

thm_2Etrasc_2EMCLAURIN
(TMK1JbdML7KT5T9CvDkz9H55mJMgSBdU2)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{2}$$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_F))$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (omega^{ty_2Enum_2Enum}) \tag{3}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (omega^{omega}) \tag{4}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \tag{5}$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 11 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 12 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 13 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{6}$$

Definition 14 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealx_2Ereal \tag{7}$$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}) \tag{8}$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{9}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{10}$$

Let $c_2Erealx_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealx_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealx_2Ereal}) \tag{11}$$

Definition 15 We define $c_2Erealx_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealx_2Ereal.(ap\ (c_2Emin_2E_40\ (t$

Let $c_2Erealx_2Etrealm_add : \iota$ be given. Assume the following.

$$c_2Erealx_2Etrealm_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal})^{ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal}) \tag{12}$$

Let $c_2Erealx_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealx_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal}) \tag{13}$$

Let $c_2Erealx_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealx_2Ereal_ABS_CLASS \in (ty_2Erealx_2Ereal^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})}) \tag{14}$$

Definition 16 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 17 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Erealax_2Etrealm_neg \in & ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal \\ & ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \end{aligned} \quad (15)$$

Definition 18 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal$

Definition 19 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Erealax_2Etrealm_inv : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Erealax_2Etrealm_inv \in & ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal \\ & ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \end{aligned} \quad (16)$$

Definition 20 We define $c_2Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal_ABS$

Let $c_2Erealax_2Etrealm_mul : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Erealax_2Etrealm_mul \in & (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal \\ & ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \end{aligned} \quad (17)$$

Definition 21 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 22 We define c_2Ereal_2E2F to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Erealax_2Etrealm_lt : \iota$ be given. Assume the following.

$$\begin{aligned} c_2Erealax_2Etrealm_lt \in & ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \end{aligned} \quad (18)$$

Definition 23 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 24 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 25 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.$

Definition 26 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ (ap\ (ap\ (c_2Ebool_2ECOND$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow & c_2Epair_2ESND \\ & A_27a\ A_27b \in (A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \end{aligned} \quad (19)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow & c_2Epair_2EFST \\ & A_27a\ A_27b \in (A_27a)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \end{aligned} \quad (20)$$

Definition 27 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda A_27c : \iota. \lambda V0f \in ((A_27c)^{A_27a})$.
Let $ty_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Emetric_2Emetric\ A0) \quad (21)$$

Let $c_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Emetric\ A_27a \in ((ty_2Emetric_2Emetric\ A_27a)^{(ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})}) \quad (22)$$

Definition 28 We define $c_2Emetric_2Emr1$ to be $(ap\ (c_2Emetric_2Emetric\ ty_2Erealax_2Ereal)\ (ap\ (c_2Epair_2EABS_prod\ : \iota \Rightarrow \iota \Rightarrow \iota)))$.
Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (23)$$

Definition 29 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap\ (c_2Emetric_2Emetric\ : \iota \Rightarrow \iota))$.
Let $c_2Enets_2Etendsto : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Enets_2Etendsto\ A_27a \in (((2^{A_27a})^{A_27a})^{(ty_2Epair_2Eprod\ (ty_2Emetric_2Emetric\ : \iota \Rightarrow \iota))}) \quad (24)$$

Let $c_2Emetric_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Edist\ A_27a \in ((ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})^{(c_2Emetric_2Edist\ : \iota \Rightarrow \iota)}) \quad (25)$$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Etopology_2Etopology\ A0) \quad (26)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Etopology\ A_27a \in ((ty_2Etopology_2Etopology\ A_27a)^{(2^{(2^{A_27a})})}) \quad (27)$$

Definition 30 We define $c_2Emetric_2Emtop$ to be $\lambda A_27a : \iota. \lambda V0m \in (ty_2Emetric_2Emetric\ A_27a). (ap\ (c_2Emetric_2Emetric\ : \iota \Rightarrow \iota))$.

Let $c_2Enets_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Enets_2Etends\ A_27a\ A_27b \in (((2^{(ty_2Epair_2Eprod\ (ty_2Etopology_2Etopology\ A_27a)\ ((2^{A_27b})^{A_27b}))})^{A_27a})^{(A_27a)^{A_27b}}) \quad (28)$$

Definition 31 We define $c_2Elim_2Etends_real_real$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal})$.

Definition 32 We define c_2Elim_2Ediff to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). \lambda V1l \in ty_2Erealax_2Ereal$.

Definition 33 We define $c_2Elim_2Edifferentiable$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).\lambda V$

Definition 34 We define $c_2Elim_2Econtl$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).\lambda V1x \in ty$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (29)$$

Let $c_2Earithmetic_2EFACT : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EFACT \in (ty_2Enum_2Enum^{ty_2Enum_2Enum}) \quad (30)$$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (31)$$

Definition 35 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic$

Let $c_2Ereal_2Epow : \iota$ be given. Assume the following.

$$c_2Ereal_2Epow \in ((ty_2Erealax_2Ereal^{ty_2Enum_2Enum})^{ty_2Erealax_2Ereal}) \quad (32)$$

Definition 36 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Definition 37 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic$

Definition 38 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Ereal_2Esum : \iota$ be given. Assume the following.

$$c_2Ereal_2Esum \in ((ty_2Erealax_2Ereal^{(ty_2Erealax_2Ereal^{ty_2Enum_2Enum})})^{(ty_2Epair_2Eprod ty_2Enum_2Enum)}) \quad (33)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & ((ap (ap c_2Earithmetic_2E_2B c_2Enum_2E0) V0m) = V0m) \wedge (((ap (\\ & ap c_2Earithmetic_2E_2B V0m) c_2Enum_2E0) = V0m) \wedge (((ap (ap c_2Earithmetic_2E_2B \\ & (ap c_2Enum_2ESUC V0m)) V1n) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B \\ & V0m) V1n))) \wedge ((ap (ap c_2Earithmetic_2E_2B V0m) (ap c_2Enum_2ESUC \\ & V1n)) = (ap c_2Enum_2ESUC (ap (ap c_2Earithmetic_2E_2B V0m) V1n))))))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & (ap (ap c_2Earithmetic_2E_2B V0m) V1n) = (ap (ap c_2Earithmetic_2E_2B \\ & V1n) V0m)))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad \forall V2p \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2B V0m) \\
& (ap (ap c_2Earithmetic_2E_2B V1n) V2p)) = (ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap (ap c_2Earithmetic_2E_2B V0m) V1n)) V2p))))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. ((V0m = c_2Enum_2E0) \vee (\exists V1n \in \\
& \quad ty_2Enum_2Enum. (V0m = (ap c_2Enum_2ESUC V1n))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad \forall V2p \in ty_2Enum_2Enum. (((p (ap (ap c_2Eprim_rec_2E_3C \\
& V0m) V1n)) \wedge (p (ap (ap c_2Eprim_rec_2E_3C V1n) V2p))) \Rightarrow (p (ap (ap \\
& \quad c_2Eprim_rec_2E_3C V0m) V2p))))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (((ap (ap c_2Earithmetic_2E_2D \\
& c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge ((ap (ap c_2Earithmetic_2E_2D \\
& \quad V0m) c_2Enum_2E0) = V0m)))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. ((ap c_2Enum_2ESUC V0m) = (ap (ap \\
& c_2Earithmetic_2E_2B V0m) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. ((ap (ap c_2Earithmetic_2E_2D (\\
& ap c_2Enum_2ESUC V0m)) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) = V0m))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad (p (ap (ap c_2Eprim_rec_2E_3C V1n) V0m)) \Rightarrow (\exists V2p \in ty_2Enum_2Enum. \\
& (V0m = (ap (ap c_2Earithmetic_2E_2B V1n) (ap (ap c_2Earithmetic_2E_2B \\
& \quad V2p) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))))))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0a \in ty_2Enum_2Enum. (\forall V1c \in ty_2Enum_2Enum. (\\
& (ap (ap c_2Earithmetic_2E_2D (ap (ap c_2Earithmetic_2E_2B V0a) \\
& \quad V1c)) V1c) = V0a))
\end{aligned} \tag{43}$$

Assume the following.

$$(\forall V0c \in ty_2Enum_2Enum.((ap (ap c_2Earithmic_2E_2D V0c) V0c) = c_2Enum_2E0)) \quad (44)$$

Assume the following.

$$(((ap c_2Earithmic_2EFACT c_2Enum_2E0) = (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO))) \wedge (\forall V0n \in ty_2Enum_2Enum.((ap c_2Earithmic_2EFACT (ap c_2Enum_2ESUC V0n)) = (ap (ap c_2Earithmic_2E_2A (ap c_2Enum_2ESUC V0n)) (ap c_2Earithmic_2EFACT V0n)))))) \quad (45)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmic_2EFACT V0n)))) \quad (46)$$

Assume the following.

$$True \quad (47)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2))))) \quad (48)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \quad (49)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \wedge ((p V1t2) \wedge (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \wedge (p V2t3))))) \quad (50)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t))))) \quad (51)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t))))) \quad (52)$$

Assume the following.

$$(\forall V0t \in 2.((\neg (\neg (p V0t))) \Leftrightarrow (p V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (53)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. ((V0x = V0x) \Leftrightarrow True)) \quad (54)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (55)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (56)$$

Assume the following.

$$(\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). (\forall V1l \in ty_2Erealax_2Ereal. (\forall V2m \in ty_2Erealax_2Ereal. (\forall V3x \in ty_2Erealax_2Ereal. (((p\ (ap\ (ap\ (ap\ c_2Elim_2Ediff1\ V0f)\ V1l)\ V3x)) \wedge (p\ (ap\ (ap\ (ap\ c_2Elim_2Ediff1\ V0f)\ V2m)\ V3x))) \Rightarrow (V1l = V2m)))))) \quad (57)$$

Assume the following.

$$(\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). (\forall V1l \in ty_2Erealax_2Ereal. (\forall V2x \in ty_2Erealax_2Ereal. ((p\ (ap\ (ap\ c_2Elim_2Ediff1\ V0f)\ V1l)\ V2x)) \Rightarrow (p\ (ap\ (ap\ c_2Elim_2Econt1\ V0f)\ V2x)))))) \quad (58)$$

Assume the following.

$$(\forall V0k \in ty_2Erealax_2Ereal. (\forall V1x \in ty_2Erealax_2Ereal. (p\ (ap\ (ap\ (ap\ c_2Elim_2Ediff1\ (\lambda V2x \in ty_2Erealax_2Ereal. V0k))\ (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))\ V1x)))) \quad (59)$$

Assume the following.

$$(\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). (\forall V1g \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). (\forall V2l \in ty_2Erealax_2Ereal. (\forall V3m \in ty_2Erealax_2Ereal. (\forall V4x \in ty_2Erealax_2Ereal. (((p\ (ap\ (ap\ (ap\ c_2Elim_2Ediff1\ V0f)\ V2l)\ V4x)) \wedge (p\ (ap\ (ap\ (ap\ c_2Elim_2Ediff1\ V1g)\ V3m)\ V4x))) \Rightarrow (p\ (ap\ (ap\ (ap\ c_2Elim_2Ediff1\ (\lambda V5x \in ty_2Erealax_2Ereal. (ap\ (ap\ c_2Erealax_2Ereal_add\ (ap\ V0f\ V5x))\ (ap\ V1g\ V5x))))\ (ap\ (ap\ c_2Erealax_2Ereal_add\ V2l)\ V3m))\ V4x)))))) \quad (60)$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1g \in \\
& (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V2l \in ty_2Erealax_2Ereal. \\
& (\forall V3m \in ty_2Erealax_2Ereal.(\forall V4x \in ty_2Erealax_2Ereal. \\
& (((p (ap (ap (ap (ap c_2Elim_2Ediff1 V0f) V2l) V4x)) \wedge (p (ap (ap (ap c_2Elim_2Ediff1 \\
V1g) V3m) V4x))) \Rightarrow (p (ap (ap (ap c_2Elim_2Ediff1 (\lambda V5x \in ty_2Erealax_2Ereal. \\
(ap (ap c_2Erealax_2Ereal_mul (ap V0f V5x)) (ap V1g V5x)))) (ap \\
(ap c_2Erealax_2Ereal_add (ap (ap c_2Erealax_2Ereal_mul V2l) \\
(ap V1g V4x))) (ap (ap c_2Erealax_2Ereal_mul V3m) (ap V0f V4x)))) \\
V4x)))))))))
\end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1g \in \\
& (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V2l \in ty_2Erealax_2Ereal. \\
& (\forall V3m \in ty_2Erealax_2Ereal.(\forall V4x \in ty_2Erealax_2Ereal. \\
& (((p (ap (ap (ap c_2Elim_2Ediff1 V0f) V2l) V4x)) \wedge (p (ap (ap (ap c_2Elim_2Ediff1 \\
V1g) V3m) V4x))) \Rightarrow (p (ap (ap (ap c_2Elim_2Ediff1 (\lambda V5x \in ty_2Erealax_2Ereal. \\
(ap (ap c_2Ereal_2Ereal_sub (ap V0f V5x)) (ap V1g V5x)))) (ap (ap \\
c_2Ereal_2Ereal_sub V2l) V3m)) V4x)))))))))
\end{aligned} \tag{62}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1g \in \\
& (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V2l \in ty_2Erealax_2Ereal. \\
& (\forall V3m \in ty_2Erealax_2Ereal.(\forall V4x \in ty_2Erealax_2Ereal. \\
& (((p (ap (ap (ap c_2Elim_2Ediff1 V0f) V2l) (ap V1g V4x))) \wedge (p (ap (\\
ap (ap c_2Elim_2Ediff1 V1g) V3m) V4x))) \Rightarrow (p (ap (ap (ap c_2Elim_2Ediff1 \\
(\lambda V5x \in ty_2Erealax_2Ereal.(ap V0f (ap V1g V5x)))) (ap (ap c_2Erealax_2Ereal_mul \\
V2l) V3m)) V4x)))))))))
\end{aligned} \tag{63}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.(p (ap (ap (ap c_2Elim_2Ediff1 \\
& (\lambda V1x \in ty_2Erealax_2Ereal.V1x)) (ap c_2Ereal_2Ereal_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \\
& V0x)))
\end{aligned} \tag{64}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum.(\forall V1x \in ty_2Erealax_2Ereal. \\
& (p (ap (ap (ap c_2Elim_2Ediff1 (\lambda V2x \in ty_2Erealax_2Ereal.(\\
ap (ap c_2Ereal_2Epow V2x) V0n))) (ap (ap c_2Erealax_2Ereal_mul \\
(ap c_2Ereal_2Ereal_of_num V0n)) (ap (ap c_2Ereal_2Epow V1x) \\
(ap (ap c_2Earithmetic_2E_2D V0n) (ap c_2Earithmetic_2ENUMERAL \\
(ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) V1x)))
\end{aligned} \tag{65}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1g \in \\
& (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V2l \in ty_2Erealax_2Ereal. \\
& (\forall V3m \in ty_2Erealax_2Ereal.(\forall V4x \in ty_2Erealax_2Ereal. \\
& (((p (ap (ap (ap c_2Elim_2Ediff1 V0f) V2l) V4x)) \wedge ((p (ap (ap (ap c_2Elim_2Ediff1 \\
& V1g) V3m) V4x)) \wedge (\neg((ap V1g V4x) = (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)))))) \Rightarrow (p (ap (ap (ap c_2Elim_2Ediff1 (\lambda V5x \in ty_2Erealax_2Ereal. \\
& (ap (ap c_2Ereal_2E_2F (ap V0f V5x)) (ap V1g V5x)))) (ap (ap c_2Ereal_2E_2F \\
& (ap (ap c_2Ereal_2Ereal_sub (ap (ap c_2Erealax_2Ereal_mul V2l) \\
& (ap V1g V4x))) (ap (ap c_2Erealax_2Ereal_mul V3m) (ap V0f V4x)))) \\
& (ap (ap c_2Ereal_2Epow (ap V1g V4x)) (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO)))))) V4x))))))))) \\
& \hspace{15em} (66)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in ((ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal})^{ty_2Enum_2Enum}). \\
& (\forall V1f_27 \in ((ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal})^{ty_2Enum_2Enum}). \\
& (\forall V2m \in ty_2Enum_2Enum.(\forall V3n \in ty_2Enum_2Enum.(\\
& \forall V4x \in ty_2Erealax_2Ereal.((\forall V5r \in ty_2Enum_2Enum. \\
& (((p (ap (ap c_2Earithmetic_2E_3C_3D V2m) V5r)) \wedge (p (ap (ap c_2Eprim_rec_2E_3C \\
& V5r) (ap (ap c_2Earithmetic_2E_2B V2m) V3n)))))) \Rightarrow (p (ap (ap (ap c_2Elim_2Ediff1 \\
& (\lambda V6x \in ty_2Erealax_2Ereal.(ap (ap V0f V5r) V6x))) (ap (ap V1f_27 \\
& V5r) V4x)) V4x)))) \Rightarrow (p (ap (ap (ap c_2Elim_2Ediff1 (\lambda V7x \in ty_2Erealax_2Ereal. \\
& (ap (ap c_2Ereal_2Esum (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum \\
& ty_2Enum_2Enum) V2m) V3n)) (\lambda V8n \in ty_2Enum_2Enum.(ap (ap V0f \\
& V8n) V7x)))))) (ap (ap c_2Ereal_2Esum (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum \\
& ty_2Enum_2Enum) V2m) V3n)) (\lambda V9r \in ty_2Enum_2Enum.(ap (ap V1f_27 \\
& V9r) V4x)))) V4x))))))))) \\
& \hspace{15em} (67)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1a \in \\
& ty_2Erealax_2Ereal.(\forall V2b \in ty_2Erealax_2Ereal.(((p (\\
& ap (ap c_2Erealax_2Ereal_lt V1a) V2b)) \wedge (((ap V0f V1a) = (ap V0f \\
& V2b)) \wedge ((\forall V3x \in ty_2Erealax_2Ereal.(((p (ap (ap c_2Ereal_2Ereal_lte \\
& V1a) V3x)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte V3x) V2b)))) \Rightarrow (p (ap (\\
& ap c_2Elim_2Econt1 V0f) V3x)))) \wedge (\forall V4x \in ty_2Erealax_2Ereal. \\
& (((p (ap (ap c_2Erealax_2Ereal_lt V1a) V4x)) \wedge (p (ap (ap c_2Erealax_2Ereal_lt \\
& V4x) V2b)))) \Rightarrow (p (ap (ap c_2Elim_2Edifferentiable V0f) V4x)))))) \Rightarrow \\
& (\exists V5z \in ty_2Erealax_2Ereal.((p (ap (ap c_2Erealax_2Ereal_lt \\
& V1a) V5z)) \wedge ((p (ap (ap c_2Erealax_2Ereal_lt V5z) V2b)) \wedge (p (ap \\
& (ap (ap c_2Elim_2Ediff1 V0f) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) \\
& V5z))))))))) \\
& \hspace{15em} (68)
\end{aligned}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\neg((ap\ c_2Enum_2ESUC\ V0n) = c_2Enum_2E0))) \quad (69)$$

Assume the following.

$$(\forall V0P \in (2^{ty_2Enum_2Enum}). (((p\ (ap\ V0P\ c_2Enum_2E0)) \wedge (\forall V1n \in ty_2Enum_2Enum. ((p\ (ap\ V0P\ V1n)) \Rightarrow (p\ (ap\ V0P\ (ap\ c_2Enum_2ESUC\ V1n)))))) \Rightarrow (\forall V2n \in ty_2Enum_2Enum. (p\ (ap\ V0P\ V2n)))))) \quad (70)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (\neg(p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0n)\ V0n)))) \quad (71)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ c_2Enum_2E0)\ (ap\ c_2Enum_2ESUC\ V0n)))) \quad (72)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum. (p\ (ap\ (ap\ c_2Eprim_rec_2E_3C\ V0n)\ (ap\ c_2Enum_2ESUC\ V0n)))) \quad (73)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. ((ap\ (ap\ c_2Erealax_2Ereal_add\ V0x)\ V1y) = (ap\ (ap\ c_2Erealax_2Ereal_add\ V1y)\ V0x)))) \quad (74)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal. ((ap\ (ap\ c_2Erealax_2Ereal_add\ (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))\ V0x) = V0x)) \quad (75)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. (\forall V2z \in ty_2Erealax_2Ereal. (((p\ (ap\ (ap\ c_2Erealax_2Ereal_lt\ V0x)\ V1y)) \wedge (p\ (ap\ (ap\ c_2Erealax_2Ereal_lt\ V1y)\ V2z))) \Rightarrow (p\ (ap\ (ap\ c_2Erealax_2Ereal_lt\ V0x)\ V2z)))))) \quad (76)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. ((ap\ (ap\ c_2Erealax_2Ereal_mul\ V0x)\ V1y) = (ap\ (ap\ c_2Erealax_2Ereal_mul\ V1y)\ V0x)))) \quad (77)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
V0x) (ap (ap c_2Erealax_2Ereal_mul V1y) V2z)) = (ap (ap c_2Erealax_2Ereal_mul \\
& (ap (ap c_2Erealax_2Ereal_mul V0x) V1y)) V2z))))))
\end{aligned} \tag{78}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL (\\
ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) V0x) = V0x))
\end{aligned} \tag{79}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((\neg(V0x = (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0))) \Rightarrow ((ap (ap c_2Erealax_2Ereal_mul (ap c_2Erealax_2Einv \\
V0x)) V0x) = (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))))))
\end{aligned} \tag{80}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add \\
& V0x) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) = V0x))
\end{aligned} \tag{81}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
V0x) (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) = V0x))
\end{aligned} \tag{82}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (((ap (ap c_2Erealax_2Ereal_add V0x) V1y) = V1y) \Leftrightarrow (V0x = (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0))))))
\end{aligned} \tag{83}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x) = (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)))
\end{aligned} \tag{84}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
V0x) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) = (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)))
\end{aligned} \tag{85}$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. ((p (ap (ap c_2Erealax_2Ereal_lt V0x) V1y)) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte V0x) V1y)))))) \quad (86)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. (\forall V2z \in ty_2Erealax_2Ereal.(((p (ap (ap c_2Ereal_2Ereal_lte V0x) V1y)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte V1y) V2z))) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte V0x) V2z))))))) \quad (87)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Ereal_2Ereal_sub V0x) V0x) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))) \quad (88)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. (((ap (ap c_2Ereal_2Ereal_sub V0x) V1y) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) \Leftrightarrow (V0x = V1y)))) \quad (89)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((\neg (V0x = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))) \Rightarrow ((ap c_2Erealax_2Einv (ap c_2Erealax_2Einv V0x)) = V0x))) \quad (90)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(((p (ap (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x)) \Rightarrow (p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) (ap c_2Erealax_2Einv V0x)))))) \quad (91)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.((p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num V0m)) (ap c_2Ereal_2Ereal_of_num V1n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n)))) \quad (92)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(((ap c_2Ereal_2Ereal_of_num V0m) = (ap c_2Ereal_2Ereal_of_num V1n)) \Leftrightarrow (V0m = V1n)))) \quad (93)$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\
& \quad (ap (ap c_2Erealx_2Ereal_mul (ap c_2Ereal_2Ereal_of_num \\
& \quad V0m)) (ap c_2Ereal_2Ereal_of_num V1n)) = (ap c_2Ereal_2Ereal_of_num \\
& \quad (ap (ap c_2Earithmetic_2E_2A V0m) V1n))))))
\end{aligned} \tag{94}$$

Assume the following.

$$\begin{aligned}
& ((ap c_2Erealx_2Einv (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) = (ap c_2Ereal_2Ereal_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))))
\end{aligned} \tag{95}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealx_2Ereal. ((ap (ap c_2Ereal_2E_2F (ap \\
& c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x) = (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)))
\end{aligned} \tag{96}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealx_2Ereal. ((ap (ap c_2Ereal_2Ereal_sub \\
& V0x) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) = V0x))
\end{aligned} \tag{97}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealx_2Ereal. (\forall V1y \in ty_2Erealx_2Ereal. \\
& (\forall V2z \in ty_2Erealx_2Ereal. ((V0x = (ap (ap c_2Ereal_2Ereal_sub \\
& V1y) V2z)) \Leftrightarrow ((ap (ap c_2Erealx_2Ereal_add V0x) V2z) = V1y))))))
\end{aligned} \tag{98}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealx_2Ereal. (\forall V1y \in ty_2Erealx_2Ereal. \\
& (\forall V2z \in ty_2Erealx_2Ereal. (((ap (ap c_2Ereal_2Ereal_sub \\
& V0x) V1y) = V2z) \Leftrightarrow (V0x = (ap (ap c_2Erealx_2Ereal_add V2z) V1y))))))
\end{aligned} \tag{99}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealx_2Ereal. (\forall V1y \in ty_2Erealx_2Ereal. \\
& (((\neg(V0x = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))) \wedge (\neg(V1y = \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)))) \Rightarrow ((ap c_2Erealx_2Einv \\
& (ap (ap c_2Erealx_2Ereal_mul V0x) V1y)) = (ap (ap c_2Erealx_2Ereal_mul \\
& (ap c_2Erealx_2Einv V0x)) (ap c_2Erealx_2Einv V1y))))))
\end{aligned} \tag{100}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealx_2Ereal. ((p (ap (ap c_2Erealx_2Ereal_lt \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x)) \Rightarrow (\neg(V0x = (ap \\
& c_2Ereal_2Ereal_of_num c_2Enum_2E0))))))
\end{aligned} \tag{101}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\neg((ap\ c_2Ereal_2Ereal_of_num \\
& (ap\ c_2Earithmetic_2EFACT\ V0n)) = (ap\ c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0))))
\end{aligned} \tag{102}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0x \in ty_2Erealax_2Ereal. ((ap\ (ap\ c_2Ereal_2Epow\ V0x) \\
& c_2Enum_2E0) = (ap\ c_2Ereal_2Ereal_of_num\ (ap\ c_2Earithmetic_2ENUMERAL \\
& \quad (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO)))))) \wedge (\forall V1x \in \\
& ty_2Erealax_2Ereal. (\forall V2n \in ty_2Enum_2Enum. ((ap\ (ap\ c_2Ereal_2Epow \\
& \quad V1x)\ (ap\ c_2Enum_2ESUC\ V2n)) = (ap\ (ap\ c_2Erealax_2Ereal_mul\ V1x) \\
& \quad (ap\ (ap\ c_2Ereal_2Epow\ V1x)\ V2n))))))
\end{aligned} \tag{103}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((ap\ (ap\ c_2Ereal_2Epow\ (ap\ c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0))\ (ap\ c_2Enum_2ESUC\ V0n)) = (ap\ c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0)))
\end{aligned} \tag{104}$$

Assume the following.

$$\begin{aligned}
& (\forall V0c \in ty_2Erealax_2Ereal. (\forall V1n \in ty_2Enum_2Enum. \\
& \quad ((\neg(V0c = (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))) \Rightarrow (\neg((ap \\
& (ap\ c_2Ereal_2Epow\ V0c)\ V1n) = (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))))))
\end{aligned} \tag{105}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap\ (ap\ c_2Ereal_2Epow\ V0x) \\
& (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO))) = \\
& \quad (ap\ (ap\ c_2Erealax_2Ereal_mul\ V0x)\ V0x)))
\end{aligned} \tag{106}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum. (\forall V1f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}). \\
& \quad ((ap\ (ap\ c_2Ereal_2Esum\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Enum_2Enum \\
& \quad ty_2Enum_2Enum)\ V0n)\ c_2Enum_2E0))\ V1f) = (ap\ c_2Ereal_2Ereal_of_num \\
& \quad \quad c_2Enum_2E0)))))) \wedge (\forall V2n \in ty_2Enum_2Enum. (\forall V3m \in \\
& \quad ty_2Enum_2Enum. (\forall V4f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}). \\
& \quad ((ap\ (ap\ c_2Ereal_2Esum\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Enum_2Enum \\
& \quad ty_2Enum_2Enum)\ V2n)\ (ap\ c_2Enum_2ESUC\ V3m)))\ V4f) = (ap\ (ap\ c_2Erealax_2Ereal_add \\
& \quad (ap\ (ap\ c_2Ereal_2Esum\ (ap\ (ap\ (c_2Epair_2E_2C\ ty_2Enum_2Enum \\
& \quad ty_2Enum_2Enum)\ V2n)\ V3m))\ V4f))\ (ap\ V4f\ (ap\ (ap\ c_2Earithmetic_2E_2B \\
& \quad \quad V2n)\ V3m)))))))))
\end{aligned} \tag{107}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(\forall V1n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Ereal_2Esum (ap (ap (c_2Epair_2E_2C \\
& \quad ty_2Enum_2Enum ty_2Enum_2Enum) V1n) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) V0f) = (\\
& \quad \quad ap V0f V1n))))
\end{aligned} \tag{108}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(\forall V1n \in \\
& \quad ty_2Enum_2Enum.(\forall V2k \in ty_2Enum_2Enum.((ap (ap c_2Ereal_2Esum \\
& \quad (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum ty_2Enum_2Enum) c_2Enum_2E0 \\
& \quad V1n)) (\lambda V3m \in ty_2Enum_2Enum.(ap V0f (ap (ap c_2Earithmetic_2E_2B \\
& \quad V3m) V2k)))) = (ap (ap c_2Ereal_2Ereal_sub (ap (ap c_2Ereal_2Esum \\
& \quad (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum ty_2Enum_2Enum) c_2Enum_2E0 \\
& \quad (ap (ap c_2Earithmetic_2E_2B V1n) V2k))) V0f)) (ap (ap c_2Ereal_2Esum \\
& \quad (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum ty_2Enum_2Enum) c_2Enum_2E0 \\
& \quad V2k)) V0f))))))
\end{aligned} \tag{109}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\
& \quad (ap (ap c_2Ereal_2Esum (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum \\
& \quad ty_2Enum_2Enum) V0m) V1n)) (\lambda V2r \in ty_2Enum_2Enum.(ap c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0))) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))))
\end{aligned} \tag{110}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \tag{111}$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \tag{112}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))
\end{aligned} \tag{113}$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& \quad ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False))))
\end{aligned} \tag{114}$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \tag{115}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \Leftrightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee ((p \ V1q) \vee (p \ V2r))) \wedge (((p \ V0p) \vee ((\neg(\\
& p \ V2r)) \vee (\neg(p \ V1q)))) \wedge (((p \ V1q) \vee ((\neg(p \ V2r)) \vee (\neg(p \ V0p)))) \wedge ((p \ V2r) \vee \\
& ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{116}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \wedge (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee ((\neg(p \ V1q)) \vee (\neg(p \ V2r)))) \wedge (((p \ V1q) \vee \\
& (\neg(p \ V0p))) \wedge ((p \ V2r) \vee (\neg(p \ V0p)))))))))
\end{aligned} \tag{117}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \ V0p) \Leftrightarrow (\\
& (p \ V1q) \Rightarrow (p \ V2r))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge (((p \ V0p) \vee (\neg(p \ V2r))) \wedge ((\\
& \neg(p \ V1q)) \vee ((p \ V2r) \vee (\neg(p \ V0p))))))))))
\end{aligned} \tag{118}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee \\
& (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))
\end{aligned} \tag{119}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p)))
\end{aligned} \tag{120}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (\neg(p \ V1q))))
\end{aligned} \tag{121}$$

Theorem 1

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1diff \in \\
& ((ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal})ty_2Enum_2Enum). \\
& (\forall V2h \in ty_2Erealax_2Ereal.(\forall V3n \in ty_2Enum_2Enum. \\
& ((p (ap (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) V2h)) \wedge (p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) \\
& V3n)) \wedge ((ap V1diff c_2Enum_2E0) = V0f) \wedge (\forall V4m \in ty_2Enum_2Enum. \\
& (\forall V5t \in ty_2Erealax_2Ereal.(((p (ap (ap c_2Eprim_rec_2E_3C \\
& V4m) V3n)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) V5t)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte V5t) V2h)))))) \Rightarrow \\
& (p (ap (ap (ap c_2Elim_2Ediff (ap V1diff V4m)) (ap (ap V1diff (ap \\
& c_2Enum_2ESUC V4m)) V5t)) V5t)))))) \Rightarrow (\exists V6t \in ty_2Erealax_2Ereal. \\
& ((p (ap (ap c_2Erealax_2Ereal_lt (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) V6t)) \wedge (p (ap (ap c_2Erealax_2Ereal_lt V6t) V2h)) \wedge \\
& ((ap V0f V2h) = (ap (ap c_2Erealax_2Ereal_add (ap (ap c_2Ereal_2Esum \\
& (ap (ap (c_2Epair_2E_2C ty_2Enum_2Enum ty_2Enum_2Enum) c_2Enum_2E0) \\
& V3n)) (\lambda V7m \in ty_2Enum_2Enum.(ap (ap c_2Erealax_2Ereal_mul \\
& (ap (ap c_2Ereal_2E_2F (ap (ap V1diff V7m) (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0))) (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2EFACT \\
& V7m)))) (ap (ap c_2Ereal_2Epow V2h) V7m)))))) (ap (ap c_2Erealax_2Ereal_mul \\
& (ap (ap c_2Ereal_2E_2F (ap (ap V1diff V3n) V6t)) (ap c_2Ereal_2Ereal_of_num \\
& (ap c_2Earithmetic_2EFACT V3n)))) (ap (ap c_2Ereal_2Epow V2h) \\
& V3n)))))))))
\end{aligned}$$