

# thm\_2Etransc\_2EDIVISION\_\_LT\_\_GEN (TMb- WJFFCrAXKiWsibHqm8H3qMDuNPjofFJE)

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Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda 27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_F$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \quad (2)$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \quad (3)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \quad (4)$$

**Definition 8** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num (c\_2Enum\_2ESUC\_REP m))$

**Definition 9** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge P x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 10** We define  $c\_Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ V0P\ (ap\ (c\_Emin\_2E\_40$

**Definition 11** We define  $c\_Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 12** We define  $c\_Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

**Definition 13** We define  $c\_Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{5}$$

**Definition 14** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 15** We define  $c\_Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

**Definition 16** We define  $c\_Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum. (ap\ (ap\ c\_2Earithmetic$

**Definition 17** We define  $c\_Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum. V0x$ .

**Definition 18** We define  $c\_Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 19** We define  $c\_Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{7}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{8}$$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{9}$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax}) \tag{10}$$

**Definition 20** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal. (ap\ (c\_Emin\_2E\_40\ (t$

Let  $c\_2Erealax\_2Etreal\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal)}) \tag{11}$$

**Definition 21** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$ .

**Definition 22** We define  $c\_2Etransc\_2Edsize$  to be  $\lambda V0D \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(ap (c\_2E$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_2Epair\_2EABS\_prod \\ A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \end{aligned} \quad (12)$$

**Definition 23** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2E$

Let  $c\_2Etransc\_2Edivision : \iota$  be given. Assume the following.

$$c\_2Etransc\_2Edivision \in ((2^{(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})})^{(ty\_2Epair\_2Eprod ty\_2Erealax\_2Ereal ty\_2Ereal)} \quad (13)$$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\ ((ap (ap c\_2Earithmetic\_2E\_2B c\_2Enum\_2E0) V0m) = V0m) \wedge (((ap ( \\ ap c\_2Earithmetic\_2E\_2B V0m) c\_2Enum\_2E0) = V0m) \wedge (((ap (ap c\_2Earithmetic\_2E\_2B \\ (ap c\_2Enum\_2ESUC V0m)) V1n) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B \\ V0m) V1n)))) \wedge (((ap (ap c\_2Earithmetic\_2E\_2B V0m) (ap c\_2Enum\_2ESUC \\ V1n)) = (ap c\_2Enum\_2ESUC (ap (ap c\_2Earithmetic\_2E\_2B V0m) V1n)))))))))) \end{aligned} \quad (14)$$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\ (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D (ap c\_2Enum\_2ESUC V0m)) V1n))) \Rightarrow \\ (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n)))) \end{aligned} \quad (15)$$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty\_2Enum\_2Enum.((ap c\_2Enum\_2ESUC V0m) = (ap (ap \\ c\_2Earithmetic\_2E\_2B V0m) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\ c\_2Earithmetic\_2EZERO)))))) \end{aligned} \quad (16)$$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\ (p (ap (ap c\_2Eprim\_rec\_2E\_3C V0m) V1n))) \Rightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\ V0m) V1n)))) \end{aligned} \quad (17)$$

Assume the following.

$$\begin{aligned} (\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.( \\ (p (ap (ap c\_2Eprim\_rec\_2E\_3C V1n) V0m))) \Rightarrow (\exists V2p \in ty\_2Enum\_2Enum. \\ (V0m = (ap (ap c\_2Earithmetic\_2E\_2B V1n) (ap (ap c\_2Earithmetic\_2E\_2B \\ V2p) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\ c\_2Earithmetic\_2EZERO)))))))))) \end{aligned} \quad (18)$$

Assume the following.

$$True \quad (19)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a. (\forall V1y \in A\_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (20)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg( \\ & p\ V0t)))))) \quad (21) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0P \in (2^{ty\_2Enum\_2Enum}). (((p\ (ap\ V0P\ c\_2Enum\_2E0)) \wedge \\ & (\forall V1n \in ty\_2Enum\_2Enum. ((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ c\_2Enum\_2ESUC \\ & V1n)))))) \Rightarrow (\forall V2n \in ty\_2Enum\_2Enum. (p\ (ap\ V0P\ V2n)))))) \quad (22) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealx\_2Ereal. (\forall V1y \in ty\_2Erealx\_2Ereal. \\ & (\forall V2z \in ty\_2Erealx\_2Ereal. (((p\ (ap\ (ap\ c\_2Erealx\_2Ereal\_lt \\ & V0x\ V1y)) \wedge (p\ (ap\ (ap\ c\_2Erealx\_2Ereal\_lt\ V1y\ V2z))) \Rightarrow (p\ (ap \\ & (ap\ c\_2Erealx\_2Ereal\_lt\ V0x\ V2z)))))) \quad (23) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0D \in (ty\_2Erealx\_2Ereal^{ty\_2Enum\_2Enum}). (\forall V1a \in \\ & ty\_2Erealx\_2Ereal. (\forall V2b \in ty\_2Erealx\_2Ereal. ((p\ (ap \\ & (ap\ c\_2Etrasc\_2Edivision\ (ap\ (ap\ (c\_2Epair\_2E\_2C\ ty\_2Erealx\_2Ereal \\ & ty\_2Erealx\_2Ereal)\ V1a\ V2b))\ V0D)) \Leftrightarrow (((ap\ V0D\ c\_2Enum\_2E0) = \\ & V1a) \wedge ((\forall V3n \in ty\_2Enum\_2Enum. ((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C \\ & V3n)\ (ap\ c\_2Etrasc\_2Esize\ V0D))) \Rightarrow (p\ (ap\ (ap\ c\_2Erealx\_2Ereal\_lt \\ & (ap\ V0D\ V3n))\ (ap\ V0D\ (ap\ c\_2Enum\_2ESUC\ V3n)))))) \wedge (\forall V4n \in \\ & ty\_2Enum\_2Enum. ((p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3E\_3D\ V4n)\ (ap\ c\_2Etrasc\_2Esize \\ & V0D))) \Rightarrow ((ap\ V0D\ V4n) = V2b)))))) \quad (24) \end{aligned}$$

### Theorem 1

$$\begin{aligned} & (\forall V0D \in (ty\_2Erealx\_2Ereal^{ty\_2Enum\_2Enum}). (\forall V1a \in \\ & ty\_2Erealx\_2Ereal. (\forall V2b \in ty\_2Erealx\_2Ereal. (\forall V3m \in \\ & ty\_2Enum\_2Enum. (\forall V4n \in ty\_2Enum\_2Enum. (((p\ (ap\ (ap\ c\_2Etrasc\_2Edivision \\ & (ap\ (ap\ (c\_2Epair\_2E\_2C\ ty\_2Erealx\_2Ereal\ ty\_2Erealx\_2Ereal \\ & V1a\ V2b))\ V0D)) \wedge ((p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V3m)\ V4n)) \wedge (p \\ & (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ V4n)\ (ap\ c\_2Etrasc\_2Esize\ V0D)))))) \Rightarrow \\ & (p\ (ap\ (ap\ c\_2Erealx\_2Ereal\_lt\ (ap\ V0D\ V3m))\ (ap\ V0D\ V4n)))))) \quad (25) \end{aligned}$$