

thm_2Etransc_2ETAN__SEC
(TMJmL24tV4koAoYjUk2VRbT9YYGjEypDNY2)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_21$ to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{3}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax_2Ereal}) \tag{4}$$

Definition 3 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap\ P\ x))$ then (the $(\lambda x.x \in A \wedge p)$ of type $\iota \Rightarrow \iota$).

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})))$

Definition 5 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (ty$

Let $c_2Erealax_2Etreall_inv : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_inv \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal}) \tag{5}$$

Let $c_2Erealax_2Etreal_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)) \quad (6)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}} \quad (7)$$

Definition 6 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 7 We define $c_2Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal_ABS)$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (8)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (9)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum)^{\omega} \quad (10)$$

Definition 8 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 9 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 10 We define $c_2Emin_2E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow p\ Q)$ of type ι .

Definition 11 We define c_2Ebool_2E7E to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E3D_3D_3E\ V0t)\ c_2Ebool_2E21))$

Let $c_2Erealax_2Etreal_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_mul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)) \quad (11)$$

Definition 12 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (12)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (13)$$

Definition 13 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num)$

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (14)$$

Definition 14 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B))$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (15)$$

Definition 15 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Definition 16 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2Earithmetic_2E_2B))$

Definition 17 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Let $c_2Ereal_2Epow : \iota$ be given. Assume the following.

$$c_2Ereal_2Epow \in ((ty_2Erealax_2Ereal^{ty_2Enum_2Enum})^{ty_2Erealax_2Ereal}) \quad (16)$$

Let $c_2Erealax_2Etrealm_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_add \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}) \quad (17)$$

Definition 18 We define $c_2Erealax_2Etrealm_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Earithmetic_2EFACT : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EFACT \in (ty_2Enum_2Enum^{ty_2Enum_2Enum}) \quad (18)$$

Let $c_2Earithmetic_2EDIV : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EDIV \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (19)$$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}) \quad (20)$$

Definition 19 We define $c_2Erealax_2Etrealm_neg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap c_2Erealax_2Ereal$

Definition 20 We define $c_2Ereal_2E_2F$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Let $c_2Earithmetic_2EEVEN : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEVEN \in (2^{ty_2Enum_2Enum}) \quad (21)$$

Definition 41 We define $c_Etransc_Etan$ to be $\lambda V0x \in ty_Erealax_Ereal.(ap (ap c_Ereal_E2F (ap c$

Assume the following.

$$True \tag{34}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \tag{35}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{36}$$

Assume the following.

$$(\forall V0x \in ty_Erealax_Ereal.(\forall V1y \in ty_Erealax_Ereal. ((ap (ap c_Erealax_Ereal_add V0x) V1y) = (ap (ap c_Erealax_Ereal_add V1y) V0x)))) \tag{37}$$

Assume the following.

$$(\forall V0x \in ty_Erealax_Ereal.((ap (ap c_Erealax_Ereal_mul (ap c_Ereal_Ereal_of_num (ap c_Earithmic_ENUMERAL (ap c_Earithmic_EBIT1 c_Earithmic_EZERO)))) V0x) = V0x)) \tag{38}$$

Assume the following.

$$(\forall V0x \in ty_Erealax_Ereal.(\forall V1y \in ty_Erealax_Ereal.(\forall V2z \in ty_Erealax_Ereal.((ap (ap c_Erealax_Ereal_mul (ap (ap c_Erealax_Ereal_add V0x) V1y)) V2z) = (ap (ap c_Erealax_Ereal_add (ap (ap c_Erealax_Ereal_mul V0x) V2z)) (ap (ap c_Erealax_Ereal_mul V1y) V2z)))))) \tag{39}$$

Assume the following.

$$(\forall V0x \in ty_Erealax_Ereal.((\neg(V0x = (ap c_Ereal_Ereal_of_num c_Eenum_E0))) \Rightarrow ((ap (ap c_Ereal_E2F V0x) V0x) = (ap c_Ereal_Ereal_of_num (ap c_Earithmic_ENUMERAL (ap c_Earithmic_EBIT1 c_Earithmic_EZERO)))))) \tag{40}$$

Assume the following.

$$(\forall V0c \in ty_Erealax_Ereal.(\forall V1n \in ty_Eenum_Eenum. ((\neg(V0c = (ap c_Ereal_Ereal_of_num c_Eenum_E0))) \Rightarrow (\neg((ap (ap c_Ereal_E2F V0c) V1n) = (ap c_Ereal_Ereal_of_num c_Eenum_E0)))))) \tag{41}$$

Assume the following.

$$\begin{aligned}
& (\forall V0c \in ty_2Erealax_2Ereal. ((\neg(V0c = (ap\ c_2Ereal_2Ereal_of_num \\
& \quad c_2Enum_2E0))) \Rightarrow (\forall V1n \in ty_2Enum_2Enum. ((ap\ c_2Erealax_2Einv \\
& (ap\ (ap\ c_2Ereal_2Epow\ V0c)\ V1n)) = (ap\ (ap\ c_2Ereal_2Epow\ (ap\ c_2Erealax_2Einv \\
& \quad V0c))\ V1n))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1x \in ty_2Erealax_2Ereal. \\
& (\forall V2y \in ty_2Erealax_2Ereal. ((ap\ (ap\ c_2Ereal_2Epow\ (ap \\
& (ap\ c_2Erealax_2Ereal_mul\ V1x)\ V2y))\ V0n) = (ap\ (ap\ c_2Erealax_2Ereal_mul \\
& (ap\ (ap\ c_2Ereal_2Epow\ V1x)\ V0n))\ (ap\ (ap\ c_2Ereal_2Epow\ V2y)\ V0n))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((ap\ (ap\ c_2Erealax_2Ereal_add \\
& (ap\ (ap\ c_2Ereal_2Epow\ (ap\ c_2Etransc_2Esin\ V0x))\ (ap\ c_2Earithmetic_2ENUMERAL \\
& \quad (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO))))\ (ap\ (ap \\
& \quad c_2Ereal_2Epow\ (ap\ c_2Etransc_2Ecos\ V0x))\ (ap\ c_2Earithmetic_2ENUMERAL \\
& (ap\ c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO)))) = (ap\ c_2Ereal_2Ereal_of_num \\
& (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))))))
\end{aligned} \tag{44}$$

Theorem 1

$$\begin{aligned}
& (\forall V0x \in ty_2Erealax_2Ereal. ((\neg((ap\ c_2Etransc_2Ecos\ V0x) = \\
& (ap\ c_2Ereal_2Ereal_of_num\ c_2Enum_2E0))) \Rightarrow ((ap\ (ap\ c_2Erealax_2Ereal_add \\
& (ap\ c_2Ereal_2Ereal_of_num\ (ap\ c_2Earithmetic_2ENUMERAL\ (\\
& \quad ap\ c_2Earithmetic_2EBIT1\ c_2Earithmetic_2EZERO))))\ (ap\ (ap\ c_2Ereal_2Epow \\
& (ap\ c_2Etransc_2Etan\ V0x))\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap\ c_2Earithmetic_2EBIT2 \\
& \quad c_2Earithmetic_2EZERO)))) = (ap\ (ap\ c_2Ereal_2Epow\ (ap\ c_2Erealax_2Einv \\
& (ap\ c_2Etransc_2Ecos\ V0x))\ (ap\ c_2Earithmetic_2ENUMERAL\ (ap \\
& \quad c_2Earithmetic_2EBIT2\ c_2Earithmetic_2EZERO))))))
\end{aligned}$$