

thm_2Etransc_2EEXP__NEG__MUL (TM- RQsV3RJqicCAc2DfesxwNCDRHwuS22kM7)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_2E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 8 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Definition 9 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 10 We define c_Enum_ESUC to be $\lambda V0m \in ty_Enum_Enum.(ap\ c_Enum_EABS_num$

Let $c_Earithmic_E_B : \iota$ be given. Assume the following.

$$c_Earithmic_E_B \in ((ty_Enum_Enum^{ty_Enum_Enum})^{ty_Enum_Enum}) \quad (6)$$

Definition 11 We define $c_Earithmic_EBIT1$ to be $\lambda V0n \in ty_Enum_Enum.(ap\ (ap\ c_Earithmic$

Definition 12 We define $c_Earithmic_ENUMERAL$ to be $\lambda V0x \in ty_Enum_Enum.V0x$.

Let $ty_Erealx_Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_Erealx_Ereal \quad (7)$$

Let $c_Ereal_Ereal_of_num : \iota$ be given. Assume the following.

$$c_Ereal_Ereal_of_num \in (ty_Erealx_Ereal^{ty_Enum_Enum}) \quad (8)$$

Let $ty_Ehreal_Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_Ehreal_Ehreal \quad (9)$$

Let $ty_Epair_Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_Epair_Eprod\ A0\ A1) \quad (10)$$

Let $c_Erealx_Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_Erealx_Ereal_REP_CLASS \in ((2^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)})^{ty_Erealx_Ereal}) \quad (11)$$

Definition 13 We define $c_Emin_E_40$ to be $\lambda A.\lambda P \in 2^A$.if $(\exists x \in A.p\ (ap\ P\ x))$ then $(the\ (\lambda x.x \in A \wedge P\ x))$ of type $\iota \Rightarrow \iota$.

Definition 14 We define $c_Erealx_Ereal_REP$ to be $\lambda V0a \in ty_Erealx_Ereal.(ap\ (c_Emin_E_40\ (the\ (\lambda x.x \in A \wedge P\ x))))$

Let $c_Erealx_Etrealm_neg : \iota$ be given. Assume the following.

$$c_Erealx_Etrealm_neg \in ((ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)^{ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal}) \quad (12)$$

Let $c_Erealx_Etrealm_eq : \iota$ be given. Assume the following.

$$c_Erealx_Etrealm_eq \in ((2^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)})^{ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal}) \quad (13)$$

Let $c_Erealx_Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_Erealx_Ereal_ABS_CLASS \in (ty_Erealx_Ereal^{(2^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)})}) \quad (14)$$

Definition 15 We define $c_Erealax_Ereal_ABS$ to be $\lambda V0r \in (ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)$

Definition 16 We define $c_Erealax_Ereal_neg$ to be $\lambda V0T1 \in ty_Erealax_Ereal$. (ap $c_Erealax_Ereal$)

Let $c_Erealax_Etrealm_add : \iota$ be given. Assume the following.

$$c_Erealax_Etrealm_add \in (((ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal) (ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal) (ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal))) (ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal) (ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal) (ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal) (15)$$

Definition 17 We define $c_Erealax_Ereal_add$ to be $\lambda V0T1 \in ty_Erealax_Ereal$. $\lambda V1T2 \in ty_Erealax_Ereal$

Let $c_Ereal_Epow : \iota$ be given. Assume the following.

$$c_Ereal_Epow \in ((ty_Erealax_Ereal^{ty_Eenum_Eenum})^{ty_Erealax_Ereal}) (16)$$

Let $c_Earithmetic_EFACT : \iota$ be given. Assume the following.

$$c_Earithmetic_EFACT \in (ty_Eenum_Eenum^{ty_Eenum_Eenum}) (17)$$

Let $c_Erealax_Etrealm_inv : \iota$ be given. Assume the following.

$$c_Erealax_Etrealm_inv \in ((ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal) (ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal) (ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)) (18)$$

Definition 18 We define $c_Erealax_Ereal_Einv$ to be $\lambda V0T1 \in ty_Erealax_Ereal$. (ap $c_Erealax_Ereal_ABS$)

Let $c_Erealax_Etrealm_mul : \iota$ be given. Assume the following.

$$c_Erealax_Etrealm_mul \in (((ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal) (ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal) (ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal))) (ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal) (ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal) (ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal) (19)$$

Definition 19 We define $c_Erealax_Ereal_mul$ to be $\lambda V0T1 \in ty_Erealax_Ereal$. $\lambda V1T2 \in ty_Erealax_Ereal$

Let $c_Epair_EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_Epair_EABS_prod\ A_27a\ A_27b \in ((ty_Epair_Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) (20)$$

Definition 20 We define $c_Epair_E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b$. (ap $c_Epair_EABS_prod$)

Let $c_Ereal_Esum : \iota$ be given. Assume the following.

$$c_Ereal_Esum \in ((ty_Erealax_Ereal^{(ty_Erealax_Ereal^{ty_Eenum_Eenum})})^{ty_Epair_Eprod\ ty_Eenum_Eenum}) (21)$$

Definition 21 We define $c_Ebool_E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}))$. (ap $V0P$) (ap $c_Emin_E_40$)

Definition 22 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 23 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 24 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21) 2) (\lambda V2t \in 2)))$

Definition 25 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 26 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Erealax_2Etrealt_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealt_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (22)$$

Definition 27 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 28 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 29 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(ap (c_2Ebool_2E_21) 2) (\lambda V3t3 \in 2))))$

Definition 30 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap (ap (ap (c_2Ebool_2ECOND) 2) 2) 2)$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (23)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (24)$$

Definition 31 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a})^{A_27b})$

Let $ty_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Emetric_2Emetric\ A0) \quad (25)$$

Let $c_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Emetric\ A_27a \in ((ty_2Emetric_2Emetric\ A_27a)^{(ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})}) \quad (26)$$

Definition 32 We define $c_2Emetric_2Emr1$ to be $(ap (c_2Emetric_2Emetric\ ty_2Erealax_2Ereal) (ap (c_2Emetric_2Emetric) 2))$

Let $c_2Emetric_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Edist\ A_27a \in ((ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})) \quad (27)$$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Etopology_2Etopology\ A0) \quad (28)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Etopology\ A_27a \in ((ty_2Etopology_2Etopology\ A_27a)^{(2^{(2^A-27a)}})) \quad (29)$$

Definition 33 We define $c_2Emetric_2Emtop$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Emetric_2Emetric\ A_27a).(ap$

Let $c_2Enets_2Eextends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Enets_2Eextends\ A_27a\ A_27b \in (((2^{(ty_2Epair_2Eprod\ (ty_2Etopology_2Etopology\ A_27a)\ (2^{A-27b})^{A-27b}}))^{A_27a})^{(A_27a^{A-27b})}) \quad (30)$$

Definition 34 We define $c_2Eseq_2E_2D_2D_3E$ to be $\lambda V0x \in (ty_2Erealax_2Ereal^{ty_2Eenum_2Eenum}).\lambda V1x$

Definition 35 We define c_2Eseq_2Esums to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Eenum_2Eenum}).\lambda V1s \in ty_2E$

Definition 36 We define $c_2Eseq_2Esuminf$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Eenum_2Eenum}).(ap\ (c_2E$

Definition 37 We define $c_2Etransc_2Eexp$ to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ c_2Eseq_2Esuminf\ (\lambda V1$

Assume the following.

$$True \quad (31)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\\ & (p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Erealax_2Ereal.((ap\ (ap\ c_2Erealax_2Ereal_add \\ & V0x)\ (ap\ c_2Ereal_2Ereal_of_num\ c_2Eenum_2E0)) = V0x)) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} & ((ap\ c_2Etransc_2Eexp\ (ap\ c_2Ereal_2Ereal_of_num\ c_2Eenum_2E0)) = \\ & (ap\ c_2Ereal_2Ereal_of_num\ (ap\ c_2Earithmetic_2ENUMERAL\ (\\ & ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap (ap c_2Erealax_2Ereal_mul (ap c_2Etransc_2Eexp (ap (ap c_2Erealax_2Ereal_add \\
& V0x) V1y))) (ap c_2Etransc_2Eexp (ap c_2Erealax_2Ereal_neg V0x))) = \\
& (ap c_2Etransc_2Eexp V1y))))))
\end{aligned} \tag{35}$$

Theorem 1

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& (ap c_2Etransc_2Eexp V0x)) (ap c_2Etransc_2Eexp (ap c_2Erealax_2Ereal_neg \\
& V0x))) = (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))))))
\end{aligned}$$