

thm_2Etransc_2ESQRT__MONO__LE (TMXp- kun6AQZWEBp1K2bwmHarDTWfCpRbULg)

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Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \tag{1}$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{2}$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \tag{3}$$

Definition 1 We define c_2Emin_2E3D to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{4}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \tag{5}$$

Definition 3 We define c_2Ebool_2ET to be $(ap\ (ap\ (c_2Emin_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

Definition 4 We define c_2Ebool_2E21 to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c_2Emin_2E3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V1x \in 2.V1x))$

Definition 5 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num\ (c_2Enum_2E0\ c_2Enum_2EREP_num\ c_2Enum_2ESUC_REP))$

Let $c_2Earithmetic_2E2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \tag{6}$$

Definition 6 We define $c_Earithmetic_EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_Earithmetic_E$

Definition 7 We define c_Ebool_EF to be $(ap (c_Ebool_E21 2) (\lambda V0t \in 2.V0t))$.

Definition 8 We define $c_Emin_E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 9 We define c_Ebool_E7E to be $(\lambda V0t \in 2.(ap (ap c_Emin_E3D_3D_3E V0t) c_Ebool_EF$

Definition 10 We define $c_Earithmetic_EZERO$ to be c_Enum_E0 .

Definition 11 We define $c_Earithmetic_EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_Earithmetic_E$

Definition 12 We define $c_Earithmetic_ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $ty_2Erealx_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealx_2Ereal \tag{7}$$

Let $c_2Ereal_2Epow : \iota$ be given. Assume the following.

$$c_2Ereal_2Epow \in ((ty_2Erealx_2Ereal^{ty_2Enum_2Enum})^{ty_2Erealx_2Ereal}) \tag{8}$$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealx_2Ereal^{ty_2Enum_2Enum}) \tag{9}$$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{10}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{11}$$

Let $c_2Erealx_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealx_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealx_2Ereal}) \tag{12}$$

Definition 13 We define c_Emin_E40 to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$ then (the $(\lambda x.x \in A \wedge P x)$ of type $\iota \Rightarrow \iota$).

Definition 14 We define $c_2Erealx_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealx_2Ereal.(ap (c_Emin_E40 (t$

Let $c_2Erealx_2Etreal_lt : \iota$ be given. Assume the following.

$$c_2Erealx_2Etreal_lt \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)}) \tag{13}$$

Definition 15 We define $c_2Erealax_2Ereal_lt$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal.$

Definition 16 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.))$

Definition 17 We define $c_2Etransc_2Eroot$ to be $\lambda V0n \in ty_2Enum_2Enum.\lambda V1x \in ty_2Erealax_2Ereal.$

Definition 18 We define $c_2Etransc_2Esqrt$ to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap (ap c_2Etransc_2Eroot ($

Definition 19 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.$

Assume the following.

$$\begin{aligned} & ((ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT2 c_2Earithmetic_2EZERO)) = \\ & (ap c_2Enum_2ESUC (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 \\ & c_2Earithmetic_2EZERO)))) \end{aligned} \tag{14}$$

Assume the following.

$$True \tag{15}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t)) \Leftrightarrow (p V0t))) \tag{16}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\ & p V0t)))))) \end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty_2Enum_2Enum.(\forall V1x \in ty_2Erealax_2Ereal. \\ & (\forall V2y \in ty_2Erealax_2Ereal.(((p (ap (ap c_2Ereal_2Ereal_lte \\ & (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V1x)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte \\ & V1x) V2y))) \Rightarrow (p (ap (ap c_2Ereal_2Ereal_lte (ap (ap c_2Etransc_2Eroot \\ & (ap c_2Enum_2ESUC V0n)) V1x)) (ap (ap c_2Etransc_2Eroot (ap c_2Enum_2ESUC \\ & V0n)) V2y)))))) \end{aligned} \tag{18}$$

Theorem 1

$$\begin{aligned} & (\forall V0x \in ty_2Erealax_2Ereal.(\forall V1y \in ty_2Erealax_2Ereal. \\ & (((p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Ereal_2Ereal_of_num \\ & c_2Enum_2E0)) V0x)) \wedge (p (ap (ap c_2Ereal_2Ereal_lte V0x) V1y))) \Rightarrow \\ & (p (ap (ap c_2Ereal_2Ereal_lte (ap c_2Etransc_2Esqrt V0x)) (ap \\ & c_2Etransc_2Esqrt V1y)))))) \end{aligned}$$