

thm_2Etransc_2ESQRT__DIV (TM- buH3VxtGNsBUH28WVwLtbCe7hFvegFjh)

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Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 3 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E_3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E_3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 5 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (c_2Enum_2ESUC_REP\ m))$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{6}$$

Definition 6 We define $c_Earithmetic_EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_Earithmetic_EBIT1 V0n) V0n)$.

Definition 7 We define c_Ebool_EF to be $(ap (c_Ebool_21 2) (\lambda V0t \in 2.V0t))$.

Definition 8 We define $c_Emin_E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 9 We define c_Ebool_E7E to be $(\lambda V0t \in 2.(ap (ap c_Emin_E3D_3D_3E V0t) c_Ebool_21) V0t)$.

Definition 10 We define $c_Earithmetic_EZERO$ to be c_2Enum_2E0 .

Definition 11 We define $c_Earithmetic_EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_Earithmetic_EBIT1 V0n) V0n)$.

Definition 12 We define $c_Earithmetic_ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \quad (7)$$

Let $c_2Ereal_2Epow : \iota$ be given. Assume the following.

$$c_2Ereal_2Epow \in ((ty_2Erealax_2Ereal^{ty_2Enum_2Enum})^{ty_2Erealax_2Ereal}) \quad (8)$$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (9)$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \quad (10)$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (11)$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \quad (12)$$

Definition 13 We define c_Emin_E40 to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A)$) of type $\iota \Rightarrow \iota$.

Definition 14 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_Emin_E40 (ap P x)) V0a)$.

Let $c_2Erealax_2Etreallt : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreallt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \quad (13)$$

Definition 15 We define $c_Erealax_Ereal_lt$ to be $\lambda V0T1 \in ty_Erealax_Ereal.\lambda V1T2 \in ty_Erealax_Ereal.$

Definition 16 We define $c_Ebool_E_E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_E_E21) 2) (\lambda V2t \in 2.))$

Definition 17 We define $c_Etransc_Eroot$ to be $\lambda V0n \in ty_Eenum_Eenum.\lambda V1x \in ty_Erealax_Ereal.$

Definition 18 We define $c_Etransc_Esqrt$ to be $\lambda V0x \in ty_Erealax_Ereal.(ap (ap c_Etransc_Eroot (ap$

Let $c_Erealax_Etrealm_inv : \iota$ be given. Assume the following.

$$c_Erealax_Etrealm_inv \in ((ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal) (ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)) (ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal) \quad (14)$$

Let $c_Erealax_Etrealm_eq : \iota$ be given. Assume the following.

$$c_Erealax_Etrealm_eq \in ((2^{(ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)} (ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)) (ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)) \quad (15)$$

Let $c_Erealax_Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_Erealax_Ereal_ABS_CLASS \in (ty_Erealax_Ereal)^{(2^{(ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)} (ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal))} \quad (16)$$

Definition 19 We define $c_Erealax_Ereal_ABS$ to be $\lambda V0r \in (ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)$

Definition 20 We define $c_Erealax_Einv$ to be $\lambda V0T1 \in ty_Erealax_Ereal.(ap c_Erealax_Ereal_ABS$

Let $c_Erealax_Etrealm_mul : \iota$ be given. Assume the following.

$$c_Erealax_Etrealm_mul \in (((ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal) (ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)) (ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)) (ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal) \quad (17)$$

Definition 21 We define $c_Erealax_Ereal_mul$ to be $\lambda V0T1 \in ty_Erealax_Ereal.\lambda V1T2 \in ty_Erealax_Ereal.$

Definition 22 We define $c_Ereal_E_E2F$ to be $\lambda V0x \in ty_Erealax_Ereal.\lambda V1y \in ty_Erealax_Ereal.$

Definition 23 We define $c_Ereal_Ereal_lte$ to be $\lambda V0x \in ty_Erealax_Ereal.\lambda V1y \in ty_Erealax_Ereal.$

Assume the following.

$$((ap c_Earithmic_EENUMERAL (ap c_Earithmic_EEBIT2 c_Earithmic_EZERO)) = (ap c_Eenum_EESUC (ap c_Earithmic_EENUMERAL (ap c_Earithmic_EEBIT1 c_Earithmic_EZERO)))) \quad (18)$$

Assume the following.

$$True \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \quad (20)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\
& p V0t))))))
\end{aligned} \tag{21}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1x \in ty_2Erealx_2Ereal. \\
& (\forall V2y \in ty_2Erealx_2Ereal. (((p (ap (ap (ap c_2Ereal_2Ereal_lte \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V1x)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte \\
& (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V2y))) \Rightarrow ((ap (ap c_2Etransc_2Eroot \\
& (ap c_2Enum_2ESUC V0n)) (ap (ap c_2Ereal_2E_2F V1x) V2y)) = (ap (\\
& ap c_2Ereal_2E_2F (ap (ap c_2Etransc_2Eroot (ap c_2Enum_2ESUC \\
& V0n)) V1x)) (ap (ap c_2Etransc_2Eroot (ap c_2Enum_2ESUC V0n)) V2y))))))
\end{aligned} \tag{22}$$

Theorem 1

$$\begin{aligned}
& (\forall V0x \in ty_2Erealx_2Ereal. (\forall V1y \in ty_2Erealx_2Ereal. \\
& (((p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) V0x)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\
& c_2Enum_2E0)) V1y))) \Rightarrow ((ap c_2Etransc_2Esqrt (ap (ap c_2Ereal_2E_2F \\
& V0x) V1y)) = (ap (ap c_2Ereal_2E_2F (ap c_2Etransc_2Esqrt V0x)) \\
& (ap c_2Etransc_2Esqrt V1y))))))
\end{aligned}$$