

thm_2Eucord_2Eucord__sup__exists__lemma
(TMUQJsjZmpY-
WFpp85bSsvk24QBFpHw8VsD)

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Definition 1 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \text{ (ap } P \ x)) \text{ of type } \iota \Rightarrow \iota.$

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota.$

Definition 3 We define `c_2Ebool_2EIN` to be $\lambda A. 27a : \iota. (\lambda V0x \in A. 27a. (\lambda V1f \in (2^{A-27a}). (\text{ap } V1f \ V0x)))$

Definition 4 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow q)$ of type $\iota.$

Definition 5 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

Definition 6 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a}))))$

Definition 7 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2)) (\lambda V2t \in 2. V2t)))$

Definition 8 We define `c_2Ebool_2E_3F` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } V0P \ (\text{ap } (\text{c_2Emin_2E_40 } A))))$

Definition 9 We define `c_2Epred__set_2ESURJ` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0f \in (A. 27b^{A-27a}). \lambda V1s \in (2^{A-27b})$

Definition 10 We define `c_2Epred__set_2EINJ` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0f \in (A. 27b^{A-27a}). \lambda V1s \in (2^{A-27b})$

Definition 11 We define `c_2Epred__set_2EBIJ` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0f \in (A. 27b^{A-27a}). \lambda V1s \in (2^{A-27b})$

Definition 12 We define `c_2Ecardinal_2Ecardeq` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda V0s1 \in (2^{A-27a}). \lambda V1s2 \in (2^{A-27b})$

Definition 13 We define `c_2Ecombin_2ES` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda A. 27c : \iota. (\lambda V0f \in ((A. 27c^{A-27b})^{A-27a}))$

Definition 14 We define `c_2Ecombin_2Eo` to be $\lambda A. 27a : \iota. \lambda A. 27b : \iota. \lambda A. 27c : \iota. \lambda V0f \in (A. 27b^{A-27c}). \lambda V1g \in (A. 27c^{A-27b})$

Definition 15 We define `c_2Emarker_2EAbbrev` to be $\lambda V0x \in 2. V0x.$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \quad (1)$$

Let $ty_2Esum_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Esum_2Esum\ A0\ A1) \quad (2)$$

Let $ty_2Ewellorder_2Ewellorder : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Ewellorder_2Ewellorder\ A0) \quad (3)$$

Let $ty_2Eordinal_2Eordinal : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eordinal_2Eordinal\ A0) \quad (4)$$

Let $c_2Eordinal_2Eordinal_REP_CLASS : \iota \Rightarrow \iota$ be given. Assume the following.

$$A_27a \in ((2^{(ty_2Ewellorder_2Ewellorder\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A_27a))})^{(ty_2Eordinal_2Eordinal\ A_27a)}) \quad (5)$$

Definition 16 We define $c_2Eordinal_2Eordinal_REP$ to be $\lambda A_27a : \iota. \lambda V0a \in (ty_2Eordinal_2Eordinal\ A_27a)$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (6)$$

Let $c_2Ewellorder_2Ewellorder_REP : \iota \Rightarrow \iota$ be given. Assume the following.

$$A_27a \in ((2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})^{(ty_2Ewellorder_2Ewellorder\ A_27a)}) \quad (7)$$

Definition 17 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E.21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 18 We define $c_2Ebool_2E.7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E.3D_3D_3E\ V0t)\ c_2Ebool_2E.7E))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (8)$$

Definition 19 We define $c_2Epair_2E.2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b.(ap\ (c_2Epair_2E.2C\ V0x\ V1y))$

Definition 34 We define $c_2Epred_set_2EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in$

Definition 35 We define $c_2Epred_set_2EBIGUNION$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(ap (c_2Epred_set_2E$

Definition 36 We define $c_2Eordinal_2Eoleast$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(ty_2Eordinal_2Eordinal A_27a)}).$

Definition 37 We define $c_2Eordinal_2Esup$ to be $\lambda A_27a : \iota.\lambda V0ordset \in (2^{(ty_2Eordinal_2Eordinal A_27a)}).$

Definition 38 We define $c_2Eordinal_2Edclose$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{(ty_2Eordinal_2Eordinal A_27a)}).$

Definition 39 We define $c_2Epred_set_2EINSERT$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1s \in (2^{A_27a}).$

Definition 40 We define $c_2Epred_set_2EEMPTY$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EF).$

Definition 41 We define $c_2Epred_set_2EDIFF$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).$

Definition 42 We define $c_2Epred_set_2EDELETE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1x \in A_27a.$

Definition 43 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).$

Definition 44 We define $c_2Epred_set_2EFINITE$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).$

Definition 45 We define $c_2Epred_set_2EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2EET).$

Definition 46 We define $c_2Epred_set_2Ecountable$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).$

Definition 47 We define $c_2Ecardinal_2Ecardleq$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0s1 \in (2^{A_27a}).\lambda V1s2 \in (2^{A_27b}).$

Assume the following.

$$True \tag{13}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \tag{14}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p V0t))) \tag{15}$$

Assume the following.

$$(\forall V0t \in 2.((p V0t) \vee \neg(p V0t))) \tag{16}$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p V0t) \Leftrightarrow (p V1x)))) \tag{17}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \wedge True) \Leftrightarrow (p \ V0t)) \wedge (((False \wedge (p \ V0t)) \Leftrightarrow False) \wedge (((p \ V0t) \wedge False) \Leftrightarrow False) \wedge (((p \ V0t) \wedge (p \ V0t)) \Leftrightarrow (p \ V0t)))))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (19)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t)) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (20)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.(V0x = V0x)) \quad (21)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.((V0x = V0x) \Leftrightarrow True)) \quad (22)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0x \in A.27a.(\forall V1y \in A.27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (23)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (24)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).((\neg(\exists V1x \in A.27a.(p \ (ap \ V0P \ V1x)))) \Leftrightarrow (\forall V2x \in A.27a.(\neg(p \ (ap \ V0P \ V2x)))))) \quad (25)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in 2.(((\forall V2x \in A.27a.(p \ (ap \ V0P \ V2x))) \wedge (p \ V1Q)) \Leftrightarrow (\forall V3x \in A.27a.((p \ (ap \ V0P \ V3x)) \wedge (p \ V1Q)))))) \quad (26)$$

Assume the following.

$$\forall A.27a.nonempty \ A.27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A.27a}).((\forall V2x \in A.27a.((p \ V0P) \vee (p \ (ap \ V1Q \ V2x)))) \Leftrightarrow ((p \ V0P) \vee (\forall V3x \in A.27a.(p \ (ap \ V1Q \ V3x)))))) \quad (27)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{A.27a}).(\forall V1Q \in \\ & 2.((\forall V2x \in A.27a.((p\ (ap\ V0P\ V2x)) \Rightarrow (p\ V1Q))) \Leftrightarrow ((\exists V3x \in \\ & A.27a.(p\ (ap\ V0P\ V3x)) \Rightarrow (p\ V1Q)))))) \end{aligned} \quad (28)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(\\ & p\ V0A)) \vee (\neg(p\ V1B)))))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A)) \wedge (\neg(p\ V1B)))))) \end{aligned} \quad (29)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V0A) \vee (\\ & (p\ V1B) \wedge (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C)))))) \end{aligned} \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p\ V1B) \wedge \\ & (p\ V2C)) \vee (p\ V0A)) \Leftrightarrow (((p\ V1B) \vee (p\ V0A)) \wedge ((p\ V2C) \vee (p\ V0A)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((p\ V0A) \Rightarrow (p\ V1B)) \Leftrightarrow ((\neg(p\ V0A)) \vee \\ & (p\ V1B)))))) \end{aligned} \quad (32)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow \\ & ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \end{aligned} \quad (33)$$

Assume the following.

$$\begin{aligned} & (\forall V0x \in 2.(\forall V1x.27 \in 2.(\forall V2y \in 2.(\forall V3y.27 \in \\ & 2.(((p\ V0x) \Leftrightarrow (p\ V1x.27)) \wedge ((p\ V1x.27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y.27)))))) \Rightarrow \\ & (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x.27) \Rightarrow (p\ V3y.27)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0f \in (2^{A.27a}).(\forall V1v \in \\ & A.27a.((\forall V2x \in A.27a.((V2x = V1v) \Rightarrow (p\ (ap\ V0f\ V2x)))) \Leftrightarrow (p\ (\\ & ap\ V0f\ V1v)))))) \end{aligned} \quad (35)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}).(p\ (ap\ (\\ & ap\ (c.2Ecardinal.2Ecardeq\ A.27a\ A.27a)\ V0s)\ V0s))) \end{aligned} \quad (36)$$

Assume the following.

$$\begin{aligned} & \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ & \forall V0s \in (2^{A.27a}).(\forall V1t \in (2^{A.27b}).((p\ (ap\ (ap\ (c.2Ecardinal.2Ecardeq \\ & A.27a\ A.27b)\ V0s)\ V1t)) \Leftrightarrow (p\ (ap\ (ap\ (c.2Ecardinal.2Ecardeq\ A.27b \\ & A.27a)\ V1t)\ V0s)))))) \end{aligned} \quad (37)$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& \quad nonempty\ A.27c \Rightarrow (\forall V0s \in (2^{A.27a}). (\forall V1t \in (2^{A.27b}). \\
& \quad (\forall V2u \in (2^{A.27c}). (((p\ (ap\ (ap\ (c.2Ecardinal_2Ecardleq \\
& \quad A.27a\ A.27b)\ V0s)\ V1t)) \wedge (p\ (ap\ (ap\ (c.2Ecardinal_2Ecardleq\ A.27b \\
& \quad A.27c)\ V1t)\ V2u))) \Rightarrow (p\ (ap\ (ap\ (c.2Ecardinal_2Ecardleq\ A.27a\ A.27c) \\
& \quad V0s)\ V2u))))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0s \in (2^{A.27a}). (\forall V1t \in (2^{A.27b}). (((p\ (ap\ (ap\ (c.2Ecardinal_2Ecardleq \\
& \quad A.27a\ A.27b)\ V0s)\ V1t)) \wedge (p\ (ap\ (ap\ (c.2Ecardinal_2Ecardleq\ A.27b \\
& \quad A.27a)\ V1t)\ V0s))) \Rightarrow (p\ (ap\ (ap\ (c.2Ecardinal_2Ecardleq\ A.27a\ A.27b) \\
& \quad V0s)\ V1t))))))
\end{aligned} \tag{39}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1s \in \\
& \quad (2^{A.27a}). ((\neg(p\ (ap\ (c.2Epred_set_2EFINITE\ A.27a)\ V1s))) \Rightarrow (\\
& \quad p\ (ap\ (ap\ (c.2Ecardinal_2Ecardeq\ A.27a\ A.27a)\ (ap\ (ap\ (c.2Epred_set_2EINSERT \\
& \quad A.27a)\ V0x)\ V1s))\ V1s))))))
\end{aligned} \tag{40}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow \forall A.27c. \\
& \quad nonempty\ A.27c \Rightarrow \forall A.27d.nonempty\ A.27d \Rightarrow (\forall V0s1 \in (\\
& \quad 2^{A.27a}). (\forall V1s2 \in (2^{A.27b}). (\forall V2t1 \in (2^{A.27c}). \\
& \quad (\forall V3t2 \in (2^{A.27d}). (((p\ (ap\ (ap\ (c.2Ecardinal_2Ecardeq \\
& \quad A.27a\ A.27b)\ V0s1)\ V1s2)) \wedge (p\ (ap\ (ap\ (c.2Ecardinal_2Ecardeq\ A.27c \\
& \quad A.27d)\ V2t1)\ V3t2))) \Rightarrow ((p\ (ap\ (ap\ (c.2Ecardinal_2Ecardleq\ A.27a \\
& \quad A.27c)\ V0s1)\ V2t1)) \Leftrightarrow (p\ (ap\ (ap\ (c.2Ecardinal_2Ecardleq\ A.27b\ A.27d) \\
& \quad V1s2)\ V3t2)))))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\neg(p\ (ap\ (c.2Epred_set_2EFINITE \\
& \quad (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A.27a))\ (c.2Epred_set_2EUNIV \\
& \quad (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A.27a))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\
& \quad \forall V0f \in (A.27b^{A.27a}). (\forall V1s \in (2^{A.27a}). (p\ (ap\ (ap\ (\\
& \quad c.2Ecardinal_2Ecardleq\ A.27b\ A.27a)\ (ap\ (ap\ (c.2Epred_set_2EIMAGE \\
& \quad A.27a\ A.27b)\ V0f)\ V1s))\ V1s))))))
\end{aligned} \tag{43}$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in (2^{A-27a}).(\forall V1y \in \\ (2^{A-27a}).((p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET\ A_27a)\ V0x)\ V1y))) \Rightarrow \end{aligned} \quad (44)$$

$$(p\ (ap\ (ap\ (c_2Ecardinal_2Ecardleq\ A_27a\ A_27a)\ V0x)\ V1y))))$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow \forall A_27c. \\ nonempty\ A_27c \Rightarrow (\forall V0f \in (A_27c^{A-27a}).(\forall V1s \in (2^{A-27a}). \\ (\forall V2t \in (2^{A-27b}).((p\ (ap\ (ap\ (c_2Ecardinal_2Ecardleq\ A_27a \\ A_27b)\ V1s)\ V2t))) \Rightarrow (p\ (ap\ (ap\ (c_2Ecardinal_2Ecardleq\ A_27c\ A_27b) \\ (ap\ (ap\ (c_2Epred_set_2EIMAGE\ A_27a\ A_27c)\ V0f)\ V1s))\ V2t)))))) \end{aligned} \quad (45)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A-27a}).((p\ (ap \\ (c_2Epred_set_2Ecountable\ A_27a)\ V0s)) \Leftrightarrow (p\ (ap\ (ap\ (c_2Ecardinal_2Ecardleq \\ A_27a\ ty_2Enum_2Enum)\ V0s)\ (c_2Epred_set_2EUNIV\ ty_2Enum_2Enum)))))) \end{aligned} \quad (46)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0s \in (2^{A-27a}).(\forall V1t \in (2^{A-27b}).((\neg(p\ (ap\ (ap\ (\\ c_2Ecardinal_2Ecardleq\ A_27b\ A_27a)\ V1t)\ V0s)))) \Leftrightarrow ((p\ (ap\ (ap\ (c_2Ecardinal_2Ecardleq \\ A_27a\ A_27b)\ V0s)\ V1t)) \wedge (\neg(p\ (ap\ (ap\ (c_2Ecardinal_2Ecardleq\ A_27a \\ A_27b)\ V0s)\ V1t)))))) \end{aligned} \quad (47)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ \forall V0k \in (2^{A-27a}).(\forall V1s1 \in (2^{(2^{A-27b})}).(((\neg(p\ (\\ ap\ (c_2Epred_set_2EFINITE\ A_27a)\ V0k)))) \wedge ((p\ (ap\ (ap\ (c_2Ecardinal_2Ecardleq \\ (2^{A-27b})\ A_27a)\ V1s1)\ V0k)) \wedge (\forall V2e \in (2^{A-27b}).((p\ (ap\ (\\ ap\ (c_2Ebool_2EIN\ (2^{A-27b})\ V2e)\ V1s1)) \Rightarrow (p\ (ap\ (ap\ (c_2Ecardinal_2Ecardleq \\ A_27b\ A_27a)\ V2e)\ V0k)))))) \Rightarrow (p\ (ap\ (ap\ (c_2Ecardinal_2Ecardleq \\ A_27b\ A_27a)\ (ap\ (c_2Epred_set_2EBIGUNION\ A_27b)\ V1s1))\ V0k)))))) \end{aligned} \quad (48)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in (ty_2Eordinal_2Eordinal \\ A_27a).(\forall V1w \in (ty_2Eordinal_2Eordinal\ A_27a).((p\ (ap \\ (ap\ (c_2Ebool_2EIN\ (ty_2Eordinal_2Eordinal\ A_27a))\ V0x)\ (ap\ (\\ c_2Eordinal_2Epreds\ A_27a)\ V1w))) \Leftrightarrow (p\ (ap\ (ap\ (c_2Eordinal_2Eordlt \\ A_27a)\ V0x)\ V1w)))))) \end{aligned} \quad (49)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0b \in (ty_2Eordinal_2Eordinal \\ A_27a).(\forall V1a \in (ty_2Eordinal_2Eordinal\ A_27a).(\neg(p (\\ ap (ap (c_2Eordinal_2Eordlt\ A_27a)\ V0b)\ V1a))) \Leftrightarrow ((p (ap (ap (c_2Eordinal_2Eordlt \\ A_27a)\ V1a)\ V0b)) \vee (V1a = V0b)))))) \end{aligned} \quad (50)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in (ty_2Eordinal_2Eordinal \\ A_27a).(\forall V1y \in (ty_2Eordinal_2Eordinal\ A_27a).(\forall V2z \in \\ (ty_2Eordinal_2Eordinal\ A_27a).(((p (ap (ap (c_2Eordinal_2Eordlt \\ A_27a)\ V0x)\ V1y)) \wedge (\neg(p (ap (ap (c_2Eordinal_2Eordlt\ A_27a)\ V2z) \\ V1y)))))) \Rightarrow (p (ap (ap (c_2Eordinal_2Eordlt\ A_27a)\ V0x)\ V2z)))))) \end{aligned} \quad (51)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{(ty_2Eordinal_2Eordinal\ A_27a)}). \\ ((p (ap (ap (c_2Ecardinal_2Ecardleq (ty_2Eordinal_2Eordinal \\ A_27a)\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A_27a))\ V0s) (c_2Epred_set_2EUNIV \\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A_27a)))) \Rightarrow (\forall V1a \in (ty_2Eordinal_2Eordinal \\ A_27a).((p (ap (ap (c_2Eordinal_2Eordlt\ A_27a)\ V1a) (ap (c_2Eordinal_2Esup \\ A_27a)\ V0s))) \Leftrightarrow (\exists V2b \in (ty_2Eordinal_2Eordinal\ A_27a). \\ ((p (ap (ap (c_2Ebool_2EIN (ty_2Eordinal_2Eordinal\ A_27a))\ V2b) \\ V0s)) \wedge (p (ap (ap (c_2Eordinal_2Eordlt\ A_27a)\ V1a)\ V2b)))))))))) \end{aligned} \quad (52)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0b \in (ty_2Eordinal_2Eordinal \\ A_27a).(\forall V1s \in (2^{(ty_2Eordinal_2Eordinal\ A_27a)}).((\\ (p (ap (ap (c_2Ecardinal_2Ecardleq (ty_2Eordinal_2Eordinal\ A_27a) \\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A_27a))\ V1s) (c_2Epred_set_2EUNIV \\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A_27a)))) \wedge (p (ap (ap (c_2Ebool_2EIN \\ (ty_2Eordinal_2Eordinal\ A_27a))\ V0b)\ V1s))) \Rightarrow (\neg(p (ap (ap (c_2Eordinal_2Eordlt \\ A_27a) (ap (c_2Eordinal_2Esup\ A_27a)\ V1s))\ V0b)))))) \end{aligned} \quad (53)$$

Assume the following.

$$\begin{aligned} \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{(ty_2Eordinal_2Eordinal\ A_27a)}). \\ ((p (ap (ap (c_2Ecardinal_2Ecardleq (ty_2Eordinal_2Eordinal \\ A_27a) (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A_27a))\ V0s) (c_2Epred_set_2EUNIV \\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A_27a)))) \Rightarrow ((ap (c_2Eordinal_2Epreds \\ A_27a) (ap (c_2Eordinal_2Esup\ A_27a)\ V0s)) = (ap (c_2Eordinal_2Edclose \\ A_27a)\ V0s)))) \end{aligned} \quad (54)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{(ty_2Eordinal_2Eordinal\ A_27a)}). \\ & ((ap\ (c_2Eordinal_2Edclose\ A_27a)\ V0s) = (ap\ (c_2Epred_set_2EBIGUNION \\ & (ty_2Eordinal_2Eordinal\ A_27a))\ (ap\ (ap\ (c_2Epred_set_2EIMAGE \\ & (ty_2Eordinal_2Eordinal\ A_27a)\ (2^{(ty_2Eordinal_2Eordinal\ A_27a)})) \\ & (c_2Eordinal_2Epreds\ A_27a))\ V0s)))) \end{aligned} \quad (55)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in \\ & A_27b. (((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap \\ & (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \end{aligned} \quad (56)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0f \in ((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}). (\forall V1v \in \\ & A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1v)\ (ap\ (c_2Epred_set_2EGSPEC \\ & A_27a\ A_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A_27b. ((ap\ (ap\ (c_2Epair_2E_2C \\ & A_27a\ 2)\ V1v)\ c_2Ebool_2ET) = (ap\ V0f\ V2x)))))) \end{aligned} \quad (57)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ (c_2Epred_set_2EUNIV\ A_27a)))) \quad (58)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0s \in (2^{A_27a}). (\forall V1x \in \\ & A_27a. (\forall V2y \in A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1x) \\ & (ap\ (ap\ (c_2Epred_set_2EDELETE\ A_27a)\ V0s)\ V2y))) \Leftrightarrow ((p\ (ap\ (ap \\ & (c_2Ebool_2EIN\ A_27a)\ V1x)\ V0s)) \wedge \neg (V1x = V2y)))))) \end{aligned} \quad (59)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1s \in \\ & (2^{A_27a}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ V1s)) \Rightarrow ((ap\ (ap \\ & (c_2Epred_set_2EINSERT\ A_27a)\ V0x)\ (ap\ (ap\ (c_2Epred_set_2EDELETE \\ & A_27a)\ V1s)\ V0x)) = V1s)))) \end{aligned} \quad (60)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \forall V0y \in A_27b. (\forall V1s \in (2^{A_27a}). (\forall V2f \in (A_27b^{A_27a}). \\ & ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27b)\ V0y)\ (ap\ (ap\ (c_2Epred_set_2EIMAGE \\ & A_27a\ A_27b)\ V2f)\ V1s))) \Leftrightarrow (\exists V3x \in A_27a. ((V0y = (ap\ V2f\ V3x)) \wedge \\ & (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V3x)\ V1s)))))) \end{aligned} \quad (61)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow \forall A.27b.nonempty\ A.27b \Rightarrow (\\ \forall V0x \in A.27a. (\forall V1s \in (2^{A.27a}). ((p\ (ap\ (ap\ (c.2Ebool.2EIN \\ A.27a)\ V0x)\ V1s))) \Rightarrow (\forall V2f \in (A.27b^{A.27a}). (p\ (ap\ (ap\ (c.2Ebool.2EIN \\ A.27b)\ (ap\ V2f\ V0x))\ (ap\ (ap\ (c.2Epred_set.2EIMAGE\ A.27a\ A.27b) \\ V2f)\ V1s)))))) \end{aligned} \quad (62)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0x \in A.27a. (\forall V1s \in \\ (2^{A.27a}). ((p\ (ap\ (c.2Epred_set.2EFINITE\ A.27a)\ (ap\ (ap\ (c.2Epred_set.2EDELETE \\ A.27a)\ V1s)\ V0x))) \Leftrightarrow (p\ (ap\ (c.2Epred_set.2EFINITE\ A.27a)\ V1s)))) \end{aligned} \quad (63)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}). (\forall V1t \in \\ (2^{A.27a}). ((p\ (ap\ (c.2Epred_set.2Ecountable\ A.27a)\ V0s)) \wedge \\ (p\ (ap\ (ap\ (c.2Epred_set.2ESUBSET\ A.27a)\ V1t)\ V0s))) \Rightarrow (p\ (ap\ (c.2Epred_set.2Ecountable \\ A.27a)\ V1t)))) \end{aligned} \quad (64)$$

Assume the following.

$$\begin{aligned} \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0s \in (2^{A.27a}). ((p\ (ap \\ (c.2Epred_set.2EFINITE\ A.27a)\ V0s)) \Rightarrow (p\ (ap\ (c.2Epred_set.2Ecountable \\ A.27a)\ V0s)))) \end{aligned} \quad (65)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (66)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (67)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \end{aligned} \quad (68)$$

Assume the following.

$$\begin{aligned} (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))) \end{aligned} \quad (69)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (70)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(\\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{71}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{72}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{73}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\\
& \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{74}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{75}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \tag{76}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{77}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V0p)))) \tag{78}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \vee (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{79}$$

Assume the following.

$$(\forall V0p \in 2. ((\neg(\neg(p V0p))) \Rightarrow (p V0p))) \tag{80}$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow & (\neg(p\ (ap\ (c_2Epred_set_2Ecountable \\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ (ty_2Esum_2Esum\ A_{.27a}\ (2^{ty_2Enum_2Enum})))) \\ (c_2Epred_set_2EUNIV\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ (ty_2Esum_2Esum \\ A_{.27a}\ (2^{ty_2Enum_2Enum}))))))) \end{aligned} \quad (81)$$

Assume the following.

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow & (p\ (ap\ (ap\ (c_2Ecardinal_2Ecardleq \\ ty_2Enum_2Enum\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ (ty_2Esum_2Esum \\ A_{.27a}\ (2^{ty_2Enum_2Enum}))))\ (c_2Epred_set_2EUNIV\ ty_2Enum_2Enum)) \\ (c_2Epred_set_2EUNIV\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ (ty_2Esum_2Esum \\ A_{.27a}\ (2^{ty_2Enum_2Enum})))))) \end{aligned} \quad (82)$$

Theorem 1

$$\begin{aligned} \forall A_{.27a}.nonempty\ A_{.27a} \Rightarrow & (p\ (ap\ (ap\ (c_2Ecardinal_2Ecardleq \\ (ty_2Eordinal_2Eordinal\ (ty_2Esum_2Esum\ A_{.27a}\ (2^{ty_2Enum_2Enum}))) \\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ (ty_2Esum_2Esum\ A_{.27a}\ (2^{ty_2Enum_2Enum})))) \\ (ap\ (c_2Epred_set_2EGSPEC\ (ty_2Eordinal_2Eordinal\ (ty_2Esum_2Esum \\ A_{.27a}\ (2^{ty_2Enum_2Enum})))\ (ty_2Eordinal_2Eordinal\ (ty_2Esum_2Esum \\ A_{.27a}\ (2^{ty_2Enum_2Enum}))))\ (\lambda V0a \in (ty_2Eordinal_2Eordinal \\ (ty_2Esum_2Esum\ A_{.27a}\ (2^{ty_2Enum_2Enum}))).(ap\ (ap\ (c_2Epair_2E_2C \\ (ty_2Eordinal_2Eordinal\ (ty_2Esum_2Esum\ A_{.27a}\ (2^{ty_2Enum_2Enum}))) \\ 2)\ V0a)\ (ap\ (c_2Epred_set_2Ecountable\ (ty_2Eordinal_2Eordinal \\ (ty_2Esum_2Esum\ A_{.27a}\ (2^{ty_2Enum_2Enum}))))\ (ap\ (c_2Eordinal_2Epreds \\ (ty_2Esum_2Esum\ A_{.27a}\ (2^{ty_2Enum_2Enum})))\ V0a))))))\ (c_2Epred_set_2EUNIV \\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ (ty_2Esum_2Esum\ A_{.27a}\ (2^{ty_2Enum_2Enum})))))) \end{aligned}$$