thm_2Etransc_2EDIVISION__UBOUND (TMWK-TNt4H7t6EPDLssN3WDjzP6m3bNkEZPp)

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Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{2}$$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{3}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^{A\_27a}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21$\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ $c_2Emin_2E_3D_3D_3E$\ V0t)\ $c_2Ebool_2EF

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21$\ 2)\ (\lambda V2t \in 2$

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \qquad (5)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \qquad (6)$$

Definition 8 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Definition 9 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \land p$
of type $\iota \Rightarrow \iota$.

Definition 10 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40

Definition 11 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 12 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in$

Definition 13 *We define* c_2Earithmetic_2E_3C_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 14 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 15 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 16 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \qquad (7)$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod \\ A0\ A1)$$
$$\qquad (8)$$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \qquad (9)$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax}$$
$$\qquad (10)$$

Definition 17 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ ($c_2Emin_2E_40 $(t$

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh}$$
$$\qquad (11)$$

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Definition 18 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_$

Definition 19 *We define* c_2Etransc_2Edsize *to be* $\lambda V0D \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(ap$ (c_2

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{12}$$

Definition 20 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP).$

Definition 21 *We define* c_2Ereal_2Ereal__lte *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod$$
$$A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{13}$$

Definition 22 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap$ (c_2

Let $c\_2Etransc\_2Edivision : \iota$ be given. Assume the following.

$$c\_2Etransc\_2Edivision \in ((2^{(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})})^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Ereal} \tag{14}$$

Assume the following.

$$((\forall V0m \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D$$
$$\text{c\_2Enum\_2E0})\ V0m) = \text{c\_2Enum\_2E0})) \wedge (\forall V1m \in ty\_2Enum\_2Enum.$$
$$(\forall V2n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ ($$
$$ap\ \text{c\_2Enum\_2ESUC}\ V1m))\ V2n) = (ap\ (ap\ (ap\ (\text{c\_2Ebool\_2ECOND}\ ty\_2Enum\_2Enum)$$
$$(ap\ (ap\ \text{c\_2Eprim\_\_rec\_2E\_3C}\ V1m)\ V2n))\ \text{c\_2Enum\_2E0})\ (ap\ \text{c\_2Enum\_2ESUC}$$
$$(ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V1m)\ V2n))))))) \tag{15}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$(ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$V1n)\ V0m)))) \tag{16}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.($$
$$(p\ (ap\ (ap\ \text{c\_2Earithmetic\_2E\_3C\_3D}\ (ap\ \text{c\_2Enum\_2ESUC}\ V0n))\ (ap$$
$$\text{c\_2Enum\_2ESUC}\ V1m))) \Leftrightarrow (p\ (ap\ (ap\ \text{c\_2Earithmetic\_2E\_3C\_3D}\ V0n)$$
$$V1m))))) \tag{17}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$((\neg(p\ (ap\ (ap\ \text{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ V1n))) \wedge (\neg(V0m = V1n))) \Rightarrow$$
$$(p\ (ap\ (ap\ \text{c\_2Eprim\_\_rec\_2E\_3C}\ V1n)\ V0m))))) \tag{18}$$

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Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ V1n))\lor(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\\V1n)\ V0m)))))$$

$$(19)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\\V0m)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0m))))$$

$$(20)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\(\neg(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ V1n)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\\V1n)\ V0m)))))$$

$$(21)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(((ap\ (ap\ c\_2Earithmetic\_2E\_2D\\\mathsf{c\_2Enum\_2E0})\ V0m) = \mathsf{c\_2Enum\_2E0})\land((ap\ (ap\ c\_2Earithmetic\_2E\_2D\\V0m)\ \mathsf{c\_2Enum\_2E0}) = V0m)))$$

$$(22)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(\\(ap\ (ap\ c\_2Earithmetic\_2E\_2D\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n))\ (ap\ \mathsf{c\_2Enum\_2ESUC}\\V1m)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0n)\ V1m))))$$

$$(23)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\((ap\ (ap\ c\_2Earithmetic\_2E\_2D\ V0m)\ V1n) = \mathsf{c\_2Enum\_2E0})\Leftrightarrow(p\ (ap\ (\\ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V1n)))))$$

$$(24)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\\forall V2p \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\\V0m)\ V1n))\land(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V1n)\ V2p)))\Rightarrow(p\ (\\ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\ V0m)\ V2p))))))$$

$$(25)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\\V0m)\ V0m)))$$

$$(26)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ V0m)\ V1n))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3C\_3D}\\V0m)\ V1n)))))$$

$$(27)$$

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Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(\\ p \ (ap \ (ap \ \mathsf{c\_2Earithmetic\_2E\_3C\_3D} \ (ap \ (ap \ c\_2Earithmetic\_2E\_2D \\ V0n) \ V1m)) \ V0n)))) \tag{28}$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\ ((ap \ (ap \ c\_2Earithmetic\_2E\_2D \ V0m) \ V1n) = V0m) \Leftrightarrow ((V0m = \mathsf{c\_2Enum\_2E0}) \vee \\ (V1n = \mathsf{c\_2Enum\_2E0}))))) \tag{29}$$

Assume the following.

$$(\forall V0a \in ty\_2Enum\_2Enum.(\forall V1c \in ty\_2Enum\_2Enum.(\\ (ap \ (ap \ c\_2Earithmetic\_2E\_2D \ (ap \ (ap \ c\_2Earithmetic\_2E\_2B \ V0a) \\ V1c)) \ V1c) = V0a))) \tag{30}$$

Assume the following.

$$(\forall V0b \in ty\_2Enum\_2Enum.(\forall V1c \in ty\_2Enum\_2Enum.(\\ (p \ (ap \ (ap \ \mathsf{c\_2Earithmetic\_2E\_3C\_3D} \ V1c) \ V0b)) \Rightarrow (\forall V2a \in ty\_2Enum\_2Enum. \\ ((ap \ (ap \ c\_2Earithmetic\_2E\_2D \ V2a) \ (ap \ (ap \ c\_2Earithmetic\_2E\_2D \\ V0b) \ V1c)) = (ap \ (ap \ c\_2Earithmetic\_2E\_2D \ (ap \ (ap \ c\_2Earithmetic\_2E\_2B \\ V2a) \ V1c)) \ V0b)))))) \tag{31}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(\\ (p \ (ap \ (ap \ \mathsf{c\_2Earithmetic\_2E\_3E\_3D} \ V0n) \ V1m)) \Leftrightarrow (p \ (ap \ (ap \ \mathsf{c\_2Earithmetic\_2E\_3C\_3D} \\ V1m) \ V0n))))) \tag{32}$$

Assume the following.
$$True \tag{33}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p \ V0t1) \Rightarrow (p \ V1t2)) \Rightarrow (((p \\ V1t2) \Rightarrow (p \ V0t1)) \Rightarrow ((p \ V0t1) \Leftrightarrow (p \ V1t2)))))) \tag{34}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p \ V0t))) \tag{35}$$

Assume the following.

$$(\forall V0t \in 2.((p \ V0t) \vee (\neg (p \ V0t)))) \tag{36}$$

Assume the following.

$$\forall A\_27a.nonempty \ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in \\ A\_27a.(p \ V0t)) \Leftrightarrow (p \ V0t))) \tag{37}$$

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Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge \\ (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{38}$$

Assume the following.

$$(\forall V0t \in 2.(((True \vee (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \vee True) \Leftrightarrow True) \wedge \\ (((False \vee (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \vee False) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \vee \\ (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{39}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow \\ True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t))))))))) \tag{40}$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ ((\neg False) \Leftrightarrow True))) \tag{41}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow \\ True)) \tag{42}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in \\ A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{43}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow \\ (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (\\ p\ V0t)))))))) \tag{44}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in \\ A\_27a.(((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2ET})\ V0t1) \\ V1t2) = V0t1) \wedge ((ap\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2ECOND}\ A\_27a)\ \mathsf{c\_2Ebool\_2EF}) \\ V0t1)\ V1t2) = V1t2)))) \tag{45}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\neg ((ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n) = \mathsf{c\_2Enum\_2E0}))) \tag{46}$$

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Assume the following.

$$(\forall V0P \in (2^{ty\_2Enum\_2Enum}).(((p\ (ap\ V0P\ \mathsf{c\_2Enum\_2E0}))\wedge$$
$$(\forall V1n \in ty\_2Enum\_2Enum.((p\ (ap\ V0P\ V1n))\Rightarrow(p\ (ap\ V0P\ (ap\ \mathsf{c\_2Enum\_2ESUC}$$
$$V1n)))))))\Rightarrow(\forall V2n \in ty\_2Enum\_2Enum.(p\ (ap\ V0P\ V2n)))))) \tag{47}$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\neg(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}$$
$$V0n)\ \mathsf{c\_2Enum\_2E0})))) \tag{48}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.$$
$$((p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ V0x)\ V1y))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}$$
$$V0x)\ V1y))))) \tag{49}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.$$
$$(\forall V2z \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}$$
$$V0x)\ V1y))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V1y)\ V2z)))\Rightarrow(p\ (ap\ ($$
$$ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V0x)\ V2z)))))) \tag{50}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.$$
$$((V0x = V1y)\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V0x)\ V1y))))) \tag{51}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t)))\Leftrightarrow(p\ V0t))) \tag{52}$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A)\Rightarrow((\neg(p\ V0A))\Rightarrow False))) \tag{53}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A)\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow$$
$$(((p\ V0A)\Rightarrow False)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{54}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A))\vee(p\ V1B)))\Rightarrow False)\Leftrightarrow$$
$$((p\ V0A)\Rightarrow((\neg(p\ V1B))\Rightarrow False))))) \tag{55}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A))\Rightarrow False)\Rightarrow(((p\ V0A)\Rightarrow False)\Rightarrow False))) \tag{56}$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p)\Leftrightarrow($$
$$(p\ V1q)\Leftrightarrow(p\ V2r)))\Leftrightarrow(((p\ V0p)\vee((p\ V1q)\vee(p\ V2r)))\wedge(((p\ V0p)\vee((\neg($$
$$p\ V2r))\vee(\neg(p\ V1q))))\wedge(((p\ V1q)\vee((\neg(p\ V2r))\vee(\neg(p\ V0p))))\wedge((p\ V2r)\vee$$
$$((\neg(p\ V1q))\vee(\neg(p\ V0p)))))))))))))$$

$$(57)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(p\ V0p)))) \qquad (58)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.((\neg((p\ V0p)\Rightarrow(p\ V1q)))\Rightarrow(\neg(p\ V1q))))) \qquad (59)$$

Assume the following.

$$(\forall V0D \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V1a \in$$
$$ty\_2Erealax\_2Ereal.(\forall V2b \in ty\_2Erealax\_2Ereal.((p\ (ap$$
$$(ap\ c\_2Etransc\_2Edivision\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal$$
$$ty\_2Erealax\_2Ereal)\ V1a)\ V2b))\ V0D))\Leftrightarrow(((ap\ V0D\ \mathsf{c\_2Enum\_2E0})=$$
$$V1a)\wedge((\forall V3n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_rec\_2E\_3C}$$
$$V3n)\ (ap\ c\_2Etransc\_2Edsize\ V0D)))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}$$
$$(ap\ V0D\ V3n))\ (ap\ V0D\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V3n))))))\wedge(\forall V4n \in$$
$$ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Earithmetic\_2E\_3E\_3D}\ V4n)\ (ap\ c\_2Etransc\_2Edsize$$
$$V0D)))\Rightarrow((ap\ V0D\ V4n)=V2b)))))))))))$$

$$(60)$$

Assume the following.

$$(\forall V0D \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V1a \in$$
$$ty\_2Erealax\_2Ereal.(\forall V2b \in ty\_2Erealax\_2Ereal.((p\ (ap$$
$$(ap\ c\_2Etransc\_2Edivision\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal$$
$$ty\_2Erealax\_2Ereal)\ V1a)\ V2b))\ V0D))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}$$
$$V1a)\ V2b))))))$$

$$(61)$$

Theorem 1

$$(\forall V0D \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V1a \in$$
$$ty\_2Erealax\_2Ereal.(\forall V2b \in ty\_2Erealax\_2Ereal.((p\ (ap$$
$$(ap\ c\_2Etransc\_2Edivision\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal$$
$$ty\_2Erealax\_2Ereal)\ V1a)\ V2b))\ V0D))\Rightarrow(\forall V3r \in ty\_2Enum\_2Enum.$$
$$(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ V0D\ V3r))\ V2b)))))))$$