

thm_2Etransc_2ESIN__COS__SQ (TMTzETpF- SpXRMKv7PM5qDTNLhXmo9pNcEom)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_21$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_21$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_21))$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{3}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})\ ty_2Erealax_2Ereal) \tag{4}$$

Definition 7 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p))$ of type $\iota \Rightarrow \iota$.

Definition 8 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (ty_2Erealax_2Ereal_REP_CLASS a)))$

Definition 15 We define c_Enum_ESUC to be $\lambda V0m \in ty_Enum_Enum.(ap\ c_Enum_EABS_num$

Let $c_Arithmetic_E_B : \iota$ be given. Assume the following.

$$c_Arithmetic_E_B \in ((ty_Enum_Enum^{ty_Enum_Enum})^{ty_Enum_Enum}) \quad (14)$$

Definition 16 We define $c_Arithmetic_EBIT2$ to be $\lambda V0n \in ty_Enum_Enum.(ap\ (ap\ c_Arithmetic$

Definition 17 We define $c_Arithmetic_ENUMERAL$ to be $\lambda V0x \in ty_Enum_Enum.V0x$.

Let $c_Real_Epow : \iota$ be given. Assume the following.

$$c_Real_Epow \in ((ty_Realax_Real^{ty_Enum_Enum})^{ty_Realax_Real}) \quad (15)$$

Let $c_Real_Real_of_num : \iota$ be given. Assume the following.

$$c_Real_Real_of_num \in (ty_Realax_Real^{ty_Enum_Enum}) \quad (16)$$

Let $c_Realax_Etreal_lt : \iota$ be given. Assume the following.

$$c_Realax_Etreal_lt \in ((2^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)})(ty_Epair_Eprod\ ty_Ehreal_Ehreal)) \quad (17)$$

Definition 18 We define $c_Realax_Etreal_lt$ to be $\lambda V0T1 \in ty_Realax_Real.\lambda V1T2 \in ty_Realax_Real$

Definition 19 We define $c_Ebool_E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_Ebool_E_21\ 2)\ (\lambda V2t \in 2$

Definition 20 We define $c_Etransc_Eroot$ to be $\lambda V0n \in ty_Enum_Enum.\lambda V1x \in ty_Realax_Real$

Definition 21 We define $c_Etransc_Esqrt$ to be $\lambda V0x \in ty_Realax_Real.(ap\ (ap\ c_Etransc_Eroot\ ($

Definition 22 We define $c_Arithmetic_EBIT1$ to be $\lambda V0n \in ty_Enum_Enum.(ap\ (ap\ c_Arithmetic$

Let $c_Arithmetic_EFACT : \iota$ be given. Assume the following.

$$c_Arithmetic_EFACT \in (ty_Enum_Enum^{ty_Enum_Enum}) \quad (18)$$

Let $c_Arithmetic_EDIV : \iota$ be given. Assume the following.

$$c_Arithmetic_EDIV \in ((ty_Enum_Enum^{ty_Enum_Enum})^{ty_Enum_Enum}) \quad (19)$$

Let $c_Realax_Etreal_inv : \iota$ be given. Assume the following.

$$c_Realax_Etreal_inv \in ((ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)^{(ty_Epair_Eprod\ ty_Ehreal_Ehreal\ ty_Ehreal_Ehreal)}) \quad (20)$$

Definition 23 We define c_Realax_Einv to be $\lambda V0T1 \in ty_Realax_Real.(ap\ c_Realax_Real_ABS$

Let $c_2Erealax_2Etrealmul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealmul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)))(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal) \quad (21)$$

Definition 24 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 25 We define $c_2Ereal_2E_2F$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (22)$$

Definition 26 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (23)$$

Definition 27 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2E$

Let $c_2Ereal_2Esum : \iota$ be given. Assume the following.

$$c_2Ereal_2Esum \in ((ty_2Erealax_2Ereal^{(ty_2Erealax_2Ereal^{ty_2Enum_2Enum})})(ty_2Epair_2Eprod\ ty_2Enum_2Enum)) \quad (24)$$

Definition 28 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 29 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Definition 30 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Definition 31 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 32 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Definition 33 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Definition 34 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ (ap\ (ap\ (c_2Ebool_2ECOND$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (25)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (26)$$

Definition 43 We define $c_2Etransc_2Epi$ to be $(ap (ap c_2Erealax_2Ereal_mul (ap c_2Ereal_2Ereal_of_num$

Assume the following.

$$True \quad (34)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \wedge (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \wedge True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \wedge (p V0t)) \Leftrightarrow False) \wedge (((p V0t) \wedge False) \Leftrightarrow False) \wedge \\ & (((p V0t) \wedge (p V0t)) \Leftrightarrow (p V0t)))))) \end{aligned} \quad (35)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (36)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg \\ & (p V0t)))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\ & (\forall V2z \in ty_2Erealax_2Ereal.((V0x = (ap (ap c_2Ereal_2Ereal_sub \\ & V1y) V2z)) \Leftrightarrow ((ap (ap c_2Erealax_2Ereal_add V0x) V2z) = V1y)))))) \end{aligned} \quad (38)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\ & (((ap (ap c_2Ereal_2Epow V0x) (ap c_2Earithmetic_2ENUMERAL (\\ & ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO))) = V1y) \wedge (p \\ & (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0) \\ & V0x))) \Rightarrow (V0x = (ap c_2Etransc_2Esqrt V1y)))))) \end{aligned} \quad (39)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Erealax_2Ereal_add \\ & (ap (ap c_2Ereal_2Epow (ap c_2Etransc_2Esin V0x) (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO)))) (ap (ap \\ & c_2Ereal_2Epow (ap c_2Etransc_2Ecos V0x) (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO)))))) = (ap c_2Ereal_2Ereal_of_num \\ & (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))))) \end{aligned} \quad (40)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty_2Erealax_2Ereal.(((p (ap (ap c_2Ereal_2Ereal_lte \\ & (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0) V0x)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte \\ & V0x) c_2Etransc_2Epi))) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\ & c_2Enum_2E0) (ap c_2Etransc_2Esin V0x)))))) \end{aligned} \quad (41)$$

Theorem 1

$$\begin{aligned} & (\forall V0x \in ty_2Erealax_2Ereal.(((p (ap (ap c_2Ereal_2Ereal_lte \\ & (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte \\ & V0x) c_2Etransc_2Epi))) \Rightarrow ((ap c_2Etransc_2Esin V0x) = (ap c_2Etransc_2Esqrt \\ & (ap (ap c_2Ereal_2Ereal_sub (ap c_2Ereal_2Ereal_of_num (ap \\ & c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) \\ & (ap (ap c_2Ereal_2Epow (ap c_2Etransc_2Ecos V0x)) (ap c_2Earithmetic_2ENUMERAL \\ & (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO)))))))))) \end{aligned}$$