thm_2Eucord_2EUnum__cardle__ucinf (TM-MgCuC8s9HW91zb4Qx8JQakWoxug7YE7PC)

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Definition 1 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 2 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 3 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 6 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 7 *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x)))$

Definition 8 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 9 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}\ A$

Definition 10 *We define* c_2Epred__set_2ESURJ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in ($

Definition 11 *We define* c_2Epred__set_2EINJ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in (2^A$

Definition 12 *We define* c_2Epred__set_2EBIJ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in (2^A$

Definition 13 *We define* c_2Ecardinal_2Ecardeq *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0s1 \in (2^{A\_27a}).\lambda V1s2 \in (2^{A}$

Definition 14 *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in$

Definition 15 *We define* c_2Epred__set_2EUNIV *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.\mathsf{c\_2Ebool\_2ET})$.

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \tag{1}$$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \tag{2}$$

Definition 16 *We define* c_2Ecardinal_2Ecardleq *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0s1 \in (2^{A\_27a}).\lambda V1s2 \in (2^{A$

Definition 17 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Assume the following.

$$True \tag{3}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{4}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{5}$$

Assume the following.

$$(\forall V0t \in 2.(((True \lor (p\ V0t)) \Leftrightarrow True) \land ((((p\ V0t) \lor True) \Leftrightarrow True) \land (((False \lor (p\ V0t)) \Leftrightarrow (p\ V0t)) \land ((((p\ V0t) \lor False) \Leftrightarrow (p\ V0t)) \land (((p\ V0t) \lor (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{6}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \land (((\neg True) \Leftrightarrow False) \land ((\neg False) \Leftrightarrow True))) \tag{7}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0s1 \in (2^{A\_27a}).(\forall V1s2 \in (2^{A\_27b}).((p\ (ap\ (ap\ (\mathsf{c\_2Ecardinal\_2Ecardleq}\ A\_27a\ A\_27b)\ V0s1)\ V1s2)) \Leftrightarrow ((\neg(p\ (ap\ (ap\ (\mathsf{c\_2Ecardinal\_2Ecardleq}\ A\_27b\ A\_27a)\ V1s2)\ V0s1))) \lor (p\ (ap\ (ap\ (\mathsf{c\_2Ecardinal\_2Ecardeq}\ A\_27a\ A\_27b)\ V0s1)\ V1s2)))))) \tag{8}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\neg(p\ (ap\ (ap\ (\mathsf{c\_2Ecardinal\_2Ecardleq}\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ (ty\_2Esum\_2Esum\ A\_27a\ (2^{ty\_2Enum\_2Enum})))\ ty\_2Enum\_2Enum)\ (\mathsf{c\_2Epred\_\_set\_2EUNIV}\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ (ty\_2Esum\_2Esum\ A\_27a\ (2^{ty\_2Enum\_2Enum})))))\ (\mathsf{c\_2Epred\_\_set\_2EUNIV}\ ty\_2Enum\_2Enum)))) \tag{9}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ecardinal\_2Ecardleq}\ ty\_2Enum\_2Enum\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ (ty\_2Esum\_2Esum\ A\_27a\ (2^{ty\_2Enum\_2Enum})))))\ (\mathsf{c\_2Epred\_\_set\_2EUNIV}\ ty\_2Enum\_2Enum))\ (\mathsf{c\_2Epred\_\_set\_2EUNIV}\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ (ty\_2Esum\_2Esum\ A\_27a\ (2^{ty\_2Enum\_2Enum}))))))$$