

# thm\_2Etransc\_2ESQRT\_\_POW2 (TM- cQV2mj2McEYZ4J9By6dyBWsoLL46CaFKs)

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Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \tag{1}$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{2}$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum^{\omega}) \tag{3}$$

**Definition 1** We define  $c\_2Emin\_2E3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Enum\_2EREP\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EREP\_num \in (\omega^{ty\_2Enum\_2Enum}) \tag{4}$$

Let  $c\_2Enum\_2ESUC\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2ESUC\_REP \in (\omega^{\omega}) \tag{5}$$

**Definition 3** We define  $c\_2Ebool\_2ET$  to be  $(ap\ (ap\ (c\_2Emin\_2E3D\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1x))$

**Definition 4** We define  $c\_2Ebool\_2E21$  to be  $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap\ (ap\ (c\_2Emin\_2E3D\ (2^{A-27a}))\ (\lambda V1x \in 2.V1x))\ (\lambda V1x \in 2.V1x))$

**Definition 5** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num\ (c\_2Enum\_2E0\ m))$

Let  $c\_2Earithmetic\_2E2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \tag{6}$$

**Definition 6** We define  $c\_Earithmic\_EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_Earithmic\_EBIT1 V0n) c\_Earithmic\_EBIT1)$ .

**Definition 7** We define  $c\_Ebool\_EF$  to be  $(ap (c\_Ebool\_EF 2) (\lambda V0t \in 2.V0t))$ .

**Definition 8** We define  $c\_Emin\_E3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 9** We define  $c\_Ebool\_E7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_Emin\_E3D\_3D\_3E V0t) c\_Ebool\_EF V0t))$ .

**Definition 10** We define  $c\_Ebool\_E2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_Ebool\_E21 2) (\lambda V2t \in 2.V2t) V1t2) V0t1))$ .

**Definition 11** We define  $c\_Earithmic\_EZERO$  to be  $c\_2Enum\_2E0$ .

**Definition 12** We define  $c\_Earithmic\_EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap (ap c\_Earithmic\_EBIT2 V0n) c\_Earithmic\_EBIT2)$ .

**Definition 13** We define  $c\_Earithmic\_ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{7}$$

Let  $c\_2Ereal\_2Epow : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})^{ty\_2Erealax\_2Ereal}) \tag{8}$$

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \tag{9}$$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{10}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{11}$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal}) \tag{12}$$

**Definition 14** We define  $c\_Emin\_E40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) then (the (\lambda x.x \in A \wedge P x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 15** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_Emin\_E40 (ap P a) a) V0a)$ .

Let  $c\_2Erealax\_2Etreal\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etreal\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \tag{13}$$

**Definition 16** We define  $c\_2Erealax\_2Ereal\_lte$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal.$

**Definition 17** We define  $c\_2Etransc\_2Eroot$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.\lambda V1x \in ty\_2Erealax\_2Ereal.$

**Definition 18** We define  $c\_2Etransc\_2Esqrt$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap (ap c\_2Etransc\_2Eroot ($

**Definition 19** We define  $c\_2Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.$

Assume the following.

$$\begin{aligned} & ((ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO)) = \\ & (ap c\_2Enum\_2ESUC (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT1 \\ & c\_2Earithmetic\_2EZERO)))) \end{aligned} \tag{14}$$

Assume the following.

$$True \tag{15}$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p \\ & V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \end{aligned} \tag{16}$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg( \\ & p V0t)))))) \end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in ty\_2Erealax\_2Ereal.(p (ap (ap c\_2Ereal\_2Ereal\_lte \\ & (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) (ap (ap c\_2Ereal\_2Epow \\ & V0x) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 \\ & c\_2Earithmetic\_2EZERO)))))) \end{aligned} \tag{18}$$

Assume the following.

$$\begin{aligned} & (\forall V0n \in ty\_2Enum\_2Enum.(\forall V1x \in ty\_2Erealax\_2Ereal. \\ & ((p (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Ereal\_2Ereal\_of\_num \\ & c\_2Enum\_2E0)) V1x)) \Rightarrow ((ap (ap c\_2Ereal\_2Epow (ap (ap c\_2Etransc\_2Eroot \\ & (ap c\_2Enum\_2ESUC V0n)) V1x)) (ap c\_2Enum\_2ESUC V0n)) = V1x)))) \end{aligned} \tag{19}$$

**Theorem 1**

$$\begin{aligned} & (\forall V0x \in ty\_2Erealax\_2Ereal.(((ap (ap c\_2Ereal\_2Epow (ap \\ & c\_2Etransc\_2Esqrt V0x)) (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 \\ & c\_2Earithmetic\_2EZERO))) = V0x) \Leftrightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte \\ & (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) V0x)))) \end{aligned}$$