

# thm\_2Etransc\_2EEXP\_\_CONVERGES (TMYd63UnJu7od7Xk9H434d7dtaPanfsCNfQ)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_2T$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_2F$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p \Rightarrow q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_2Emin\_2E\_3D\_3D\_3E V0t) c\_2Ebool\_2E\_2F))$

**Definition 7** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t)))$

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{2}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{3}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{4}$$

**Definition 8** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap c\_2Enum\_2EABS\_num (ap c\_2Enum\_2EREP\_num (ap c\_2Enum\_2ESUC\_REP m)))$

**Definition 9** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge P x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 10** We define  $c\_Ebool\_2E\_3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ V0P\ (ap\ (c\_Emin\_2E\_40$

**Definition 11** We define  $c\_Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

**Definition 12** We define  $c\_Ebool\_2E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

**Definition 13** We define  $c\_Earithmetic\_2E\_3C\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum. \lambda V1n \in ty\_2Enum\_2Enum$

Let  $c\_Earithmetic\_2E\_2A : \iota$  be given. Assume the following.

$$c\_Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (5)$$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (6)$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0. nonempty\ A0 \Rightarrow \forall A1. nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (7)$$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (8)$$

Let  $c\_Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal}) \quad (9)$$

**Definition 14** We define  $c\_Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal. (ap\ (c\_Emin\_2E\_40\ ($

Let  $c\_Erealax\_2Etrealm\_inv : \iota$  be given. Assume the following.

$$c\_Erealax\_2Etrealm\_inv \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (10)$$

Let  $c\_Erealax\_2Etrealm\_eq : \iota$  be given. Assume the following.

$$c\_Erealax\_2Etrealm\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (11)$$

Let  $c\_Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})} \quad (12)$$

**Definition 15** We define  $c\_Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 16** We define  $c\_Erealax\_2Einv$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal. (ap\ c\_Erealax\_2Ereal\_ABS$

Let  $c\_2Erealax\_2Etreal\_mul : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)))(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal) \quad (13)$$

**Definition 17** We define  $c\_2Erealax\_2Ereal\_mul$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal.$

**Definition 18** We define  $c\_2Ereal\_2E\_2F$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.$

Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})ty\_2Enum\_2Enum) \quad (14)$$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (15)$$

**Definition 19** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{(2^{A\_27b})^{A\_27a}}) \quad (16)$$

**Definition 20** We define  $c\_2Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2Epair\_2EABS\_prod\ A\_27a\ A\_27b)\ x\ y)$

Let  $c\_2Ereal\_2Esum : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Esum \in ((ty\_2Erealax\_2Ereal^{(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})})ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum) \quad (17)$$

**Definition 21** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.$

**Definition 22** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum.$

Let  $c\_2Erealax\_2Etreal\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)) \quad (18)$$

**Definition 23** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal.$

Let  $c\_2Erealax\_2Etreal\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)))(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal) \quad (19)$$

**Definition 24** We define  $c\_2Erealx\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealx\_2Ereal.\lambda V1T2 \in ty\_2Erealx\_2Ereal$

**Definition 25** We define  $c\_2Ereal\_2Ereal\_sub$  to be  $\lambda V0x \in ty\_2Erealx\_2Ereal.\lambda V1y \in ty\_2Erealx\_2Ereal$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealx\_2Ereal^{ty\_2Enum\_2Enum}) \quad (20)$$

Let  $c\_2Erealx\_2Etrealt : \iota$  be given. Assume the following.

$$c\_2Erealx\_2Etrealt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)) \quad (21)$$

**Definition 26** We define  $c\_2Erealx\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealx\_2Ereal.\lambda V1T2 \in ty\_2Erealx\_2Ereal$

**Definition 27** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealx\_2Ereal.\lambda V1y \in ty\_2Erealx\_2Ereal$

**Definition 28** We define  $c\_2Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.$

**Definition 29** We define  $c\_2Ereal\_2Eabs$  to be  $\lambda V0x \in ty\_2Erealx\_2Ereal.(ap\ (ap\ (ap\ (c\_2Ebool\_2ECOND$

Let  $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND \\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \end{aligned} \quad (22)$$

Let  $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST \\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \end{aligned} \quad (23)$$

**Definition 30** We define  $c\_2Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27a$

Let  $ty\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Emetric\_2Emetric\ A0) \quad (24)$$

Let  $c\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Emetric\ A\_27a \in ((ty\_2Emetric\_2Emetric \\ A\_27a)^{(ty\_2Erealx\_2Ereal^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}})) \end{aligned} \quad (25)$$

**Definition 31** We define  $c\_2Emetric\_2Emr1$  to be  $(ap\ (c\_2Emetric\_2Emetric\ ty\_2Erealx\_2Ereal)\ (ap\ (c$

Let  $c\_2Emetric\_2Edist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Edist\ A\_27a \in ((ty\_2Erealx\_2Ereal^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}})) \quad (26)$$

Let  $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \quad (27)$$

Let  $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{2^A-27a}})) \quad (28)$$

**Definition 32** We define  $c\_2Emetric\_2Emtop$  to be  $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Emetric\_2Emetric\ A\_27a).(ap$

Let  $c\_2Enets\_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Enets\_2Etends\ A\_27a\ A\_27b \in (((2^{(ty\_2Epair\_2Eprod\ (ty\_2Etopology\_2Etopology\ A\_27a)\ ((2^{A\_27b})^{A\_27b}))})_{A\_27a})(A\_27a^{A\_27b})) \quad (29)$$

**Definition 33** We define  $c\_2Eseq\_2E\_2D\_2D\_3E$  to be  $\lambda V0x \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1x$

**Definition 34** We define  $c\_2Eseq\_2Esums$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1s \in ty\_2$

**Definition 35** We define  $c\_2Eseq\_2Esummable$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(ap\ (c\_2E$

**Definition 36** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

**Definition 37** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

**Definition 38** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Ereal\_2Epow : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})^{ty\_2Erealax\_2Ereal}) \quad (30)$$

Let  $c\_2Earithmetic\_2EFACT : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EFACT \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}) \quad (31)$$

**Definition 39** We define  $c\_2Eseq\_2Esuminf$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(ap\ (c\_2E$

**Definition 40** We define  $c\_2Etransc\_2Eexp$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ c\_2Eseq\_2Esuminf\ (\lambda V1$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ c\_2Enum\_2E0)\ V0n))) \quad (32)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ V0m)\ (ap\ c\_2Enum\_2ESUC\ V0m)))) \quad (33)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.((ap\ c\_2Enum\_2ESUC\ V0m) = (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO)))))) \quad (34)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\forall V2p \in ty\_2Enum\_2Enum.(((p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ V0m)\ V1n)) \wedge (p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ V1n)\ V2p))) \Rightarrow (p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ V0m)\ V2p)))))) \quad (35)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\forall V1m \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3E\_3D\ V0n)\ V1m)) \Leftrightarrow (p\ (ap\ (ap\ c\_2Earithmetic\_2E\_3C\_3D\ V1m)\ V0n)))) \quad (36)$$

Assume the following.

$$(((ap\ c\_2Earithmetic\_2EFACT\ c\_2Enum\_2E0) = (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO))) \wedge (\forall V0n \in ty\_2Enum\_2Enum.((ap\ c\_2Earithmetic\_2EFACT\ (ap\ c\_2Enum\_2ESUC\ V0n)) = (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ c\_2Enum\_2ESUC\ V0n))\ (ap\ c\_2Earithmetic\_2EFACT\ V0n)))))) \quad (37)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ c\_2Enum\_2E0)\ (ap\ c\_2Earithmetic\_2EFACT\ V0n)))) \quad (38)$$

Assume the following.

$$True \quad (39)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (40)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (41)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (42)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (( \\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \end{aligned} \quad (43)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge ((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (44)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in \\ & A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg( \\ & p V0t)))))) \end{aligned} \quad (46)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(\neg((ap\ c\_2Enum\_2ESUC\ V0n) = c\_2Enum\_2E0))) \quad (47)$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ c\_2Eprim\_rec\_2E\_3C\ c\_2Enum\_2E0) \\ & (ap\ c\_2Enum\_2ESUC\ V0n)))) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. \\ & ((ap\ (ap\ c\_2Erealax\_2Ereal\_mul\ V0x)\ V1y) = (ap\ (ap\ c\_2Erealax\_2Ereal\_mul \\ & V1y)\ V0x)))) \end{aligned} \quad (49)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. \\ & (\forall V2z \in ty\_2Erealax\_2Ereal.((ap\ (ap\ c\_2Erealax\_2Ereal\_mul \\ & V0x)\ (ap\ (ap\ c\_2Erealax\_2Ereal\_mul\ V1y)\ V2z)) = (ap\ (ap\ c\_2Erealax\_2Ereal\_mul \\ & (ap\ (ap\ c\_2Erealax\_2Ereal\_mul\ V0x)\ V1y))\ V2z)))))) \end{aligned} \quad (50)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. \\ & ((p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lte\ V0x)\ V1y)) \Rightarrow (p\ (ap\ (ap\ c\_2Ereal\_2Ereal\_lte \\ & V0x)\ V1y)))))) \end{aligned} \quad (51)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2z \in ty\_2Erealax\_2Ereal. ((p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& V0x) V1y)) \wedge (p (ap (ap c\_2Ereal\_2Ereal\_lte V1y) V2z))) \Rightarrow (p (ap ( \\
& ap c\_2Ereal\_2Ereal\_lte V0x) V2z))))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)) (ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Erealax\_2Ereal\_lt V0x) V1y)) \Rightarrow (\neg(V0x = V1y))))))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& (p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Ereal\_2Ereal\_of\_num \\
& V0m)) (ap c\_2Ereal\_2Ereal\_of\_num V1n))) \Leftrightarrow (p (ap (ap c\_2Earithmetic\_2E\_3C\_3D \\
& V0m) V1n))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& (p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& V0m)) (ap c\_2Ereal\_2Ereal\_of\_num V1n))) \Leftrightarrow (p (ap (ap c\_2Eprim\_rec\_2E\_3C \\
& V0m) V1n))))))
\end{aligned} \tag{56}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& ((ap c\_2Ereal\_2Ereal\_of\_num V0m) = (ap c\_2Ereal\_2Ereal\_of\_num \\
& V1n)) \Leftrightarrow (V0m = V1n))))))
\end{aligned} \tag{57}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty\_2Enum\_2Enum. (\forall V1n \in ty\_2Enum\_2Enum. ( \\
& (ap (ap c\_2Erealax\_2Ereal\_mul (ap c\_2Ereal\_2Ereal\_of\_num \\
& V0m)) (ap c\_2Ereal\_2Ereal\_of\_num V1n)) = (ap c\_2Ereal\_2Ereal\_of\_num \\
& (ap (ap c\_2Earithmetic\_2E\_2A V0m) V1n))))))
\end{aligned} \tag{58}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) V0x)) \Rightarrow (\exists V1y \in \\
& ty\_2Erealax\_2Ereal. ((p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)) V1y)) \wedge (p (ap (ap c\_2Erealax\_2Ereal\_lt V1y) V0x))))))
\end{aligned} \tag{59}$$



Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (((\neg(V0x = (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0))) \wedge (\neg(V1y = \\
& (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0)))) \Rightarrow ((ap\ c\_2Erealax\_2Ereal\_Einv \\
& (ap\ (ap\ c\_2Erealax\_2Ereal\_mul\ V0x)\ V1y)) = (ap\ (ap\ c\_2Erealax\_2Ereal\_mul \\
& (ap\ c\_2Erealax\_2Ereal\_Einv\ V0x))\ (ap\ c\_2Erealax\_2Ereal\_Einv\ V1y))))))
\end{aligned} \tag{60}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2z \in ty\_2Erealax\_2Ereal. ((p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lte \\
& (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0)\ V2z)) \Rightarrow ((p\ (ap\ (ap \\
& c\_2Ereal\_2Ereal\_lte\ (ap\ (ap\ c\_2Erealax\_2Ereal\_mul\ V0x)\ V2z)) \\
& (ap\ (ap\ c\_2Erealax\_2Ereal\_mul\ V1y)\ V2z))) \Leftrightarrow (p\ (ap\ (ap\ c\_2Ereal\_2Ereal\_lte \\
& V0x)\ V1y))))))
\end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2z \in ty\_2Erealax\_2Ereal. (((p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lte \\
& (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0)\ V0x)) \wedge (p\ (ap\ (ap\ c\_2Ereal\_2Ereal\_lte \\
& V1y)\ (ap\ (ap\ c\_2Erealax\_2Ereal\_mul\ V2z)\ V0x))) \Rightarrow (p\ (ap\ (ap\ c\_2Ereal\_2Ereal\_lte \\
& (ap\ (ap\ c\_2Ereal\_2E\_2F\ V1y)\ V0x))\ V2z))))))
\end{aligned} \tag{62}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (\forall V2z \in ty\_2Erealax\_2Ereal. (((p\ (ap\ (ap\ c\_2Ereal\_2Ereal\_lte \\
& (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0)\ V0x)) \wedge (p\ (ap\ (ap\ c\_2Ereal\_2Ereal\_lte \\
& V1y)\ V2z))) \Rightarrow (p\ (ap\ (ap\ c\_2Ereal\_2Ereal\_lte\ (ap\ (ap\ c\_2Erealax\_2Ereal\_mul \\
& V1y)\ V0x))\ (ap\ (ap\ c\_2Erealax\_2Ereal\_mul\ V2z)\ V0x))))))
\end{aligned} \tag{63}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (p\ (ap\ (ap\ c\_2Ereal\_2Ereal\_lte \\
& (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0)\ (ap\ c\_2Ereal\_2Eabs \\
& V0x))))
\end{aligned} \tag{64}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((ap\ c\_2Ereal\_2Eabs\ (ap\ (ap\ c\_2Erealax\_2Ereal\_mul\ V0x)\ V1y)) = \\
& (ap\ (ap\ c\_2Erealax\_2Ereal\_mul\ (ap\ c\_2Ereal\_2Eabs\ V0x))\ (ap\ c\_2Ereal\_2Eabs \\
& V1y))))
\end{aligned} \tag{65}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((\neg(V0x = (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0))) \Leftrightarrow (p\ (ap\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0))\ (ap\ c\_2Ereal\_2Eabs\ V0x)))))) \quad (66)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((\neg(V0x = (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0))) \Rightarrow ((ap\ c\_2Ereal\_2Eabs\ (ap\ c\_2Erealax\_2Einv\ V0x)) = (ap\ c\_2Erealax\_2Einv\ (ap\ c\_2Ereal\_2Eabs\ V0x)))))) \quad (67)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(((ap\ c\_2Ereal\_2Eabs\ V0x) = V0x) \Leftrightarrow (p\ (ap\ (ap\ c\_2Ereal\_2Ereal\_lte\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0))\ V0x)))) \quad (68)$$

Assume the following.

$$(\forall V0c \in ty\_2Erealax\_2Ereal.(\forall V1m \in ty\_2Enum\_2Enum.(\forall V2n \in ty\_2Enum\_2Enum.((ap\ c\_2Ereal\_2Epow\ V0c)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V1m)\ V2n)) = (ap\ (ap\ c\_2Erealax\_2Ereal\_mul\ (ap\ (ap\ c\_2Ereal\_2Epow\ V0c)\ V1m))\ (ap\ (ap\ c\_2Ereal\_2Epow\ V0c)\ V2n)))))) \quad (69)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ c\_2Ereal\_2Epow\ V0x)\ (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO))) = V0x)) \quad (70)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0))\ V0x)) \Rightarrow (\forall V1y \in ty\_2Erealax\_2Ereal.(\exists V2n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ V1y)\ (ap\ (ap\ c\_2Erealax\_2Ereal\_mul\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ V2n))\ V0x))))))) \quad (71)$$

Assume the following.

$$(\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).((p\ (ap\ c\_2Eseq\_2Esummable\ V0f)) \Rightarrow (p\ (ap\ (ap\ c\_2Eseq\_2Esums\ V0f)\ (ap\ c\_2Eseq\_2Esuminf\ V0f)))))) \quad (72)$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V1c \in \\
& ty\_2Erealax\_2Ereal.(\forall V2N \in ty\_2Enum\_2Enum.(((p (ap (ap \\
& c\_2Erealax\_2Ereal\_lt V1c) (ap c\_2Ereal\_2Ereal\_of\_num (ap \\
& c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))))))\wedge \\
& (\forall V3n \in ty\_2Enum\_2Enum.((p (ap (ap c\_2Earithmetic\_2E\_3E\_3D \\
& V3n) V2N))\Rightarrow(p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Ereal\_2Eabs \\
& (ap V0f (ap c\_2Enum\_2ESUC V3n)))) (ap (ap c\_2Erealax\_2Ereal\_mul \\
& V1c) (ap c\_2Ereal\_2Eabs (ap V0f V3n))))))))\Rightarrow(p (ap c\_2Eseq\_2Esummable \\
& V0f))))))
\end{aligned} \tag{73}$$

**Theorem 1**

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal.(p (ap (ap c\_2Eseq\_2Esums (\lambda V1n \in \\
& ty\_2Enum\_2Enum.(ap (ap c\_2Erealax\_2Ereal\_mul (ap (\lambda V2n \in \\
& ty\_2Enum\_2Enum.(ap c\_2Erealax\_2Einv (ap c\_2Ereal\_2Ereal\_of\_num \\
& (ap c\_2Earithmetic\_2EFACT V2n)))) V1n)) (ap (ap c\_2Ereal\_2Epow \\
& V0x) V1n)))) (ap c\_2Etransc\_2Eexp V0x))))
\end{aligned}$$