

thm_2Etransc_2ESIN__COS__ADD (TMX- PvDyGswP7xRRZGsRotgMAYuraR1ZThcU)

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Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Earithmetic_2EODD : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EODD \in (2^{ty_2Enum_2Enum}) \tag{3}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow p \Rightarrow Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_F$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (omega^{omega}) \tag{5}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \tag{6}$$

Definition 8 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Definition 9 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.\text{if } (\exists x \in A.p\ (ap\ P\ x))$ then (the $(\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$).

Definition 10 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 11 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 12 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 13 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 14 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 15 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{7}$$

Definition 16 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 17 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 18 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ (ap\ (ap\ (c_2Ebool_2E$

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{8}$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{9}$$

Definition 19 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{10}$$

Definition 20 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 21 We define $c_2Enumeral_2EiDUB$ to be $\lambda V0x \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Let $c_2Enumeral_2EiSUB : \iota$ be given. Assume the following.

$$c_2Enumeral_2EiSUB \in (((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum})^2 \quad (11)$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum)^{ty_2Enum_2Enum})^{ty_2Enum_2Enum} \quad (12)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (13)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (14)$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (15)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (16)$$

Definition 22 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E_40\ (t$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (17)$$

Let $c_2Erealax_2Etrealmul_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (18)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (19)$$

Definition 23 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 24 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Erealax_2Etrealmul_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (20)$$

Definition 25 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$.
Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)\ (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)) \quad (21)$$

Definition 26 We define $c_2Erealax_2Ereal_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal$.

Definition 27 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$.

Let $c_2Ereal_2Epow : \iota$ be given. Assume the following.

$$c_2Ereal_2Epow \in ((ty_2Erealax_2Ereal^{ty_2Enum_2Enum})\ ty_2Erealax_2Ereal) \quad (22)$$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (23)$$

Let $c_2Erealax_2Etrealm_inv : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_inv \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)\ (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)) \quad (24)$$

Definition 28 We define $c_2Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal_ABS$.

Definition 29 We define c_2Ereal_2E2F to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$.

Let $c_2Erealax_2Etrealm_lt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})\ (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)) \quad (25)$$

Definition 30 We define $c_2Erealax_2Ereal_lte$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$.

Definition 31 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$.

Definition 32 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ (ap\ (ap\ (c_2Ebool_2ECONJ$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \quad (26)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \quad (27)$$

Definition 33 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c)^{A_27a})$.
Let $ty_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Emetric_2Emetric\ A0) \quad (28)$$

Let $c_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Emetric\ A_27a \in ((ty_2Emetric_2Emetric\ A_27a)^{(ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})}) \quad (29)$$

Definition 34 We define $c_2Emetric_2Emr1$ to be $(ap\ (c_2Emetric_2Emetric\ ty_2Erealax_2Ereal)\ (ap\ (c_2Epair_2EABS_prod\ : \iota \Rightarrow \iota \Rightarrow \iota)))$.
Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (30)$$

Definition 35 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2EABS_prod\ : \iota \Rightarrow \iota \Rightarrow \iota))$.
Let $c_2Enets_2Etendsto : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Enets_2Etendsto\ A_27a \in (((2^{A_27a})^{A_27a})^{(ty_2Epair_2Eprod\ (ty_2Emetric_2Emetric\ A_27a)\ A_27a)}) \quad (31)$$

Let $c_2Emetric_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Edist\ A_27a \in ((ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})^{(ty_2Emetric_2Edist\ A_27a)}) \quad (32)$$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Etopology_2Etopology\ A0) \quad (33)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Etopology\ A_27a \in ((ty_2Etopology_2Etopology\ A_27a)^{(2^{(2^{A_27a})})}) \quad (34)$$

Definition 36 We define $c_2Emetric_2Emtop$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Emetric_2Emetric\ A_27a).(ap\ (c_2Epair_2EABS_prod\ : \iota \Rightarrow \iota \Rightarrow \iota))$.

Let $c_2Enets_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Enets_2Etends\ A_27a\ A_27b \in (((2^{(ty_2Epair_2Eprod\ (ty_2Etopology_2Etopology\ A_27a)\ (2^{A_27b})^{A_27b}))})^{A_27a})^{(A_27a)^{A_27b}}) \quad (35)$$

Definition 37 We define $c_2Elim_2Etends_real_real$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal})$.

Definition 38 We define c_2Elim_2Ediff to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).\lambda V1l \in ty_2Erealax_2Ereal$.

Let $c_2Earithmetic_2EFACT : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EFACT \in (ty_2Enum_2Enum^{ty_2Enum_2Enum}) \quad (36)$$

Definition 39 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Definition 40 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 41 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2EFACT$

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (37)$$

Let $c_2Ereal_2Esum : \iota$ be given. Assume the following.

$$c_2Ereal_2Esum \in ((ty_2Erealax_2Ereal^{(ty_2Erealax_2Ereal^{ty_2Enum_2Enum})})^{(ty_2Epair_2Eprod ty_2Enum_2Enum)}) \quad (38)$$

Definition 42 We define $c_2Eseq_2E_2D_2D_3E$ to be $\lambda V0x \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).\lambda V1x$

Definition 43 We define c_2Eseq_2Esums to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).\lambda V1s \in ty_2Enum_2Enum$

Definition 44 We define $c_2Eseq_2Esuminf$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(ap (c_2Eseq_2Esums$

Definition 45 We define $c_2Etransc_2Esin$ to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap c_2Eseq_2Esuminf (\lambda V1n$

Definition 46 We define $c_2Etransc_2Ecos$ to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap c_2Eseq_2Esuminf (\lambda V1n$

Assume the following.

$$\begin{aligned} & ((ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO)) = \\ & (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\ & c_2Earithmetic_2EZERO)))) \end{aligned} \quad (39)$$

Assume the following.

$$True \quad (40)$$

Assume the following.

$$\begin{aligned} & ((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge \\ & ((\neg False) \Leftrightarrow True))) \end{aligned} \quad (41)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (42)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (43)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\
& p V0t))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow (\forall V0t1 \in A.27a. (\forall V1t2 \in \\
& A.27a. (((ap (ap (ap (c.2Ebool.2ECOND A.27a) c.2Ebool.2ET) V0t1) \\
& V1t2) = V0t1) \wedge ((ap (ap (ap (c.2Ebool.2ECOND A.27a) c.2Ebool.2EF \\
& V0t1) V1t2) = V1t2))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0k \in ty_2Erealax_2Ereal. (\forall V1x \in ty_2Erealax_2Ereal. \\
& (p (ap (ap (ap c.2Elim_2Ediff1 (\lambda V2x \in ty_2Erealax_2Ereal.V0k) \\
& (ap c.2Ereal_2Ereal_of_num c.2Enum_2E0) V1x))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). (\forall V1g \in \\
& (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). (\forall V2l \in ty_2Erealax_2Ereal. \\
& (\forall V3m \in ty_2Erealax_2Ereal. (\forall V4x \in ty_2Erealax_2Ereal. \\
& (((p (ap (ap (ap c.2Elim_2Ediff1 V0f) V2l) V4x)) \wedge (p (ap (ap (ap c.2Elim_2Ediff1 \\
& V1g) V3m) V4x))) \Rightarrow (p (ap (ap (ap c.2Elim_2Ediff1 (\lambda V5x \in ty_2Erealax_2Ereal. \\
& (ap (ap c.2Erealax_2Ereal_add (ap V0f V5x)) (ap V1g V5x))) (ap \\
& (ap c.2Erealax_2Ereal_add V2l) V3m)) V4x)))))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). (\forall V1g \in \\
& (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}). (\forall V2l \in ty_2Erealax_2Ereal. \\
& (\forall V3m \in ty_2Erealax_2Ereal. (\forall V4x \in ty_2Erealax_2Ereal. \\
& (((p (ap (ap (ap c.2Elim_2Ediff1 V0f) V2l) V4x)) \wedge (p (ap (ap (ap c.2Elim_2Ediff1 \\
& V1g) V3m) V4x))) \Rightarrow (p (ap (ap (ap c.2Elim_2Ediff1 (\lambda V5x \in ty_2Erealax_2Ereal. \\
& (ap (ap c.2Erealax_2Ereal_mul (ap V0f V5x)) (ap V1g V5x))) (ap \\
& (ap c.2Erealax_2Ereal_add (ap (ap c.2Erealax_2Ereal_mul V2l) \\
& (ap V1g V4x))) (ap (ap c.2Erealax_2Ereal_mul V3m) (ap V0f V4x)))))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1g \in \\
& (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V2l \in ty_2Erealax_2Ereal. \\
& (\forall V3m \in ty_2Erealax_2Ereal.(\forall V4x \in ty_2Erealax_2Ereal. \\
& (((p (ap (ap (ap (ap c_2Elim_2Ediff1 V0f) V2l) V4x)) \wedge (p (ap (ap (ap c_2Elim_2Ediff1 \\
& V1g) V3m) V4x))) \Rightarrow (p (ap (ap (ap c_2Elim_2Ediff1 (\lambda V5x \in ty_2Erealax_2Ereal. \\
& (ap (ap c_2Ereal_2Ereal_sub (ap V0f V5x)) (ap V1g V5x)))) (ap (ap \\
& c_2Ereal_2Ereal_sub V2l) V3m)) V4x)))))))))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V1g \in \\
& (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).(\forall V2l \in ty_2Erealax_2Ereal. \\
& (\forall V3m \in ty_2Erealax_2Ereal.(\forall V4x \in ty_2Erealax_2Ereal. \\
& (((p (ap (ap (ap c_2Elim_2Ediff1 V0f) V2l) (ap V1g V4x))) \wedge (p (ap (\\
& ap (ap c_2Elim_2Ediff1 V1g) V3m) V4x))) \Rightarrow (p (ap (ap (ap c_2Elim_2Ediff1 \\
& (\lambda V5x \in ty_2Erealax_2Ereal.(ap V0f (ap V1g V5x)))) (ap (ap c_2Erealax_2Ereal_mul \\
& V2l) V3m)) V4x)))))))))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal.(p (ap (ap (ap c_2Elim_2Ediff1 \\
& (\lambda V1x \in ty_2Erealax_2Ereal.V1x)) (ap c_2Ereal_2Ereal_of_num \\
& (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \\
& V0x)))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum.(\forall V1x \in ty_2Erealax_2Ereal. \\
& (p (ap (ap (ap c_2Elim_2Ediff1 (\lambda V2x \in ty_2Erealax_2Ereal.(\\
& ap (ap c_2Ereal_2Epow V2x) V0n))) (ap (ap c_2Erealax_2Ereal_mul \\
& (ap c_2Ereal_2Ereal_of_num V0n)) (ap (ap c_2Ereal_2Epow V1x) \\
& (ap (ap c_2Earithmetic_2E_2D V0n) (ap c_2Earithmetic_2ENUMERAL \\
& (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) V1x))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty_2Erealax_2Ereal^{ty_2Erealax_2Ereal}).((\forall V1x \in \\
& ty_2Erealax_2Ereal.(p (ap (ap (ap c_2Elim_2Ediff1 V0f) (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) V1x))) \Rightarrow (\forall V2x \in ty_2Erealax_2Ereal.(\forall V3y \in \\
& ty_2Erealax_2Ereal.((ap V0f V2x) = (ap V0f V3y))))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& \quad ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2B \\
& \quad (ap c_2Earithmetic_2ENUMERAL V2n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V3m)) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& \quad (ap c_2Earithmetic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& \quad ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A \\
& \quad V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& \quad (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2A (\\
& \quad ap c_2Earithmetic_2ENUMERAL V6n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V7m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2A \\
& \quad V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& \quad ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2E_2D \\
& \quad (ap c_2Earithmetic_2ENUMERAL V10n)) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V11m)) = (ap c_2Earithmetic_2ENUMERAL (ap (ap c_2Earithmetic_2E_2D \\
& \quad V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP \\
& \quad c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& \quad (ap c_2Earithmetic_2EEXP c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& \quad ty_2Enum_2Enum.((ap (ap c_2Earithmetic_2EEXP V14n) c_2Enum_2E0) = \\
& \quad (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \wedge \\
& \quad ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& \quad ((ap (ap c_2Earithmetic_2EEXP (ap c_2Earithmetic_2ENUMERAL V15n)) \\
& \quad (ap c_2Earithmetic_2ENUMERAL V16m)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap (ap c_2Earithmetic_2EEXP V15n) V16m)))))) \wedge ((ap c_2Enum_2ESUC \\
& \quad c_2Enum_2E0) = (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\
& \quad c_2Earithmetic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum. (\\
& \quad (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL V17n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Enum_2ESUC V17n)))))) \wedge ((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& \quad c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& \quad (ap c_2Earithmetic_2ENUMERAL V18n)) = (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Eprim_rec_2EPRE V18n)))))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& \quad (((ap c_2Earithmetic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmetic_2EZERO))) \wedge \\
& \quad ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmetic_2ENUMERAL \\
& \quad V20n)) \Leftrightarrow (V20n = c_2Earithmetic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& \quad (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmetic_2ENUMERAL \\
& \quad V21n) = (ap c_2Earithmetic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))))) \wedge \\
& \quad ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmetic_2ENUMERAL \\
& \quad V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V24n)))))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmetic_2ENUMERAL \\
& \quad V25n)) (ap c_2Earithmetic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3E \\
& \quad c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmetic_2EZERO) \\
& \quad V28n)))))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3E (ap c_2Earithmetic_2ENUMERAL \\
& \quad V29n)) (ap c_2Earithmetic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& \quad V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\
& \quad c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& \quad ((p (ap (ap c_2Earithmetic_2E_3C_3D (ap c_2Earithmetic_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmic_2EZERO) (ap c_2Earithmic_2EBIT1 \\
& V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmic_2EZERO) \\
& (ap c_2Earithmic_2EBIT2 V0n))) \Leftrightarrow True) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& V0n) c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmic_2EBIT1 V0n)) (ap c_2Earithmic_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmic_2EBIT2 V0n)) (ap c_2Earithmic_2EBIT2 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))) \wedge (((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmic_2EBIT1 V0n)) (ap c_2Earithmic_2EBIT2 V1m))) \Leftrightarrow \\
& (\neg (p (ap (ap c_2Eprim_rec_2E_3C V1m) V0n)))) \wedge ((p (ap (ap c_2Eprim_rec_2E_3C \\
& (ap c_2Earithmic_2EBIT2 V0n)) (ap c_2Earithmic_2EBIT1 V1m))) \Leftrightarrow \\
& (p (ap (ap c_2Eprim_rec_2E_3C V0n) V1m))))))))))
\end{aligned} \tag{55}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Enum_2Enum. (\forall V1b \in 2. (\forall V2n \in ty_2Enum_2Enum. \\
& (\forall V3m \in ty_2Enum_2Enum. (((ap (ap (ap c_2Enumeral_2EiSUB \\
& V1b) c_2Earithmic_2EZERO) V0x) = c_2Earithmic_2EZERO) \wedge (\\
& ((ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) V2n) c_2Earithmic_2EZERO) = \\
& V2n) \wedge (((ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) (ap c_2Earithmic_2EBIT1 \\
& V2n)) c_2Earithmic_2EZERO) = (ap c_2Enumeral_2EiDUB V2n)) \wedge \\
& (((ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) (ap c_2Earithmic_2EBIT1 \\
& V2n)) (ap c_2Earithmic_2EBIT1 V3m)) = (ap c_2Enumeral_2EiDUB \\
& (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) V2n) V3m))) \wedge (((ap \\
& (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) (ap c_2Earithmic_2EBIT1 \\
& V2n)) (ap c_2Earithmic_2EBIT1 V3m)) = (ap c_2Earithmic_2EBIT1 \\
& (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) V2n) V3m))) \wedge (((ap \\
& (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) (ap c_2Earithmic_2EBIT1 \\
& V2n)) (ap c_2Earithmic_2EBIT2 V3m)) = (ap c_2Earithmic_2EBIT1 \\
& (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) V2n) V3m))) \wedge (((ap \\
& (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) (ap c_2Earithmic_2EBIT1 \\
& V2n)) (ap c_2Earithmic_2EBIT2 V3m)) = (ap c_2Enumeral_2EiDUB \\
& (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) V2n) V3m))) \wedge (((ap \\
& (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) (ap c_2Earithmic_2EBIT2 \\
& V2n)) c_2Earithmic_2EZERO) = (ap c_2Earithmic_2EBIT1 V2n)) \wedge \\
& (((ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) (ap c_2Earithmic_2EBIT2 \\
& V2n)) (ap c_2Earithmic_2EBIT1 V3m)) = (ap c_2Earithmic_2EBIT1 \\
& (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) V2n) V3m))) \wedge (((ap \\
& (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) (ap c_2Earithmic_2EBIT2 \\
& V2n)) (ap c_2Earithmic_2EBIT1 V3m)) = (ap c_2Enumeral_2EiDUB \\
& (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) V2n) V3m))) \wedge (((ap \\
& (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) (ap c_2Earithmic_2EBIT2 \\
& V2n)) (ap c_2Earithmic_2EBIT2 V3m)) = (ap c_2Enumeral_2EiDUB \\
& (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2ET) V2n) V3m))) \wedge ((ap \\
& (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) (ap c_2Earithmic_2EBIT2 \\
& V2n)) (ap c_2Earithmic_2EBIT2 V3m)) = (ap c_2Earithmic_2EBIT1 \\
& (ap (ap (ap c_2Enumeral_2EiSUB c_2Ebool_2EF) V2n) V3m))))))))))))))))) \\
& \hspace{15em} (56)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& (ap c_2Earithmic_2ENUMERAL (ap (ap c_2Earithmic_2E_2D V0n) \\
& V1m)) = (ap (ap (ap (c_2Ebool_2ECOND ty_2Enum_2Enum) (ap (ap c_2Eprim_rec_2E_3C \\
& V1m) V0n)) (ap c_2Earithmic_2ENUMERAL (ap (ap (ap c_2Enumeral_2EiSUB \\
& c_2Ebool_2ET) V0n) V1m))) c_2Enum_2E0)))) \\
& \hspace{15em} (57)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x) = V0x)) \\
& \hspace{15em} (58)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap (ap c_2Erealax_2Ereal_mul V0x) V1y) = (ap (ap c_2Erealax_2Ereal_mul \\
& \quad V1y) V0x))))
\end{aligned} \tag{59}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& V0x) (ap (ap c_2Erealax_2Ereal_mul V1y) V2z)) = (ap (ap c_2Erealax_2Ereal_mul \\
& \quad (ap (ap c_2Erealax_2Ereal_mul V0x) V1y)) V2z))))))
\end{aligned} \tag{60}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL (\\
& \quad ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) V0x) = V0x))
\end{aligned} \tag{61}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add \\
& V0x) (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) = V0x))
\end{aligned} \tag{62}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& V0x) (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL \\
& \quad (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) = V0x))
\end{aligned} \tag{63}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap c_2Erealax_2Ereal_neg (ap (ap c_2Erealax_2Ereal_add V0x) \\
& V1y)) = (ap (ap c_2Erealax_2Ereal_add (ap c_2Erealax_2Ereal_neg \\
& \quad V0x)) (ap c_2Erealax_2Ereal_neg V1y))))))
\end{aligned} \tag{64}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_mul \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x) = (ap c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0)))
\end{aligned} \tag{65}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap c_2Erealax_2Ereal_neg (ap (ap c_2Erealax_2Ereal_mul V0x) \\
& V1y)) = (ap (ap c_2Erealax_2Ereal_mul (ap c_2Erealax_2Ereal_neg \\
& \quad V0x)) V1y))))))
\end{aligned} \tag{66}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& ((ap\ c_2Erealax_2Ereal_neg (ap (ap\ c_2Erealax_2Ereal_mul\ V0x) \\
& V1y)) = (ap (ap\ c_2Erealax_2Ereal_mul\ V0x) (ap\ c_2Erealax_2Ereal_neg \\
& V1y))))))
\end{aligned} \tag{67}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap\ c_2Erealax_2Ereal_neg \\
& (ap\ c_2Erealax_2Ereal_neg\ V0x)) = V0x))
\end{aligned} \tag{68}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap\ c_2Ereal_2Ereal_sub \\
& V0x) V0x) = (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)))
\end{aligned} \tag{69}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap\ c_2Ereal_2Ereal_sub \\
& (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)) V0x) = (ap\ c_2Erealax_2Ereal_neg \\
& V0x)))
\end{aligned} \tag{70}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap\ c_2Ereal_2Ereal_sub \\
& V0x) (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0)) = V0x))
\end{aligned} \tag{71}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\
& (\forall V2z \in ty_2Erealax_2Ereal. ((V0x = (ap (ap\ c_2Ereal_2Ereal_sub \\
& V1y) V2z)) \Leftrightarrow ((ap (ap\ c_2Erealax_2Ereal_add\ V0x) V2z) = V1y))))))
\end{aligned} \tag{72}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. ((ap (ap\ c_2Ereal_2Epow (ap\ c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) (ap\ c_2Enum_2ESUC\ V0n)) = (ap\ c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)))
\end{aligned} \tag{73}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap (ap\ c_2Ereal_2Epow\ V0x) \\
& (ap\ c_2Earithmic_2ENUMERAL (ap\ c_2Earithmic_2EBIT1\ c_2Earithmic_2EZERO))) = \\
& V0x))
\end{aligned} \tag{74}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \tag{75}$$

Assume the following.

$$(\forall V0A \in 2.((p \ V0A) \Rightarrow ((\neg(p \ V0A)) \Rightarrow False))) \quad (76)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p \ V0A) \vee (p \ V1B))) \Rightarrow False) \Leftrightarrow ((p \ V0A) \Rightarrow False) \Rightarrow ((\neg(p \ V1B)) \Rightarrow False)))) \quad (77)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p \ V0A)) \vee (p \ V1B))) \Rightarrow False) \Leftrightarrow ((p \ V0A) \Rightarrow ((\neg(p \ V1B)) \Rightarrow False)))) \quad (78)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p \ V0A)) \Rightarrow False) \Rightarrow (((p \ V0A) \Rightarrow False) \Rightarrow False))) \quad (79)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow (p \ V1q) \Leftrightarrow (p \ V2r)) \Leftrightarrow (((p \ V0p) \vee ((p \ V1q) \vee (p \ V2r))) \wedge (((p \ V0p) \vee (\neg(p \ V2r)) \vee (\neg(p \ V1q)))) \wedge (((p \ V1q) \vee ((\neg(p \ V2r)) \vee (\neg(p \ V0p)))) \wedge ((p \ V2r) \vee ((\neg(p \ V1q)) \vee (\neg(p \ V0p)))))))))) \quad (80)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p \ V0p) \Leftrightarrow (p \ V1q) \wedge (p \ V2r)) \Leftrightarrow (((p \ V0p) \vee ((\neg(p \ V1q)) \vee (\neg(p \ V2r)))) \wedge (((p \ V1q) \vee (\neg(p \ V0p))) \wedge ((p \ V2r) \vee (\neg(p \ V0p)))))))) \quad (81)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p)))))) \quad (82)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (p \ V0p)))) \quad (83)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((\neg((p \ V0p) \Rightarrow (p \ V1q))) \Rightarrow (\neg(p \ V1q)))) \quad (84)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(p \ (ap \ (ap \ (ap \ c_2Elim_2Ediff \ c_2Etransc_2Esin) \ (ap \ c_2Etransc_2Ecos \ V0x)) \ V0x))) \quad (85)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.(p (ap (ap (ap c_2Elim_2Ediff c_2Etrasc_2Ecos) (ap c_2Erealax_2Ereal_neg (ap c_2Etrasc_2Esin V0x))) V0x))) \quad (86)$$

Assume the following.

$$((ap c_2Etrasc_2Esin (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) \quad (87)$$

Assume the following.

$$((ap c_2Etrasc_2Ecos (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) = (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \quad (88)$$

Theorem 1

$$(\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. ((ap (ap c_2Erealax_2Ereal_add (ap (ap c_2Ereal_2Epow (ap (ap c_2Ereal_2Ereal_sub (ap c_2Etrasc_2Esin (ap (ap c_2Erealax_2Ereal_add V0x) V1y))) (ap (ap c_2Erealax_2Ereal_add (ap (ap c_2Erealax_2Ereal_mul (ap c_2Etrasc_2Esin V0x)) (ap c_2Etrasc_2Ecos V1y))) (ap (ap c_2Erealax_2Ereal_mul (ap c_2Etrasc_2Ecos V0x)) (ap c_2Etrasc_2Esin V1y)))))) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO)))) (ap (ap c_2Ereal_2Epow (ap (ap c_2Ereal_2Ereal_sub (ap c_2Etrasc_2Ecos (ap (ap c_2Erealax_2Ereal_add V0x) V1y))) (ap (ap c_2Ereal_2Ereal_sub (ap (ap c_2Erealax_2Ereal_mul (ap c_2Etrasc_2Ecos V0x)) (ap c_2Etrasc_2Ecos V1y))) (ap (ap c_2Erealax_2Ereal_mul (ap c_2Etrasc_2Esin V0x)) (ap c_2Etrasc_2Esin V1y)))))) (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO)))) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))))$$