

thm_2Eucord_2Eomega1__not__countable (TMLC66tTCXetXjd9LqZRWFqpFDZgjTjBgSQ)

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Definition 1 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 2 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a}))$

Definition 5 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 6 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t)))$

Definition 7 We define $c_2Ebool_2E_2E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Let $ty_2Eenum_2Eenum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eenum_2Eenum \tag{1}$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \tag{2}$$

Let $ty_2Ewellorder_2Ewellorder : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ewellorder_2Ewellorder\ A0) \tag{3}$$

Let $ty_2Eordinal_2Eordinal : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eordinal_2Eordinal\ A0) \tag{4}$$

Let $c_2Eordinal_2Eordinal_REP_CLASS : \iota \Rightarrow \iota$ be given. Assume the following.

$$A_27a \in ((2^{(ty_2Ewellorder_2Ewellorder\ (ty_2Esum_2Esum\ ty_2Eenum_2Eenum\ A_27a))})^{(ty_2Eordinal_2Eordinal\ A_27a)}) \tag{5}$$

Definition 8 We define $c_2Emin_2E.40$ to be $\lambda A.\lambda P \in 2^A$. **if** $(\exists x \in A.p (ap P x))$ **then** (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 9 We define $c_2Eordinal_2Eordinal_REP$ to be $\lambda A.27a : \iota.\lambda V0a \in (ty_2Eordinal_2Eordinal A.27a)$. Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow \forall A1.nonempty A1 \Rightarrow nonempty (ty_2Epair_2Eprod A0 A1) \quad (6)$$

Let $c_2Ewellorder_2Ewellorder_REP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow c_2Ewellorder_2Ewellorder_REP A.27a \in ((2^{(ty_2Epair_2Eprod A.27a A.27a)})^{(ty_2Ewellorder_2Ewellorder A.27a)}) \quad (7)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c_2Epair_2EABS_prod A.27a A.27b \in ((ty_2Epair_2Eprod A.27a A.27b)^{(2^{A.27b})^{A.27a}}) \quad (8)$$

Definition 10 We define $c_2Epair_2E.2C$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0x \in A.27a.\lambda V1y \in A.27b.(ap (c_2E.2C$

Definition 11 We define c_2Ebool_2EIN to be $\lambda A.27a : \iota.(\lambda V0x \in A.27a.(\lambda V1f \in (2^{A.27a}).(ap V1f V0x)))$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c_2Epair_2ESND A.27a A.27b \in (A.27b)^{(ty_2Epair_2Eprod A.27a A.27b)} \quad (9)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c_2Epair_2EFST A.27a A.27b \in (A.27a)^{(ty_2Epair_2Eprod A.27a A.27b)} \quad (10)$$

Definition 12 We define $c_2Epair_2EUNCURRY$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda A.27c : \iota.\lambda V0f \in ((A.27c)^{A.27b})$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow \forall A.27b.nonempty A.27b \Rightarrow c_2Epred_set_2EGSPEC A.27a A.27b \in ((2^{A.27a})^{(ty_2Epair_2Eprod A.27a 2)^{A.27b}}) \quad (11)$$

Definition 13 We define $c_2Eset_relation_2Estrict$ to be $\lambda A.27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod A.27a A.27a)})$

Definition 14 We define $c_2Ewellorder_2Eiseg$ to be $\lambda A.27a : \iota.\lambda V0w \in (ty_2Ewellorder_2Ewellorder A.27a)$

Definition 15 We define $c_2Eset_relation_2Errestrict$ to be $\lambda A.27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod A.27a A.27a)})$

Let $c_2Ewellorder_2Ewellorder_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow c_2Ewellorder_2Ewellorder_ABS \\ & A.27a \in ((ty_2Ewellorder_2Ewellorder\ A.27a)^{(2^{(ty_2Epair_2Eprod\ A.27a\ A.27a)})}) \end{aligned} \quad (12)$$

Definition 16 We define $c_2Ewellorder_2Ewobound$ to be $\lambda A.27a : \iota.\lambda V0x \in A.27a.\lambda V1w \in (ty_2Ewellorder$

Definition 17 We define $c_2Ebool_2E_3F$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A.27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40$

Definition 18 We define $c_2Eset_relation_2Erange$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A$

Definition 19 We define $c_2Eset_relation_2Edomain$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod$

Definition 20 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 21 We define $c_2Epred_set_2EUNION$ to be $\lambda A.27a : \iota.\lambda V0s \in (2^{A.27a}).\lambda V1t \in (2^{A.27a}).(ap\ (c$

Definition 22 We define $c_2Ewellorder_2EelsOf$ to be $\lambda A.27a : \iota.\lambda V0w \in (ty_2Ewellorder_2Ewellorder\ A$

Definition 23 We define $c_2Ewellorder_2Eorderiso$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0w1 \in (ty_2Ewellorder_2E$

Definition 24 We define $c_2Ewellorder_2Eorderlt$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0w1 \in (ty_2Ewellorder_2E$

Definition 25 We define $c_2Eordinal_2Eordlt$ to be $\lambda A.27a : \iota.\lambda V0T1 \in (ty_2Eordinal_2Eordinal\ A.27a).$

Definition 26 We define $c_2Eordinal_2Epreds$ to be $\lambda A.27a : \iota.\lambda V0w \in (ty_2Eordinal_2Eordinal\ A.27a).$

Definition 27 We define $c_2Epred_set_2EUNIV$ to be $\lambda A.27a : \iota.(\lambda V0x \in A.27a.c_2Ebool_2E2ET).$

Definition 28 We define $c_2Epred_set_2EINJ$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0f \in (A.27b^{A.27a}).\lambda V1s \in (2^{A$

Definition 29 We define $c_2Epred_set_2Ecountable$ to be $\lambda A.27a : \iota.\lambda V0s \in (2^{A.27a}).(ap\ (c_2Ebool_2E_3F$

Definition 30 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A.27a : \iota.\lambda A.27b : \iota.\lambda V0f \in (A.27b^{A.27a}).\lambda V1s \in$

Definition 31 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A.27a : \iota.\lambda V0P \in (2^{(2^{A.27a})}).(ap\ (c_2Epred_s$

Definition 32 We define $c_2Eordinal_2Eoleast$ to be $\lambda A.27a : \iota.\lambda V0P \in (2^{(ty_2Eordinal_2Eordinal\ A.27a)}).$

Definition 33 We define $c_2Eordinal_2Esup$ to be $\lambda A.27a : \iota.\lambda V0ordset \in (2^{(ty_2Eordinal_2Eordinal\ A.27a)}).$

Definition 34 We define $c_2Eucord_2Eomega1$ to be $\lambda A.27a : \iota.(ap\ (c_2Eordinal_2Esup\ (ty_2Esum_2Esum$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p \\ & V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \end{aligned} \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \quad (14)$$

Assume the following.

$$\begin{aligned}
& (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\
& (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow \neg(p V0t)) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow \neg(\\
& p V0t))))))
\end{aligned} \tag{15}$$

Assume the following.

$$\forall A.27a.nonempty A.27a \Rightarrow (\forall V0w \in (ty.2Eordinal.2Eordinal A.27a).(\neg(p (ap (ap (c.2Eordinal.2Eordlt A.27a) V0w) V0w)))) \tag{16}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow (\forall V0x \in (ty.2Eordinal.2Eordinal \\
& (ty.2Esum.2Esum A.27a (2^{ty.2Enum.2Enum}))).((p (ap (ap (c.2Eordinal.2Eordlt \\
& (ty.2Esum.2Esum A.27a (2^{ty.2Enum.2Enum}))) V0x) (c.2Eucord.2Eomega1 \\
& A.27a))) \Leftrightarrow (p (ap (c.2Epred_set.2Ecountable (ty.2Eordinal.2Eordinal \\
& (ty.2Esum.2Esum A.27a (2^{ty.2Enum.2Enum})))) (ap (c.2Eordinal.2Epreds \\
& (ty.2Esum.2Esum A.27a (2^{ty.2Enum.2Enum}))) V0x))))))
\end{aligned} \tag{17}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty A.27a \Rightarrow (\neg(p (ap (c.2Epred_set.2Ecountable \\
& (ty.2Eordinal.2Eordinal (ty.2Esum.2Esum A.27a (2^{ty.2Enum.2Enum})))) \\
& (ap (c.2Eordinal.2Epreds (ty.2Esum.2Esum A.27a (2^{ty.2Enum.2Enum}))) \\
& (c.2Eucord.2Eomega1 A.27a))))))
\end{aligned}$$