

thm_2Eupdate_2EAPPLY__UPDATE__ID (TM- PUokqT2zGgeh1N8UoNQSDRwkUUrBAhbuD)

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Definition 1 We define `c_2Emin_2E_3D` to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define `c_2Ebool_2E_2T` to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define `c_2Ebool_2E_21` to be $\lambda A_{.27a} : \iota.(\lambda V0P \in (2^{A_{.27a}}).(ap (ap (c_2Emin_2E_3D (2^{A_{.27a}})) P) P))$

Definition 4 We define `c_2Ebool_2E_2F` to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 6 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))))$

Definition 7 We define `c_2Emin_2E_40` to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p (ap P x)))$ of type $\iota \Rightarrow \iota$.

Definition 8 We define `c_2Ebool_2ECOND` to be $\lambda A_{.27a} : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_{.27a}.(\lambda V2t2 \in A_{.27a}.(ap (c_2Emin_2E_40) V1a) V2t2))))$

Definition 9 We define `c_2Ecombin_2EUPDATE` to be $\lambda A_{.27a} : \iota.\lambda A_{.27b} : \iota.\lambda V0a \in A_{.27a}.\lambda V1b \in A_{.27b}.(ap (c_2Emin_2E_40) V1a) V0a)$

Assume the following.

$$\begin{aligned} & \forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\\ & \forall V0f \in (A_{.27b}^{A_{.27a}}).(\forall V1a \in A_{.27a}.((ap (ap (ap (c_2Ecombin_2EUPDATE \\ & A_{.27a} A_{.27b}) V1a) (ap V0f V1a)) V0f) = V0f))) \end{aligned} \quad (1)$$

Theorem 1

$$\begin{aligned} & \forall A_{.27a}.nonempty A_{.27a} \Rightarrow \forall A_{.27b}.nonempty A_{.27b} \Rightarrow (\\ & \forall V0f \in (A_{.27b}^{A_{.27a}}).(\forall V1a \in A_{.27a}.((ap (ap (ap (c_2Ecombin_2EUPDATE \\ & A_{.27a} A_{.27b}) V1a) (ap V0f V1a)) V0f) = V0f))) \end{aligned}$$