

# thm\_2Etransc\_2EDIVISION\_\_SINGLE (TMX-ASbe1CpCLHcvYwJnjXrBHNJeQiT8uCDW)

October 26, 2020

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

**Definition 3** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{5}$$

**Definition 4** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V2x \in 2.V2x))$

**Definition 6** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ (ap\ (ap\ (c\_2Emin\_2E\_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

**Definition 7** We define  $c\_Earithmic\_EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_Earithmic\_E$

**Definition 8** We define  $c\_Earithmic\_ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

**Definition 9** We define  $c\_Ebool\_EF$  to be  $(ap (c\_Ebool\_E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 10** We define  $c\_Emin\_E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 11** We define  $c\_Ebool\_E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_Emin\_E\_3D\_3D\_3E V0t) c\_Ebool\_E\_7E$

**Definition 12** We define  $c\_Ebool\_E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_Ebool\_E\_21 2) (\lambda V2t \in$

**Definition 13** We define  $c\_Emin\_E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$  then (the  $(\lambda x.x \in A \wedge$  of type  $\iota \Rightarrow \iota$ .

**Definition 14** We define  $c\_Ebool\_E\_3F$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap V0P (ap (c\_Emin\_E\_40$

**Definition 15** We define  $c\_Eprim\_rec\_E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 16** We define  $c\_Ebool\_E\_5C\_2F$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_Ebool\_E\_21 2) (\lambda V2t \in$

**Definition 17** We define  $c\_Earithmic\_E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

**Definition 18** We define  $c\_Ebool\_ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{7}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{8}$$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{9}$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})\ ty\_2Erealax\_2Ereal) \tag{10}$$

**Definition 19** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_Emin\_E\_40 (t$

Let  $c\_2Erealax\_2Etreall\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreall\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})\ (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)) \tag{11}$$

**Definition 20** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal.$

**Definition 21** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.$

**Definition 22** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 23** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.$

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}})$$
(12)

**Definition 24** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2E$

Let  $c\_2Etransc\_2Edivision : \iota$  be given. Assume the following.

$$c\_2Etransc\_2Edivision \in ((2^{(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})})^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)})$$
(13)

Assume the following.

$$((ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)) = (ap\ c\_2Enum\_2ESUC\ c\_2Enum\_2E0))$$
(14)

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\neg(c\_2Enum\_2E0 = (ap\ c\_2Enum\_2ESUC\ V0n))))$$
(15)

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ (ap\ c\_2Enum\_2ESUC\ V0m))\ (ap\ c\_2Enum\_2ESUC\ V1n))) \Leftrightarrow (p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ V0m)\ V1n))))$$
(16)

Assume the following.

$$(p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3E\_3D\ V0n)\ V1m)) \Leftrightarrow (p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ V1m)\ V0n)))$$
(17)

Assume the following.

$$((\forall V0n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ V0n)\ c\_2Enum\_2E0)) \Leftrightarrow (V0n = c\_2Enum\_2E0))) \wedge (\forall V1m \in ty\_2Enum\_2Enum.(\forall V2n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ V1m)\ (ap\ c\_2Enum\_2ESUC\ V2n))) \Leftrightarrow ((V1m = (ap\ c\_2Enum\_2ESUC\ V2n)) \vee (p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ V1m)\ V2n)))))))$$
(18)

Assume the following.

$$True \quad (19)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p \ V0t1) \Rightarrow (p \ V1t2)) \Rightarrow (((p \ V1t2) \Rightarrow (p \ V0t1)) \Rightarrow ((p \ V0t1) \Leftrightarrow (p \ V1t2)))))) \quad (20)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p \ V0t))) \quad (21)$$

Assume the following.

$$(\forall V0t \in 2.((p \ V0t) \vee \neg(p \ V0t))) \quad (22)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p \ V0t)) \Leftrightarrow (p \ V0t))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (24)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((p \ V0t) \Rightarrow False) \Leftrightarrow \neg(p \ V0t)))))) \quad (25)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (26)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \quad (27)$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow \neg(p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p \ V0t)))))) \quad (29)$$

Assume the following.

$$\begin{aligned} \forall A\_27a. \text{nonempty } A\_27a \Rightarrow (\forall V0t1 \in A\_27a. (\forall V1t2 \in \\ A\_27a. (((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2ET) V0t1) \\ V1t2) = V0t1) \wedge ((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) c\_2Ebool\_2EF) \\ V0t1) V1t2) = V1t2)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} \forall A\_27a. \text{nonempty } A\_27a \Rightarrow (\forall V0b \in 2. (\forall V1t \in A\_27a. \\ ((ap (ap (ap (c\_2Ebool\_2ECOND A\_27a) V0b) V1t) V1t) = V1t))) \end{aligned} \quad (31)$$

Assume the following.

$$(\forall V0n \in \text{ty\_2Enum\_2Enum}. (\neg((ap c\_2Enum\_2ESUC V0n) = c\_2Enum\_2E0))) \quad (32)$$

Assume the following.

$$\begin{aligned} (\forall V0P \in (2^{\text{ty\_2Enum\_2Enum}}). (((p (ap V0P c\_2Enum\_2E0)) \wedge \\ (\forall V1n \in \text{ty\_2Enum\_2Enum}. ((p (ap V0P V1n)) \Rightarrow (p (ap V0P (ap c\_2Enum\_2ESUC \\ V1n)))))) \Rightarrow (\forall V2n \in \text{ty\_2Enum\_2Enum}. (p (ap V0P V2n)))))) \end{aligned} \quad (33)$$

Assume the following.

$$(\forall V0n \in \text{ty\_2Enum\_2Enum}. (\neg(p (ap (ap c\_2Eprim\_rec\_2E\_3C \\ V0n) c\_2Enum\_2E0)))) \quad (34)$$

Assume the following.

$$(\forall V0n \in \text{ty\_2Enum\_2Enum}. (p (ap (ap c\_2Eprim\_rec\_2E\_3C c\_2Enum\_2E0) \\ (ap c\_2Enum\_2ESUC V0n)))) \quad (35)$$

Assume the following.

$$\begin{aligned} (\forall V0x \in \text{ty\_2Erealax\_2Ereal}. (\forall V1y \in \text{ty\_2Erealax\_2Ereal}. \\ ((p (ap (ap c\_2Ereal\_2Ereal\_lte V0x) V1y)) \Leftrightarrow ((p (ap (ap c\_2Erealax\_2Ereal\_lt \\ V0x) V1y)) \vee (V0x = V1y)))))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} (\forall V0a \in \text{ty\_2Erealax\_2Ereal}. (\forall V1b \in \text{ty\_2Erealax\_2Ereal}. \\ (\forall V2D \in (\text{ty\_2Erealax\_2Ereal}^{\text{ty\_2Enum\_2Enum}}). ((p (ap (ap \\ c\_2Etransc\_2Edivision (ap (ap (c\_2Epair\_2E\_2C \text{ty\_2Erealax\_2Ereal} \\ \text{ty\_2Erealax\_2Ereal}) V0a) V1b)) V2D)) \Leftrightarrow (((ap V2D c\_2Enum\_2E0) = \\ V0a) \wedge (\exists V3N \in \text{ty\_2Enum\_2Enum}. ((\forall V4n \in \text{ty\_2Enum\_2Enum}. \\ ((p (ap (ap c\_2Eprim\_rec\_2E\_3C V4n) V3N)) \Rightarrow (p (ap (ap c\_2Erealax\_2Ereal\_lt \\ (ap V2D V4n)) (ap V2D (ap c\_2Enum\_2ESUC V4n)))))) \wedge (\forall V5n \in \\ \text{ty\_2Enum\_2Enum}. ((p (ap (ap c\_2Earithmic\_2E\_3E\_3D V5n) V3N)) \Rightarrow \\ ((ap V2D V5n) = V1b)))))))))) \end{aligned} \quad (37)$$

**Theorem 1**

$(\forall V0a \in ty\_2Erealax\_2Ereal. (\forall V1b \in ty\_2Erealax\_2Ereal. ((p (ap (ap c\_2Ereal\_2Ereal\_lte V0a) V1b)) \Rightarrow (p (ap (ap c\_2Etransc\_2Edivision (ap (ap (c\_2Epair\_2E\_2C ty\_2Erealax\_2Ereal ty\_2Erealax\_2Ereal) V0a) V1b)) (\lambda V2n \in ty\_2Enum\_2Enum. (ap (ap (ap (c\_2Ebool\_2ECOND ty\_2Erealax\_2Ereal) (ap (ap (c\_2Emin\_2E\_3D ty\_2Enum\_2Enum) V2n) c\_2Enum\_2E0)) V0a) V1b))))))))))$