

# thm\_2Etransc\_2ETAN\_\_TOTAL\_\_LEMMA (TMQHRupte6WizvQevyiAtHorvPNcYZa1vP1)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_21$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let  $ty\_2Ehreal\_2Ehreal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{1}$$

Let  $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let  $ty\_2Erealax\_2Ereal : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{3}$$

Let  $c\_2Erealax\_2Ereal\_REP\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_REP\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax\_2Ereal}) \tag{4}$$

**Definition 3** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap\ P\ x))$  then (the  $(\lambda x.x \in A \wedge p)$  of type  $\iota \Rightarrow \iota$ ).

**Definition 4** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A-27a})))$

**Definition 5** We define  $c\_2Erealax\_2Ereal\_REP$  to be  $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap (c\_2Emin\_2E\_40 (ty$

Let  $c\_2Erealax\_2Etrealm\_neg : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_neg \in ((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal}) \tag{5}$$

Let  $c\_2Erealax\_2Etrealm\_eq : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (6)$$

Let  $c\_2Erealax\_2Ereal\_ABS\_CLASS : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Ereal\_ABS\_CLASS \in (ty\_2Erealax\_2Ereal)^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})} \quad (7)$$

**Definition 6** We define  $c\_2Erealax\_2Ereal\_ABS$  to be  $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)$

**Definition 7** We define  $c\_2Erealax\_2Ereal\_neg$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.(ap\ c\_2Erealax\_2Ereal\_ABS)$

Let  $c\_2Erealax\_2Etrealm\_add : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_add \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)}) \quad (8)$$

**Definition 8** We define  $c\_2Erealax\_2Ereal\_add$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 9** We define  $c\_2Ereal\_2Ereal\_sub$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal$

Let  $c\_2Enum\_2EZERO\_REP : \iota$  be given. Assume the following.

$$c\_2Enum\_2EZERO\_REP \in \omega \quad (9)$$

Let  $ty\_2Enum\_2Enum : \iota$  be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (10)$$

Let  $c\_2Enum\_2EABS\_num : \iota$  be given. Assume the following.

$$c\_2Enum\_2EABS\_num \in (ty\_2Enum\_2Enum)^{\omega} \quad (11)$$

**Definition 10** We define  $c\_2Enum\_2E0$  to be  $(ap\ c\_2Enum\_2EABS\_num\ c\_2Enum\_2EZERO\_REP)$ .

Let  $c\_2Ereal\_2Ereal\_of\_num : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Ereal\_of\_num \in (ty\_2Erealax\_2Ereal)^{ty\_2Enum\_2Enum} \quad (12)$$

Let  $c\_2Erealax\_2Etrealm\_lt : \iota$  be given. Assume the following.

$$c\_2Erealax\_2Etrealm\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal)}) \quad (13)$$

**Definition 11** We define  $c\_2Erealax\_2Ereal\_lt$  to be  $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_2Ereal$

**Definition 12** We define  $c\_2Ebool\_2EF$  to be  $(ap\ (c\_2Ebool\_2E21\ 2)\ (\lambda V0t \in 2.V0t))$ .

**Definition 13** We define  $c\_2Emin\_2E3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o\ (p\ P \Rightarrow p\ Q)$  of type  $\iota$ .

**Definition 14** We define  $c\_Ebool\_2E\_7E$  to be  $(\lambda V0t \in 2.(ap (ap c\_Emin\_2E\_3D\_3D\_3E V0t) c\_Ebool\_2E\_7E))$

**Definition 15** We define  $c\_Ereal\_2Ereal\_lte$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Ereal.$

**Definition 16** We define  $c\_Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_Ebool\_2E\_21) 2) (\lambda V2t \in 2.))$

**Definition 17** We define  $c\_Ebool\_2ECOND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.))$

**Definition 18** We define  $c\_Ereal\_2Eabs$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap (ap (ap (c\_Ebool\_2ECOND$

Let  $c\_Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_Epair\_2ESND \\ A\_27a A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod A\_27a A\_27b)}) \end{aligned} \quad (14)$$

Let  $c\_Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_Epair\_2EFST \\ A\_27a A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod A\_27a A\_27b)}) \end{aligned} \quad (15)$$

**Definition 19** We define  $c\_Epair\_2EUNCURRY$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27a})$

Let  $ty\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty\_2Emetric\_2Emetric A0) \quad (16)$$

Let  $c\_2Emetric\_2Emetric : \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emetric\_2Emetric A\_27a \in ((ty\_2Emetric\_2Emetric \\ A\_27a)^{(ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod A\_27a A\_27a)})}) \end{aligned} \quad (17)$$

**Definition 20** We define  $c\_2Emetric\_2Emr1$  to be  $(ap (c\_2Emetric\_2Emetric ty\_2Erealax\_2Ereal) (ap (c\_2Emetric\_2Emetric$

Let  $c\_Epair\_2EABS\_prod : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\begin{aligned} \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow c\_Epair\_2EABS\_prod \\ A\_27a A\_27b \in ((ty\_2Epair\_2Eprod A\_27a A\_27b)^{(2^{A\_27b})^{A\_27a}}) \end{aligned} \quad (18)$$

**Definition 21** We define  $c\_Epair\_2E\_2C$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap (c\_2Emetric\_2Emetric$

Let  $c\_2Enets\_2Etendsto : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Enets\_2Etendsto A\_27a \in (((2^{A\_27a})^{A\_27a})^{(ty\_2Epair\_2Eprod (ty\_2Emetric\_2Emetric A\_27a) A\_27a)}) \quad (19)$$

Let  $c\_2Emetric\_2Edist : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow c\_2Emetric\_2Edist A\_27a \in ((ty\_2Erealax\_2Ereal^{(ty\_2Epair\_2Eprod A\_27a A\_27a)})^{(ty\_2Emetric\_2Edist A\_27a)}) \quad (20)$$

**Definition 22** We define  $c\_Ebool\_E\_3F$  to be  $\lambda A\_27a : \iota. (\lambda V0P \in (2^{A\_27a}). (ap\ V0P\ (ap\ (c\_Emin\_E\_40$

Let  $ty\_Etopology\_Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_Etopology\_Etopology\ A0) \quad (21)$$

Let  $c\_Etopology\_Etopology : \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_Etopology\_Etopology\ A\_27a \in ((ty\_Etopology\_Etopology\ A\_27a)^{2^{(2^{A\_27a}})})) \quad (22)$$

**Definition 23** We define  $c\_Emetric\_E\_mtop$  to be  $\lambda A\_27a : \iota. \lambda V0m \in (ty\_Emetric\_Emetric\ A\_27a). (ap$

Let  $c\_Enets\_E\_tends : \iota \Rightarrow \iota \Rightarrow \iota$  be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_Enets\_E\_tends\ A\_27a\ A\_27b \in (((2^{(ty\_Epair\_Eprod\ (ty\_Etopology\_Etopology\ A\_27a)\ (2^{A\_27b})^{A\_27b}})))^{A\_27a})^{A\_27b} \quad (23)$$

**Definition 24** We define  $c\_Elim\_E\_tends\_real\_real$  to be  $\lambda V0f \in (ty\_Erealax\_Ereal^{ty\_Erealax\_Ereal}).$

**Definition 25** We define  $c\_Elim\_E\_contl$  to be  $\lambda V0f \in (ty\_Erealax\_Ereal^{ty\_Erealax\_Ereal}). \lambda V1x \in ty$

Let  $c\_Erealax\_Ereal\_mul : \iota$  be given. Assume the following.

$$c\_Erealax\_Ereal\_mul \in (((ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)^{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal)}))^{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal)} \quad (24)$$

**Definition 26** We define  $c\_Erealax\_Ereal\_mul$  to be  $\lambda V0T1 \in ty\_Erealax\_Ereal. \lambda V1T2 \in ty\_Erealax\_Ereal.$

Let  $c\_Erealax\_Ereal\_inv : \iota$  be given. Assume the following.

$$c\_Erealax\_Ereal\_inv \in ((ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal\ ty\_Ehreal\_Ehreal)^{(ty\_Epair\_Eprod\ ty\_Ehreal\_Ehreal)}) \quad (25)$$

**Definition 27** We define  $c\_Erealax\_Einv$  to be  $\lambda V0T1 \in ty\_Erealax\_Ereal. (ap\ c\_Erealax\_Ereal\_ABS$

**Definition 28** We define  $c\_Ebool\_E\_5C\_2F$  to be  $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c\_Ebool\_E\_21\ 2)\ (\lambda V2t \in$

**Definition 29** We define  $c\_Ereal\_E\_2F$  to be  $\lambda V0x \in ty\_Erealax\_Ereal. \lambda V1y \in ty\_Erealax\_Ereal. ($

**Definition 30** We define  $c\_Elim\_E\_diff$  to be  $\lambda V0f \in (ty\_Erealax\_Ereal^{ty\_Erealax\_Ereal}). \lambda V1l \in ty\_Erealax\_Ereal. ($

Let  $c\_Enum\_E\_REP\_num : \iota$  be given. Assume the following.

$$c\_Enum\_E\_REP\_num \in (omega^{ty\_Enum\_Enum}) \quad (26)$$

Let  $c\_Enum\_E\_SUC\_REP : \iota$  be given. Assume the following.

$$c\_Enum\_E\_SUC\_REP \in (omega^{omega}) \quad (27)$$

**Definition 31** We define  $c\_2Enum\_2ESUC$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.(ap\ c\_2Enum\_2EABS\_num)$   
Let  $c\_2Earithmetic\_2E\_2B : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2B \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (28)$$

**Definition 32** We define  $c\_2Earithmetic\_2EBIT1$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2E\_2B))$

**Definition 33** We define  $c\_2Earithmetic\_2EZERO$  to be  $c\_2Enum\_2E0$ .

**Definition 34** We define  $c\_2Earithmetic\_2EBIT2$  to be  $\lambda V0n \in ty\_2Enum\_2Enum.(ap\ (ap\ c\_2Earithmetic\_2E\_2B))$

**Definition 35** We define  $c\_2Earithmetic\_2ENUMERAL$  to be  $\lambda V0x \in ty\_2Enum\_2Enum.V0x$ .

Let  $c\_2Ereal\_2Epow : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Epow \in ((ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})^{ty\_2Erealax\_2Ereal}) \quad (29)$$

Let  $c\_2Earithmetic\_2EFACT : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EFACT \in (ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum}) \quad (30)$$

Let  $c\_2Earithmetic\_2EDIV : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EDIV \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (31)$$

Let  $c\_2Earithmetic\_2EEVEN : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2EEVEN \in (2^{ty\_2Enum\_2Enum}) \quad (32)$$

Let  $c\_2Ereal\_2Esum : \iota$  be given. Assume the following.

$$c\_2Ereal\_2Esum \in ((ty\_2Erealax\_2Ereal^{(ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum})})^{(ty\_2Epair\_2Eprod\ ty\_2Enum\_2Enum)}) \quad (33)$$

**Definition 36** We define  $c\_2Eprim\_rec\_2E\_3C$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 37** We define  $c\_2Earithmetic\_2E\_3E$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 38** We define  $c\_2Earithmetic\_2E\_3E\_3D$  to be  $\lambda V0m \in ty\_2Enum\_2Enum.\lambda V1n \in ty\_2Enum\_2Enum$

**Definition 39** We define  $c\_2Eseq\_2E\_2D\_2D\_3E$  to be  $\lambda V0x \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1x \in ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}$

**Definition 40** We define  $c\_2Eseq\_2Esums$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).\lambda V1s \in ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}$

**Definition 41** We define  $c\_2Eseq\_2Esuminf$  to be  $\lambda V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}).(ap\ (c\_2Eseq\_2Esums\ f))$

**Definition 42** We define  $c\_2Etransc\_2Ecos$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ c\_2Eseq\_2Esuminf\ (\lambda V1n \in ty\_2Enum\_2Enum.2^{ty\_2Enum\_2Enum}))$

**Definition 43** We define  $c\_2Etransc\_2Epi$  to be  $(ap\ (ap\ c\_2Erealax\_2Ereal\_mul\ (ap\ c\_2Ereal\_2Ereal\_of\_m)))$

Let  $c\_2Earithmetic\_2E\_2D : \iota$  be given. Assume the following.

$$c\_2Earithmetic\_2E\_2D \in ((ty\_2Enum\_2Enum^{ty\_2Enum\_2Enum})^{ty\_2Enum\_2Enum}) \quad (34)$$

**Definition 44** We define  $c\_2Etransc\_2Esin$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ c\_2Eseq\_2Esuminf\ (\lambda V1x))$

**Definition 45** We define  $c\_2Etransc\_2Etan$  to be  $\lambda V0x \in ty\_2Erealax\_2Ereal.(ap\ (ap\ c\_2Ereal\_2E2F\ (ap\ c\_2Eseq\_2Esuminf\ (\lambda V1x))\ (ap\ c\_2Eseq\_2Esuminf\ (\lambda V1x))\ (ap\ c\_2Eseq\_2Esuminf\ (\lambda V1x))))$

Assume the following.

$$True \quad (35)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (36)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (37)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (38)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \quad (39)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (40)$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (41)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \quad (42)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow (\neg(p\ V0A)) \vee (\neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B)))))) \quad (43)$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1y0 \in \\
& \quad ty\_2Erealax\_2Ereal.(\forall V2x0 \in ty\_2Erealax\_2Ereal.((p ( \\
& \quad \quad ap (ap (ap c\_2Elim\_2Etends\_real\_real V0f) V1y0) V2x0)) \Leftrightarrow (\forall V3e \in \\
& ty\_2Erealax\_2Ereal.((p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad \quad c\_2Enum\_2E0)) V3e)) \Rightarrow (\exists V4d \in ty\_2Erealax\_2Ereal.((p (ap \\
& (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) \\
& \quad V4d)) \wedge (\forall V5x \in ty\_2Erealax\_2Ereal.(((p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& \quad (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) (ap c\_2Ereal\_2Eabs \\
& (ap (ap c\_2Ereal\_2Ereal\_sub V5x) V2x0)))) \wedge (p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& \quad (ap c\_2Ereal\_2Eabs (ap (ap c\_2Ereal\_2Ereal\_sub V5x) V2x0))) V4d))) \Rightarrow \\
& (p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Eabs (ap (ap c\_2Ereal\_2Ereal\_sub \\
& \quad (ap V0f V5x)) V1y0))) V3e)))))))))
\end{aligned} \tag{44}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1g \in \\
& (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V2l \in ty\_2Erealax\_2Ereal. \\
& \quad (\forall V3m \in ty\_2Erealax\_2Ereal.(\forall V4x \in ty\_2Erealax\_2Ereal. \\
& \quad \quad ((p (ap (ap (ap c\_2Elim\_2Etends\_real\_real V0f) V2l) V4x)) \wedge ( \\
& \quad \quad (p (ap (ap (ap c\_2Elim\_2Etends\_real\_real V1g) V3m) V4x)) \wedge (\neg ( \\
& \quad \quad V3m = (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)))))) \Rightarrow (p (ap (ap \\
& \quad (ap c\_2Elim\_2Etends\_real\_real (\lambda V5x \in ty\_2Erealax\_2Ereal. \\
& (ap (ap c\_2Ereal\_2E\_2F (ap V0f V5x)) (ap V1g V5x))) (ap (ap c\_2Ereal\_2E\_2F \\
& \quad V2l) V3m)) V4x))))))
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1l \in \\
& \quad ty\_2Erealax\_2Ereal.(\forall V2x \in ty\_2Erealax\_2Ereal.((p (ap \\
& (ap (ap c\_2Elim\_2Ediff1 V0f) V1l) V2x)) \Rightarrow (p (ap (ap c\_2Elim\_2Econt1 \\
& \quad V0f) V2x))))))
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& (\forall V0f \in (ty\_2Erealax\_2Ereal^{ty\_2Erealax\_2Ereal}).(\forall V1x \in \\
& \quad ty\_2Erealax\_2Ereal.((p (ap (ap c\_2Elim\_2Econt1 V0f) V1x)) \Leftrightarrow (p \\
& (ap (ap (ap c\_2Elim\_2Etends\_real\_real V0f) (ap V0f V1x)) V1x))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\neg ((ap c\_2Ereal\_2Ereal\_of\_num (ap c\_2Earithmetic\_2ENUMERAL \\
& (ap c\_2Earithmetic\_2EBIT1 c\_2Earithmetic\_2EZERO))) = (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad c\_2Enum\_2E0)))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((ap (ap c\_2Erealax\_2Ereal\_mul V0x) V1y) = (ap (ap c\_2Erealax\_2Ereal\_mul \\
& \quad V1y) V0x))))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (((p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad c\_2Enum\_2E0)) V0x)) \wedge (p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad c\_2Enum\_2E0)) V1y))) \Rightarrow (p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad c\_2Enum\_2E0)) (ap (ap c\_2Erealax\_2Ereal\_mul V0x) V1y))))))
\end{aligned} \tag{50}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (((ap (ap c\_2Erealax\_2Ereal\_mul V0x) V1y) = (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad c\_2Enum\_2E0)) \Leftrightarrow ((V0x = (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) \vee \\
& \quad (V1y = (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)))))
\end{aligned} \tag{51}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((\neg (p (ap (ap c\_2Ereal\_2Ereal\_lte V0x) V1y))) \Leftrightarrow (p (ap (ap c\_2Erealax\_2Ereal\_lt \\
& \quad V1y) V0x))))
\end{aligned} \tag{52}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& ((p (ap (ap c\_2Erealax\_2Ereal\_lt V0x) V1y)) \Rightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& \quad V0x) V1y))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((p (ap (ap c\_2Ereal\_2Ereal\_lte \\
& \quad (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) (ap c\_2Erealax\_2Ereal\_neg \\
& \quad V0x))) \Leftrightarrow (p (ap (ap c\_2Ereal\_2Ereal\_lte V0x) (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad c\_2Enum\_2E0)))))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (((p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& \quad c\_2Enum\_2E0)) (ap (ap c\_2Ereal\_2Ereal\_sub V0x) V1y))) \Leftrightarrow (p (ap \\
& \quad (ap c\_2Erealax\_2Ereal\_lt V1y) V0x))))))
\end{aligned} \tag{55}$$



Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((\neg(V0x = (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0))) \Rightarrow ((ap\ c\_2Erealax\_2Einv\ (ap\ c\_2Erealax\_2Einv\ V0x)) = V0x))) \quad (56)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0))\ V0x)) \Rightarrow (p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0))\ (ap\ c\_2Erealax\_2Einv\ V0x)))))) \quad (57)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ c\_2Ereal\_2Ereal\_lte\ V0x)\ (ap\ (ap\ c\_2Erealax\_2Ereal\_add\ V0x)\ V1y))) \Leftrightarrow (p\ (ap\ (ap\ c\_2Ereal\_2Ereal\_lte\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0))\ V1y)))))) \quad (58)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ c\_2Ereal\_2E2F\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0))\ V0x) = (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0))) \quad (59)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0))\ V0x)) \wedge (p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0))\ V1y))) \Rightarrow (\exists V2z \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0))\ V2z)) \wedge ((p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ V2z)\ V0x)) \wedge (p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ V2z)\ V1y)))))))))) \quad (60)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal.((ap\ (ap\ c\_2Ereal\_2Ereal\_sub\ (ap\ (ap\ c\_2Ereal\_2Ereal\_sub\ V0x)\ V1y))\ V0x) = (ap\ c\_2Erealax\_2Ereal\_neg\ V1y)))) \quad (61)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ c\_2Ereal\_2Ereal\_sub\ V0x)\ (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0)) = V0x)) \quad (62)$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (((p (ap (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)) V0x)) \wedge (p (ap (ap (ap c\_2Erealax\_2Ereal\_lt V0x) V1y)))) \Rightarrow \quad (63) \\
& (p (ap (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Erealax\_2Einv V1y)) (ap \\
& c\_2Erealax\_2Einv V0x))))))
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (\forall V1y \in ty\_2Erealax\_2Ereal. \\
& (((\neg(V0x = (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0))) \wedge (\neg(V1y = \\
& (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)))) \Rightarrow ((ap c\_2Erealax\_2Einv \\
& (ap (ap (ap c\_2Erealax\_2Ereal\_mul V0x) V1y)) = (ap (ap c\_2Erealax\_2Ereal\_mul \\
& (ap c\_2Erealax\_2Einv V0x)) (ap c\_2Erealax\_2Einv V1y)))))) \quad (64)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((p (ap (ap (ap c\_2Erealax\_2Ereal\_lt \\
& (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) V0x)) \Rightarrow (\neg(V0x = (ap \\
& c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)))))) \quad (65)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((ap c\_2Ereal\_2Eabs (ap c\_2Erealax\_2Ereal\_neg \\
& V0x)) = (ap c\_2Ereal\_2Eabs V0x))) \quad (66)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. ((\neg(V0x = (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0))) \Leftrightarrow (p (ap (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)) (ap c\_2Ereal\_2Eabs V0x)))))) \quad (67)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal. (((ap c\_2Ereal\_2Eabs V0x) = \\
& V0x) \Leftrightarrow (p (ap (ap (ap c\_2Ereal\_2Ereal\_lte (ap c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0)) V0x)))))) \quad (68)
\end{aligned}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (69)$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (70)$$

Assume the following.

$$\begin{aligned}
& (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow \\
& (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))))) \quad (71)
\end{aligned}$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (72)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (73)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (74)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q)) \vee (\neg(p V2r))) \wedge (((p V1q) \vee (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p)))))))) \quad (75)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p)))))))))) \quad (76)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p)))))) \quad (77)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p)))) \quad (78)$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. (((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))))) \quad (79)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal. (p (ap (ap (ap c\_2Elim\_2Ediff c\_2Etransc\_2Esin) (ap c\_2Etransc\_2Ecos V0x)) V0x))) \quad (80)$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal. (p (ap (ap (ap c\_2Elim\_2Ediff c\_2Etransc\_2Ecos) (ap c\_2Erealax\_2Ereal\_neg (ap c\_2Etransc\_2Esin V0x))) V0x))) \quad (81)$$

Assume the following.

$$\begin{aligned}
& ((ap\ c\_2Etransc\_2Ecos\ (ap\ (ap\ c\_2Ereal\_2E\_2F\ c\_2Etransc\_2Epi) \\
& (ap\ c\_2Ereal\_2Ereal\_of\_num\ (ap\ c\_2Earithmetic\_2ENUMERAL\ ( \\
& ap\ c\_2Earithmetic\_2EBIT2\ c\_2Earithmetic\_2EZERO)))))) = (ap\ c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0))
\end{aligned} \tag{82}$$

Assume the following.

$$\begin{aligned}
& ((p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ (ap\ c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0))\ (ap\ (ap\ c\_2Ereal\_2E\_2F\ c\_2Etransc\_2Epi)\ (ap\ c\_2Ereal\_2Ereal\_of\_num \\
& (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT2\ c\_2Earithmetic\_2EZERO)))))) \wedge \\
& (p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ (ap\ (ap\ c\_2Ereal\_2E\_2F\ c\_2Etransc\_2Epi) \\
& (ap\ c\_2Ereal\_2Ereal\_of\_num\ (ap\ c\_2Earithmetic\_2ENUMERAL\ ( \\
& ap\ c\_2Earithmetic\_2EBIT2\ c\_2Earithmetic\_2EZERO))))))\ (ap\ c\_2Ereal\_2Ereal\_of\_num \\
& (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT2\ c\_2Earithmetic\_2EZERO))))))
\end{aligned} \tag{83}$$

Assume the following.

$$\begin{aligned}
& ((ap\ c\_2Etransc\_2Esin\ (ap\ (ap\ c\_2Ereal\_2E\_2F\ c\_2Etransc\_2Epi) \\
& (ap\ c\_2Ereal\_2Ereal\_of\_num\ (ap\ c\_2Earithmetic\_2ENUMERAL\ ( \\
& ap\ c\_2Earithmetic\_2EBIT2\ c\_2Earithmetic\_2EZERO)))))) = (ap\ c\_2Ereal\_2Ereal\_of\_num \\
& (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT1\ c\_2Earithmetic\_2EZERO))))
\end{aligned} \tag{84}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt \\
& (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0))\ V0x)) \wedge (p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt \\
& V0x)\ (ap\ (ap\ c\_2Ereal\_2E\_2F\ c\_2Etransc\_2Epi)\ (ap\ c\_2Ereal\_2Ereal\_of\_num \\
& (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT2\ c\_2Earithmetic\_2EZERO)))))) \Rightarrow \\
& (p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ (ap\ c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0))\ (ap\ c\_2Etransc\_2Esin\ V0x))))
\end{aligned} \tag{85}$$

Assume the following.

$$\begin{aligned}
& (\forall V0x \in ty\_2Erealax\_2Ereal.(((p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt \\
& (ap\ c\_2Ereal\_2Ereal\_of\_num\ c\_2Enum\_2E0))\ V0x)) \wedge (p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt \\
& V0x)\ (ap\ (ap\ c\_2Ereal\_2E\_2F\ c\_2Etransc\_2Epi)\ (ap\ c\_2Ereal\_2Ereal\_of\_num \\
& (ap\ c\_2Earithmetic\_2ENUMERAL\ (ap\ c\_2Earithmetic\_2EBIT2\ c\_2Earithmetic\_2EZERO)))))) \Rightarrow \\
& (p\ (ap\ (ap\ c\_2Erealax\_2Ereal\_lt\ (ap\ c\_2Ereal\_2Ereal\_of\_num \\
& c\_2Enum\_2E0))\ (ap\ c\_2Etransc\_2Ecos\ V0x))))
\end{aligned} \tag{86}$$

**Theorem 1**

$$\begin{aligned} & (\forall V0y \in ty\_2Erealax\_2Ereal.((p (ap (ap c\_2Erealax\_2Ereal\_lt \\ & (ap c\_2Ereal\_2Ereal\_of\_num c\_2Enum\_2E0)) V0y)) \Rightarrow (\exists V1x \in \\ ty\_2Erealax\_2Ereal.((p (ap (ap c\_2Erealax\_2Ereal\_lt (ap c\_2Ereal\_2Ereal\_of\_num \\ & c\_2Enum\_2E0)) V1x)) \wedge ((p (ap (ap c\_2Erealax\_2Ereal\_lt V1x) (ap \\ & (ap c\_2Ereal\_2E\_2F c\_2Etransc\_2Epi) (ap c\_2Ereal\_2Ereal\_of\_num \\ & (ap c\_2Earithmetic\_2ENUMERAL (ap c\_2Earithmetic\_2EBIT2 c\_2Earithmetic\_2EZERO)))))) \wedge \\ & (p (ap (ap c\_2Erealax\_2Ereal\_lt V0y) (ap c\_2Etransc\_2Etan V1x)))))) \end{aligned}$$