thm_2Etransc_2ESQRT__EVEN__POW2 (TMKeNbAUj4azHpfsDsw7WP6CkmgrpPfsaU8)

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Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{1}$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{3}$$

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$
of type $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Enum\_2EREP\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EREP\_\_num \in (omega^{ty\-2Enum\-2Enum}) \tag{4}$$

Let $c\_2Enum\_2ESUC\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2ESUC\_\_REP \in (omega^{omega}) \tag{5}$$

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\-27a}).(ap\ (ap\ ($c_2Emin_2E_3D$\ (2^{A\-27}$

Definition 5 *We define* c_2Enum_2ESUC *to be* $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_\_num\ ($

Let $c\_2Earithmetic\_2E\_2B : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\-2Enum\-2Enum})^{ty\-2Enum\-2Enum}) \tag{6}$$

Definition 6 *We define* c_2Earithmetic_2EBIT1 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2$

Definition 7 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$
of type $\iota \Rightarrow \iota$.

Definition 8 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (c\_2Emin\_2E\_40\ A$

Let $c\_2Earithmetic\_2EEVEN : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \qquad (7)$$

Let $c\_2Earithmetic\_2EDIV : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(8)$$

Let $c\_2Earithmetic\_2EMOD : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2EMOD \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(9)$$

Definition 9 *We define* c_2Ebool_2EF *to be* $(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 10 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$
of type ι.

Definition 11 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ c\_2Emin\_2E\_3D\_3D\_3E\ V0t)\ c\_2Ebool\_2E$

Definition 12 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c\_2Ebool\_2E\_21\ 2)\ (\lambda V2t \in$

Definition 13 *We define* c_2Eprim__rec_2E_3C *to be* $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enu$

Let $c\_2Earithmetic\_2E\_2A : \iota$ be given. Assume the following.

$$c\_2Earithmetic\_2E\_2A \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum})$$
$$(10)$$

Definition 14 *We define* c_2Earithmetic_2EZERO *to be* c_2Enum_2E0.

Definition 15 *We define* c_2Earithmetic_2EBIT2 *to be* $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic$

Definition 16 *We define* c_2Earithmetic_2ENUMERAL *to be* $\lambda V0x \in ty\_2Enum\_2Enum.V0x$.

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \qquad (11)$$

Let $c\_2Ereal\_2Epow : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})^{ty\_2Erealax\_2Ereal})$$
$$(12)$$

Let $c\_2Ereal\_2Ereal\_of\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \qquad (13)$$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \qquad (14)$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod$$
$$A0\ A1)$$
$$(15)$$

Let $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax}$$
$$(16)$$

Definition 17 *We define* c_2Erealax_2Ereal_REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap\ ($c_2Emin_2E_40\ (t$

Let $c\_2Erealax\_2Etreal\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehr}$$
$$(17)$$

Definition 18 *We define* c_2Erealax_2Ereal_lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_$

Definition 19 *We define* c_2Etransc_2Eroot *to be* $\lambda V0n \in ty\_2Enum\_2Enum.\lambda V1x \in ty\_2Erealax\_2Erea$

Definition 20 *We define* c_2Etransc_2Esqrt *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ (ap$ c_2Etransc_2Eroot\ ($

Definition 21 *We define* c_2Ereal_2Ereal_lte *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ere$

Assume the following.

$$((ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \text{c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO})) =$$
$$(ap\ \text{c\_2Enum\_2ESUC}\ (ap\ \text{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \text{c\_2Earithmetic\_2EBIT1}$$
$$\text{c\_2Earithmetic\_2EZERO}))))$$
$$(18)$$

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.($$
$$((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ \text{c\_2Enum\_2E0})\ V0m) = V0m) \wedge (((ap\ ($$
$$ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ \text{c\_2Enum\_2E0}) = V0m) \wedge (((ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$(ap\ \text{c\_2Enum\_2ESUC}\ V0m))\ V1n) = (ap\ \text{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B$$
$$V0m)\ V1n))) \wedge ((ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ (ap\ \text{c\_2Enum\_2ESUC}$$
$$V1n)) = (ap\ \text{c\_2Enum\_2ESUC}\ (ap\ (ap\ c\_2Earithmetic\_2E\_2B\ V0m)\ V1n)))))))))$$
$$(19)$$

3

Assume the following.

$$(\forall V0m \in ty\_2Enum\_2Enum.(\forall V1n \in ty\_2Enum\_2Enum.(\\
(ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V0m)\ V1n) = (ap\ (ap\ c\_2Earithmetic\_2E\_2A\\
V1n)\ V0m))))$$

$$(20)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((p\ (ap\ c\_2Earithmetic\_2EEVEN\ V0n)) \Leftrightarrow\\
(\exists V1m \in ty\_2Enum\_2Enum.(V0n = (ap\ (ap\ c\_2Earithmetic\_2E\_2A\\
(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO})))\\
V1m)))))$$

$$(21)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\\
\mathsf{c\_2Enum\_2E0})\ V0n)) \Rightarrow (\forall V1k \in ty\_2Enum\_2Enum.((V1k = (ap\ (\\
ap\ c\_2Earithmetic\_2E\_2B\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A\ (ap\ (ap\ c\_2Earithmetic\_2EDIV\\
V1k)\ V0n))\ V0n))\ (ap\ (ap\ c\_2Earithmetic\_2EMOD\ V1k)\ V0n))) \wedge (p\ (ap\\
(ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ (ap\ (ap\ c\_2Earithmetic\_2EMOD\ V1k)\ V0n))\\
V0n))))))$$

$$(22)$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.((p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\\
\mathsf{c\_2Enum\_2E0})\ V0n)) \Rightarrow (\forall V1k \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Earithmetic\_2EMOD\\
(ap\ (ap\ c\_2Earithmetic\_2E\_2A\ V1k)\ V0n))\ V0n) = \mathsf{c\_2Enum\_2E0}))))$$

$$(23)$$

Assume the following.

$$True$$
$$(24)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\\
V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{25}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Rightarrow True) \Leftrightarrow\\
True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge ((\tag{26}\\
(p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))))))$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow\\
True)) \tag{27}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow\\
(p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg(p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(\tag{28}\\
p\ V0t)))))))$$

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \mathsf{c\_2Eprim\_\_rec\_2E\_3C}\ \mathsf{c\_2Enum\_2E0})$$
$$(ap\ \mathsf{c\_2Enum\_2ESUC}\ V0n))))$$

(29)

Assume the following.

$$(\forall V0n \in ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}$$
$$(ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num$$
$$V0n))))$$

(30)

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}$$
$$(ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V0x))\Rightarrow(\forall V1n \in$$
$$ty\_2Enum\_2Enum.(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num$$
$$\mathsf{c\_2Enum\_2E0}))\ (ap\ (ap\ c\_2Ereal\_2Epow\ V0x)\ V1n))))))$$

(31)

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1m \in ty\_2Enum\_2Enum.$$
$$(\forall V2n \in ty\_2Enum\_2Enum.((ap\ (ap\ c\_2Ereal\_2Epow\ (ap\ (ap\ c\_2Ereal\_2Epow$$
$$V0x)\ V1m))\ V2n) = (ap\ (ap\ c\_2Ereal\_2Epow\ V0x)\ (ap\ (ap\ c\_2Earithmetic\_2E\_2A$$
$$V1m)\ V2n))))))$$

(32)

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.$$
$$(((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num$$
$$\mathsf{c\_2Enum\_2E0}))\ V0x))\wedge((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num$$
$$\mathsf{c\_2Enum\_2E0}))\ V1y))\wedge((ap\ (ap\ c\_2Ereal\_2Epow\ V1y)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO}))) = V0x)))\Rightarrow$$
$$((ap\ \mathsf{c\_2Etransc\_2Esqrt}\ V0x) = V1y))))$$

(33)

Theorem 1

$$(\forall V0n \in ty\_2Enum\_2Enum.((p\ (ap\ c\_2Earithmetic\_2EEVEN\ V0n))\Rightarrow$$
$$((ap\ \mathsf{c\_2Etransc\_2Esqrt}\ (ap\ (ap\ c\_2Ereal\_2Epow\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num$$
$$(ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO}))))$$
$$V0n)) = (ap\ (ap\ c\_2Ereal\_2Epow\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ (ap$$
$$\mathsf{c\_2Earithmetic\_2ENUMERAL}\ (ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO})))$$
$$(ap\ (ap\ c\_2Earithmetic\_2EDIV\ V0n)\ (ap\ \mathsf{c\_2Earithmetic\_2ENUMERAL}$$
$$(ap\ \mathsf{c\_2Earithmetic\_2EBIT2}\ \mathsf{c\_2Earithmetic\_2EZERO}))))))))$$