 17** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_2Emin\_2E\_40 ($

Let  $c\_2Erealax\_2Etreallt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreallt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \tag{10}$$

**Definition 18** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 19** We define  $c\_2Etransc\_2Edsize$  to be  $\lambda V0D \in (ty\_2Erealax\_2Ereal)^{ty\_2Enum\_2Enum}.$ (ap (c\\_2

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \tag{11}$$

**Definition 20** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0x \in A.27a.\lambda V1y \in A.27b.(ap (c\_2Etransc\_2Edivision : \iota$  be given. Assume the following.

$$c\_2Etransc\_2Edivision \in ((2^{(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})})^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Ereal)})) \quad (12)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(p (ap (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC V0m)) V1n)) \Rightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n)))))) \quad (13)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n)) \vee (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1n) V0m)))))) \quad (14)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\neg(p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C V1n) V0m)))))) \quad (15)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n)) \Rightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0m) V1n)))))) \quad (16)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(p (ap (ap c\_2Earithmetic\_2E\_3E\_3D V0n) V1m)) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1m) V0n)))))) \quad (17)$$

Assume the following.

$$((\forall V0n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V0n) c\_2Enum\_2E0)) \Leftrightarrow (V0n = c\_2Enum\_2E0))) \wedge (\forall V1m \in ty\_2Enum\_2Enum.(\forall V2n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1m) (ap c\_2Enum\_2ESUC V2n))) \Leftrightarrow ((V1m = (ap c\_2Enum\_2ESUC V2n)) \vee (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D V1m) V2n)))))))))) \quad (18)$$

Assume the following.

$$True \quad (19)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p V0t))) \quad (21)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (23)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1y \in A.27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (24)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\ & p V0t)))))) \end{aligned} \quad (25)$$

Assume the following.

$$\begin{aligned} & (\forall V0D \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). (\forall V1a \in \\ & ty\_2Erealax\_2Ereal. (\forall V2b \in ty\_2Erealax\_2Ereal. ((p (ap \\ & (ap \ c.2Etransc.2Edivision (ap (ap (c.2Epair\_2E.2C \ ty\_2Erealax\_2Ereal \\ & ty\_2Erealax\_2Ereal) \ V1a) \ V2b)) \ V0D)) \Rightarrow ((ap \ V0D \ c.2Enum.2E0) = V1a)))))) \end{aligned} \quad (26)$$

Assume the following.

$$\begin{aligned} & (\forall V0D \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). (\forall V1a \in \\ & ty\_2Erealax\_2Ereal. (\forall V2b \in ty\_2Erealax\_2Ereal. ((p (ap \\ & (ap \ c.2Etransc.2Edivision (ap (ap (c.2Epair\_2E.2C \ ty\_2Erealax\_2Ereal \\ & ty\_2Erealax\_2Ereal) \ V1a) \ V2b)) \ V0D)) \Leftrightarrow (((ap \ V0D \ c.2Enum.2E0) = \\ & V1a) \wedge ((\forall V3n \in ty\_2Enum.2Enum. ((p (ap (ap \ c.2Eprim\_rec.2E.3C \\ & V3n) (ap \ c.2Etransc.2E.3D \ V0D))) \Rightarrow (p (ap (ap \ c.2Erealax\_2Ereal\_lt \\ & (ap \ V0D \ V3n)) (ap \ V0D (ap \ c.2Enum.2ESUC \ V3n)))))) \wedge (\forall V4n \in \\ & ty\_2Enum.2Enum. ((p (ap (ap \ c.2Earithmetic.2E.3E.3D \ V4n) (ap \ c.2Etransc.2E.3D \\ & V0D))) \Rightarrow ((ap \ V0D \ V4n) = V2b))))))))) \end{aligned} \quad (27)$$

Assume the following.

$$\begin{aligned} & (\forall V0D \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}). (\forall V1a \in \\ & ty\_2Erealax\_2Ereal. (\forall V2b \in ty\_2Erealax\_2Ereal. ((p (ap \\ & (ap \ c.2Etransc.2Edivision (ap (ap (c.2Epair\_2E.2C \ ty\_2Erealax\_2Ereal \\ & ty\_2Erealax\_2Ereal) \ V1a) \ V2b)) \ V0D)) \Rightarrow ((ap \ V0D (ap \ c.2Etransc.2E.3D \\ & V0D)) = V2b)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned}
& (\forall V0D \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V1a \in \\
& ty\_2Erealax\_2Ereal.(\forall V2b \in ty\_2Erealax\_2Ereal.((p (ap \\
& (ap c\_2Etransc\_2Edivision (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\
& ty\_2Erealax\_2Ereal) V1a) V2b)) V0D)) \Rightarrow (\forall V3n \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Eprim\_rec\_2E\_3C V3n) (ap c\_2Etransc\_2Esize V0D))) \Rightarrow \\
& (p (ap (ap c\_2Erealax\_2Ereal\_lt (ap V0D c\_2Enum\_2E0)) (ap V0D ( \\
& ap c\_2Enum\_2ESUC V3n))))))))))
\end{aligned} \tag{29}$$

Assume the following.

$$\begin{aligned}
& (\forall V0D \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V1a \in \\
& ty\_2Erealax\_2Ereal.(\forall V2b \in ty\_2Erealax\_2Ereal.((p (ap \\
& (ap c\_2Etransc\_2Edivision (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\
& ty\_2Erealax\_2Ereal) V1a) V2b)) V0D)) \Rightarrow (\forall V3n \in ty\_2Enum\_2Enum. \\
& ((p (ap (ap c\_2Eprim\_rec\_2E\_3C V3n) (ap c\_2Etransc\_2Esize V0D))) \Rightarrow \\
& (p (ap (ap c\_2Erealax\_2Ereal\_lt (ap V0D V3n)) (ap V0D (ap c\_2Etransc\_2Esize \\
& V0D))))))))))
\end{aligned} \tag{30}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0D \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V1a \in \\
& ty\_2Erealax\_2Ereal.(\forall V2b \in ty\_2Erealax\_2Ereal.(((p ( \\
& ap (ap c\_2Etransc\_2Edivision (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal \\
& ty\_2Erealax\_2Ereal) V1a) V2b)) V0D)) \wedge (\neg((ap c\_2Etransc\_2Esize \\
& V0D) = c\_2Enum\_2E0))) \Rightarrow (\forall V3n \in ty\_2Enum\_2Enum.(p (ap (ap \\
& c\_2Erealax\_2Ereal\_lt V1a) (ap V0D (ap c\_2Enum\_2ESUC V3n))))))))))
\end{aligned}$$