

thm_2Etransc_2EDIVISION__LE
(TMX16nSuQ6YhfdKvLRTX7UBHm8WpKTFCZDH)

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Definition 1 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.$ **if** $(\exists x \in A.p (ap P x))$ **then** *(the* $(\lambda x.x \in A \wedge p)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_3F$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap V0P (ap (c_2Emin_2E_40 A$

Definition 4 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ *of type* ι .

Definition 5 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V$

Definition 6 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27$

Definition 7 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Definition 8 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 9 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F$

Definition 10 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{3}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{4}$$

Definition 11 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Definition 12 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 13 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{5}$$

Definition 14 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 15 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 16 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ (c_2Ebool_2ECOND$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{6}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{7}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{8}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \tag{9}$$

Definition 17 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E_40\ ($

Let $c_2Erealax_2Etreallt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreallt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \tag{10}$$

Definition 18 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 19 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 20 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 21 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 22 We define $c_2Etransc_2Edsize$ to be $\lambda V0D \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(ap\ (c_2$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod \\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \end{aligned} \quad (11)$$

Definition 23 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2E$

Let $c_2Etransc_2Edivision : \iota$ be given. Assume the following.

$$c_2Etransc_2Edivision \in ((2^{(ty_2Erealax_2Ereal^{ty_2Enum_2Enum})})^{(ty_2Epair_2Eprod\ ty_2Erealax_2Ereal\ ty_2Ereal)}) \quad (12)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. ((V0m = c_2Enum_2E0) \vee (\exists V1n \in ty_2Enum_2Enum. (V0m = (ap\ c_2Enum_2ESUC\ V1n)))))) \quad (13)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum. (p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D\ V0m)\ V0m))) \quad (14)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (p\ (ap\ (ap\ c_2Earithmic_2E_3E_3D\ V0n)\ V1m)) \Leftrightarrow (p\ (ap\ (ap\ c_2Earithmic_2E_3C_3D\ V1m)\ V0n)))))) \quad (15)$$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((p\ V0t) \Rightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (17)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (18)$$

Assume the following.

$$(((ap\ c_2Eprim_rec_2EPRE\ c_2Enum_2E0) = c_2Enum_2E0) \wedge (\forall V0m \in ty_2Enum_2Enum. ((ap\ c_2Eprim_rec_2EPRE\ (ap\ c_2Enum_2ESUC\ V0m)) = V0m))) \quad (19)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(p (ap (ap c_2Eprim_rec_2E_3C V0n) (ap c_2Enum_2ESUC V0n)))) \quad (20)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(p (ap (ap c_2Ereal_2Ereal_lte V0x) V0x))) \quad (21)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. ((p (ap (ap c_2Erealax_2Ereal_lt V0x) V1y)) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte V0x) V1y)))))) \quad (22)$$

Assume the following.

$$(\forall V0D \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(\forall V1a \in ty_2Erealax_2Ereal.(\forall V2b \in ty_2Erealax_2Ereal.((p (ap (ap c_2Etransc_2Edivision (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V1a) V2b)) V0D)) \Leftrightarrow (((ap V0D c_2Enum_2E0) = V1a) \wedge ((\forall V3n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C V3n) (ap c_2Etransc_2Esize V0D))) \Rightarrow (p (ap (ap c_2Erealax_2Ereal_lt (ap V0D V3n)) (ap V0D (ap c_2Enum_2ESUC V3n)))))) \wedge (\forall V4n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmic_2E_3E_3D V4n) (ap c_2Etransc_2Esize V0D))) \Rightarrow ((ap V0D V4n) = V2b)))))))))) \quad (23)$$

Assume the following.

$$(\forall V0D \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(\forall V1a \in ty_2Erealax_2Ereal.(\forall V2b \in ty_2Erealax_2Ereal.((p (ap (ap c_2Etransc_2Edivision (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V1a) V2b)) V0D)) \Rightarrow (\forall V3n \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C V3n) (ap c_2Etransc_2Esize V0D))) \Rightarrow (p (ap (ap c_2Erealax_2Ereal_lt (ap V0D c_2Enum_2E0)) (ap V0D (ap c_2Enum_2ESUC V3n)))))))))) \quad (24)$$

Theorem 1

$$(\forall V0D \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(\forall V1a \in ty_2Erealax_2Ereal.(\forall V2b \in ty_2Erealax_2Ereal.((p (ap (ap c_2Etransc_2Edivision (ap (ap (c_2Epair_2E_2C ty_2Erealax_2Ereal ty_2Erealax_2Ereal) V1a) V2b)) V0D)) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte V1a) V2b))))))$$