thm_2Etransc_2ESIN__TOTAL (TMRLnmAt-tjPoTPwW3YMrsAYjCxkLBbavd5Z)

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Definition 1 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2E_3F *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ ($c_2Emin_2E_40 A

Definition 4 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D $(2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 5 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ ($c_2Emin_2E_3D $(2^{A\_27}$

Definition 6 *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V0t \in 2.V0t))$.

Definition 7 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 8 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2EF

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{1}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{3}$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{4}$$

Definition 9 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap$ (c_2Emin_2E_40 $(ty$

Let $c\_2Erealax\_2Etreal\_\_mul : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Eh} \quad (5)$$

Let $c\_2Erealax\_2Etreal\_\_eq : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \quad (6)$$

Let $c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Eh} \quad (7)$$

Definition 10 *We define* c_2Erealax_2Ereal__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty$

Definition 11 *We define* c_2Erealax_2Ereal__mul *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax$

Let $c\_2Erealax\_2Etreal\_\_neg : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (8)$$

Definition 12 *We define* c_2Erealax_2Ereal__neg *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal

Let $c\_2Erealax\_2Etreal\_\_add : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Eh} \quad (9)$$

Definition 13 *We define* c_2Erealax_2Ereal__add *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax$

Definition 14 *We define* c_2Ereal_2Ereal__sub *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2E$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \quad (10)$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (11)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (12)$$

Definition 15 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\_2Enum\_2Enum}) \tag{13}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{14}$$

Definition 16 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num$

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{15}$$

Definition 17 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Let $c\_2Erealax\_2Etreal\_\_inv : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_inv \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \tag{16}$$

Definition 18 *We define* c_2Erealax_2Einv *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap$ c_2Erealax_2Ereal__ABS

Definition 19 *We define* c_2Ereal_2E_2F *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.($

Let $c\_2Ereal\_2Epow : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})^{ty\_2Erealax\_2Ereal}) \tag{17}$$

Let $c\_2Earithmetic\_2EFACT : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EFACT \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}) \tag{18}$$

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \tag{19}$$

Definition 20 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0*.*

Definition 21 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x.$

Definition 22 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Let $c\_2Earithmetic\_2E\_2D : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{20}$$

Let $c\_2Earithmetic\_2EDIV : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{21}$$

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \tag{22}$$

Definition 23 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 24 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.($

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{23}$$

Definition 25 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Ereal\_2Esum : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Esum \in ((ty\_2Erealax\_2Ereal^{(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2En} \tag{24}$$

Definition 26 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Definition 27 *We define* c_2Earithmetic_2E_3E *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2En$

Definition 28 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Definition 29 *We define* c_2Earithmetic_2E_3E_3D *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2$

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehr} \tag{25}$$

Definition 30 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_$

Definition 31 *We define* c_2Ereal_2Ereal__lte *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Definition 32 *We define* c_2Ereal_2Eabs *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ (ap$ (c_2Ebool_2ECONI

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{26}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{27}$$

Definition 33 *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27}$

Let $ty\_2Emetric\_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Emetric\_2Emetric\ A0) \tag{28}$$

Let $c\_2Emetric\_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Emetric\ A\_27a \in ((ty\_2Emetric\_2Emetric\ A\_27a)^{(ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})}) \tag{29}$$

Definition 34 *We define* c_2Emetric_2Emr1 *to be* $(ap\ (c\_2Emetric\_2Emetric\ ty\_2Erealax\_2Ereal)\ (ap\ (c$

Let $c\_2Emetric\_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Emetric\_2Edist\ A\_27a \in ((ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27} \tag{30}$$

Let $ty\_2Etopology\_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Etopology\_2Etopology\ A0) \tag{31}$$

Let $c\_2Etopology\_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Etopology\_2Etopology\ A\_27a \in ((ty\_2Etopology\_2Etopology\ A\_27a)^{(2^{(2^{A\_27a})})}) \tag{32}$$

Definition 35 *We define* c_2Emetric_2Emtop *to be* $\lambda A\_27a : \iota.\lambda V0m \in (ty\_2Emetric\_2Emetric\ A\_27a).(ap$

Let $c\_2Enets\_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Enets\_2Etends\ A\_27a\ A\_27b \in (((2^{(ty\_2Epair\_2Eprod\ (ty\_2Etopology\_2Etopology\ A\_27a)\ ((2^{A\_27b})^{A\_27b}))})^{A\_27a})^{(A\_27a^{A\_27b})}) \tag{33}$$

Definition 36 *We define* c_2Eseq_2E_2D_2D_3E *to be* $\lambda V0x \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1x$

Definition 37 *We define* c_2Eseq_2Esums *to be* $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1s \in ty$

Definition 38 *We define* c_2Eseq_2Esuminf *to be* $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(ap\ (c\_2E$

Definition 39 *We define* c_2Etransc_2Esin *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap$ c_2Eseq_2Esuminf $(\lambda V1n$

Definition 40 *We define* c_2Etransc_2Ecos *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap$ c_2Eseq_2Esuminf $(\lambda V1n$

Definition 41 *We define* c_2Etransc_2Epi *to be* $(ap\ (ap$ c_2Erealax_2Ereal__mul $(ap\ c\_2Ereal\_2Ereal\_\_of\_\_n$

Definition 42 *We define* c_2Ebool_2E_3F_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap$ c_2Ebool_2E_2F_5C

Assume the following.

$$True \tag{34}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{35}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))))) \tag{36}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{37}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{38}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{39}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V0x)\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))) = V0x)) \tag{40}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.(\forall V2z \in ty\_2Erealax\_2Ereal.(((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ V2z) = (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V1y)\ V2z)) \Leftrightarrow (V0x = V1y))))) \tag{41}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V0x)\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))) \tag{42}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V0x)) = V0x)) \tag{43}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ V0x)\ V1y))\ V1y) = V0x))) \tag{44}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ V1y))\ V0x) = V1y))) \tag{45}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ (ap\ (ap\ \mathsf{c\_2Ereal\_2E\_2F}\ V0x)\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO})))))\ (ap\ (ap\ \mathsf{c\_2Ereal\_2E\_2F}\ V0x)\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO}))))) = V0x)) \tag{46}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.(\forall V2z \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V0x)\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ V1y)\ V2z)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ V2z))\ V1y)))))) \tag{47}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.(\forall V2z \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ V0x)\ V1y))\ V2z))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V0x)\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V2z)\ V1y))))))) \tag{48}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V0x))\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V1y)))\Leftrightarrow(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V1y)\ V0x))))) \tag{49}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V0x) = (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V0x))) \tag{50}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. (((ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V0x) = (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ V1y))\Leftrightarrow(V0x = V1y)))) \tag{51}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. ((ap\ \mathsf{c\_2Etransc\_2Ecos}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_add}\ V0x)\ V1y)) = (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_sub}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ \mathsf{c\_2Etransc\_2Ecos}\ V0x))\ (ap\ \mathsf{c\_2Etransc\_2Ecos}\ V1y)))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ \mathsf{c\_2Etransc\_2Esin}\ V0x))\ (ap\ \mathsf{c\_2Etransc\_2Esin}\ V1y)))))) \tag{52}$$

Assume the following.

$$((ap\ \mathsf{c\_2Etransc\_2Ecos}\ (ap\ (ap\ \mathsf{c\_2Ereal\_2E\_2F}\ \mathsf{c\_2Etransc\_2Epi})\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO}))))) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0})) \tag{53}$$

Assume the following.

$$((ap\ \mathsf{c\_2Etransc\_2Esin}\ (ap\ (ap\ \mathsf{c\_2Ereal\_2E\_2F}\ \mathsf{c\_2Etransc\_2Epi})\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO}))))) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))) \tag{54}$$

Assume the following.

$$(\forall V0y \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO})))))\ V0y))\wedge(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V0y)\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))))\Rightarrow (p\ (ap\ (\mathsf{c\_2Ebool\_2E\_3F\_21}\ ty\_2Erealax\_2Ereal)\ (\lambda V1x \in ty\_2Erealax\_2Ereal. (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V1x))\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V1x)\ \mathsf{c\_2Etransc\_2Epi}))\ (ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ ty\_2Erealax\_2Ereal)\ (ap\ \mathsf{c\_2Etransc\_2Ecos}\ V1x))\ V0y)))))))) \tag{55}$$

Theorem 1

$$
\begin{gathered}
(\forall V0y \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte} \\
(ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap \\
\mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))) \\
V0y))\land(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V0y)\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num \\
(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT1}\ \mathsf{c\_2Earithmetic\_2EZERO}))))))\Rightarrow \\
(p\ (ap\ (\mathsf{c\_2Ebool\_2E\_3F\_21}\ ty\_2Erealax\_2Ereal)\ (\lambda V1x \in ty\_2Erealax\_2Ereal. \\
(ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_neg} \\
(ap\ (ap\ \mathsf{c\_2Ereal\_2E\_2F}\ \mathsf{c\_2Etransc\_2Epi})\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num \\
(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO})))))) \\
V1x))\ (ap\ (ap\ \mathsf{c\_2Ebool\_2E\_2F\_5C}\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V1x) \\
(ap\ (ap\ \mathsf{c\_2Ereal\_2E\_2F}\ \mathsf{c\_2Etransc\_2Epi})\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num \\
(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO})))))) \\
(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ ty\_2Erealax\_2Ereal)\ (ap\ \mathsf{c\_2Etransc\_2Esin} \\
V1x))\ V0y))))))))
\end{gathered}
$$