thm_2Etransc_2EINTEGRAL__NULL (TMHr3YHspvoHk5gHNDDAw5DQ7FNnZ9VV8bw)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ ($c_2Emin_2E_3D $(2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2EF

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{3}$$

Definition 7 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Definition 8 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{5}$$

Definition 9 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (6)$$

Definition 10 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 11 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \quad (7)$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \quad (8)$$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \quad (9)$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \quad (10)$$

Definition 12 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 13 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap$ (c_2Emin_2E_40 $(t$

Let $c\_2Erealax\_2Etreal\_\_add : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \quad (11)$$

Let $c\_2Erealax\_2Etreal\_\_eq : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \quad (12)$$

Let $c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehr} \quad (13)$$

Definition 14 *We define* c_2Erealax_2Ereal__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty$

Definition 15 *We define* c_2Erealax_2Ereal__add *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Let $c\_2Erealax\_2Etreal\_\_mul : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \quad (14)$$

Definition 16 *We define* c_2Erealax_2Ereal__mul *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Let $c\_2Ereal\_2Esum : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Esum \in ((ty\_2Erealax\_2Ereal^{(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2En} \quad (15)$$

Let $c\_2Etransc\_2Ersum : \iota$ be given. Assume the following.

$$c\_2Etransc\_2Ersum \in ((ty\_2Erealax\_2Ereal^{(ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal})})^{(ty\_2Epair\_2Eprod\ (ty\_2Erea} \quad (16)$$

Let $c\_2Erealax\_2Etreal\_\_neg : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (17)$$

Definition 17 *We define* c_2Erealax_2Ereal__neg *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal.

Definition 18 *We define* c_2Ereal_2Ereal__sub *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (18)$$

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehr} \quad (19)$$

Definition 19 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_$

Definition 20 *We define* c_2Ereal_2Ereal__lte *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Definition 21 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 22 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Definition 23 *We define* c_2Ereal_2Eabs *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (ap$ (c_2Ebool_2ECONI

Let $c\_2Etransc\_2Efine : \iota$ be given. Assume the following.

$$c\_2Etransc\_2Efine \in ((2^{(ty\_2Epair\_2Eprod\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enu} \quad (20)$$

Let $c\_2Etransc\_2Etdiv : \iota$ be given. Assume the following.

$$c\_2Etransc\_2Etdiv \in ((2^{(ty\_2Epair\_2Eprod\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enu} \quad (21)$$

Definition 24 *We define* c_2Etransc_2Egauge *to be* $\lambda V0E \in (2^{ty\_2Erealax\_2Ereal}).\lambda V1g \in (ty\_2Erealax\_2E$

Definition 25 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40

Let $c\_2Etransc\_2EDint : \iota$ be given. Assume the following.

$$c\_2Etransc\_2EDint \in (((2^{ty\_2Erealax\_2Ereal})^{(ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal})})^{(ty\_2Epair\_2Eprod\ ty\_2Erealax} \quad (22)$$

Definition 26 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 27 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 28 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 2$)\ (\lambda V2t \in$

Definition 29 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Definition 30 *We define* c_2Etransc_2Edsize *to be* $\lambda V0D \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(ap\ ($c_2

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \quad (23)$$

Definition 31 *We define* c_2Epair_2E_2C *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ ($c_2

Let $c\_2Etransc\_2Edivision : \iota$ be given. Assume the following.

$$c\_2Etransc\_2Edivision \in ((2^{(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})})^{(ty\_2Epair\_2Eprod\ ty\_2Erealax\_2Ereal\ ty\_2Ereal} \quad (24)$$

Assume the following.

$$True \quad (25)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27b.((ap\ (\lambda V2x \in A\_27b.V0t1)\ V1t2) = V0t1))) \quad (26)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \quad (28)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{29}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{30}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{31}$$

Assume the following.

$$(p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))) \tag{32}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ V0x)\ V0x) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))) \tag{33}$$

Assume the following.

$$((ap\ \mathsf{c\_2Ereal\_2Eabs}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})) \tag{34}$$

Assume the following.

$$((\forall V0n \in ty\_2Enum\_2Enum.(\forall V1f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).((ap\ (ap\ c\_2Ereal\_2Esum\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)\ V0n)\ \mathsf{c\_2Enum\_2E0}))\ V1f) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})))) \wedge (\forall V2n \in ty\_2Enum\_2Enum.(\forall V3m \in ty\_2Enum\_2Enum.(\forall V4f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).((ap\ (ap\ c\_2Ereal\_2Esum\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)\ V2n)\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V3m)))\ V4f) = (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ (ap\ c\_2Ereal\_2Esum\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)\ V2n)\ V3m))\ V4f))\ (ap\ V4f\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V2n)\ V3m)))))))) \tag{35}$$

Assume the following.

$$(\forall V0a \in ty\_2Erealax\_2Ereal.(\forall V1b \in ty\_2Erealax\_2Ereal.(\forall V2D \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V3p \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).((p\ (ap\ (ap\ c\_2Etransc\_2Etdiv\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V0a)\ V1b))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}))\ V2D)\ V3p)))\Leftrightarrow((p\ (ap\ (ap\ c\_2Etransc\_2Edivision\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Erealax\_2Ereal\ ty\_2Erealax\_2Ereal)\ V0a)\ V1b))\ V2D))\wedge(\forall V4n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ V2D\ V4n))\ (ap\ V3p\ V4n)))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ V3p\ V4n))\ (ap\ V2D\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V4n)))))))))))) \tag{36}$$

Assume the following.

$$(\forall V0D \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V1p \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(\forall V2f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).((ap\ (ap\ c\_2Etransc\_2Ersum\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})\ (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}))\ V0D)\ V1p))\ V2f) = (ap\ (ap\ c\_2Ereal\_2Esum\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ ty\_2Enum\_2Enum\ ty\_2Enum\_2Enum)\ \mathsf{c\_2Enum\_2E0})\ (ap\ \mathsf{c\_2Etransc\_2Edsize}\ V0D)))\ (\lambda V3n \in ty\_2Enum\_2Enum.(ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ V2f\ (ap\ V1p\ V3n)))\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ (ap\ V0D\ (ap\ \mathsf{c\_2Enum\_2ESUC}\ V3n)))\ (ap\ V0D\ V3n)))))))) \tag{37}$$

Assume the following.

$$(\forall V0a \in \mathit{ty\_2Erealax\_2Ereal}.(\forall V1b \in \mathit{ty\_2Erealax\_2Ereal}.(\forall V2f \in (\mathit{ty\_2Erealax\_2Ereal}^{\mathit{ty\_2Erealax\_2Ereal}}).(\forall V3k \in \mathit{ty\_2Erealax\_2Ereal}.((p\ (ap\ (ap\ (ap\ \mathit{c\_2Etransc\_2EDint}\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ \mathit{ty\_2Erealax\_2Ereal}\ \mathit{ty\_2Erealax\_2Ereal})\ V0a)\ V1b))\ V2f)\ V3k))\Leftrightarrow(\forall V4e \in \mathit{ty\_2Erealax\_2Ereal}.((p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ (ap\ \mathit{c\_2Ereal\_2Ereal\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0}))\ V4e))\Rightarrow(\exists V5g \in (\mathit{ty\_2Erealax\_2Ereal}^{\mathit{ty\_2Erealax\_2Ereal}}).((p\ (ap\ (ap\ \mathsf{c\_2Etransc\_2Egauge}\ (\lambda V6x \in \mathit{ty\_2Erealax\_2Ereal}.(ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V0a)\ V6x))\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V6x)\ V1b))))\ V5g))\wedge(\forall V7D \in (\mathit{ty\_2Erealax\_2Ereal}^{\mathit{ty\_2Enum\_2Enum}}).(\forall V8p \in (\mathit{ty\_2Erealax\_2Ereal}^{\mathit{ty\_2Enum\_2Enum}}).(((p\ (ap\ (ap\ \mathit{c\_2Etransc\_2Etdiv}\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ \mathit{ty\_2Erealax\_2Ereal}\ \mathit{ty\_2Erealax\_2Ereal})\ V0a)\ V1b))\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (\mathit{ty\_2Erealax\_2Ereal}^{\mathit{ty\_2Enum\_2Enum}})\ (\mathit{ty\_2Erealax\_2Ereal}^{\mathit{ty\_2Enum\_2Enum}}))\ V7D)\ V8p)))\wedge(p\ (ap\ (ap\ \mathit{c\_2Etransc\_2Efine}\ V5g)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (\mathit{ty\_2Erealax\_2Ereal}^{\mathit{ty\_2Enum\_2Enum}})\ (\mathit{ty\_2Erealax\_2Ereal}^{\mathit{ty\_2Enum\_2Enum}}))\ V7D)\ V8p))))\Rightarrow(p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ (ap\ \mathsf{c\_2Ereal\_2Eabs}\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ (ap\ (ap\ \mathit{c\_2Etransc\_2Ersum}\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ (\mathit{ty\_2Erealax\_2Ereal}^{\mathit{ty\_2Enum\_2Enum}})\ (\mathit{ty\_2Erealax\_2Ereal}^{\mathit{ty\_2Enum\_2Enum}}))\ V7D)\ V8p))\ V2f))\ V3k)))\ V4e))))))))))))) \tag{38}$$

Assume the following.

$$(\forall V0D \in (\mathit{ty\_2Erealax\_2Ereal}^{\mathit{ty\_2Enum\_2Enum}}).(\forall V1a \in \mathit{ty\_2Erealax\_2Ereal}.(\forall V2b \in \mathit{ty\_2Erealax\_2Ereal}.((p\ (ap\ (ap\ \mathit{c\_2Etransc\_2Edivision}\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ \mathit{ty\_2Erealax\_2Ereal}\ \mathit{ty\_2Erealax\_2Ereal})\ V1a)\ V2b))\ V0D))\Rightarrow((V1a = V2b)\Leftrightarrow((ap\ \mathsf{c\_2Etransc\_2Edsize}\ V0D) = \mathsf{c\_2Enum\_2E0})))))) \tag{39}$$

Theorem 1

$$(\forall V0f \in (\mathit{ty\_2Erealax\_2Ereal}^{\mathit{ty\_2Erealax\_2Ereal}}).(\forall V1a \in \mathit{ty\_2Erealax\_2Ereal}.(p\ (ap\ (ap\ (ap\ \mathit{c\_2Etransc\_2EDint}\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ \mathit{ty\_2Erealax\_2Ereal}\ \mathit{ty\_2Erealax\_2Ereal})\ V1a)\ V1a))\ V0f)\ (ap\ \mathit{c\_2Ereal\_2Ereal\_\_of\_\_num}\ \mathsf{c\_2Enum\_2E0})))))$$