thm_2Eupdate_2ELIST__UPDATE__THMS (TMbHRfVr-Bup94F6NUaYkWxa9Vs5FaVyLnsh)

October 26, 2020

Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota{\Rightarrow}\iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Let $ty\_2Elist\_2Elist : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0{\Rightarrow}nonempty\ (ty\_2Elist\_2Elist\ A0) \tag{1}$$

Let $c\_2Elist\_2EAPPEND : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Elist\_2EAPPEND\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{(ty\_2Elist\_2Elist\ A\_27a)}) \tag{2}$$

Definition 3 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P{\Rightarrow}p\ Q)$ *of type* ι.

Let $c\_2Elist\_2ESNOC : \iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}c\_2Elist\_2ESNOC\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{3}$$

Definition 4 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 5 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Epair\_2Eprod : \iota{\Rightarrow}\iota{\Rightarrow}\iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0{\Rightarrow}\forall A1.nonempty\ A1{\Rightarrow}nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{4}$$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{5}$$

Definition 6 *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2E$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{6}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{7}$$

Definition 7 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 8 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 9 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(a$

Definition 10 *We define* c_2Ecombin_2EUPDATE *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0a \in A\_27a.\lambda V1b \in A\_27b$

Definition 11 *We define* c_2Ecombin_2Eo *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in (A\_27b^{A\_27c}).\lambda V1$

Let $c\_2Elist\_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ECONS\ A\_27a \in (((ty\_2Elist\_2Elist\ A\_27a)^{(ty\_2Elist\_2Elist\ A\_27a)})^{A\_27a}) \tag{8}$$

Definition 12 *We define* c_2Ecombin_2EK *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.(\lambda V0x \in A\_27a.(\lambda V1y \in A\_27b.V0x)$

Definition 13 *We define* c_2Ecombin_2ES *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.(\lambda V0f \in ((A\_27c^{A\_27b})^{A\_2}$

Definition 14 *We define* c_2Ecombin_2EI *to be* $\lambda A\_27a : \iota.(ap\ (ap\ (\mathsf{c\_2Ecombin\_2ES}\ A\_27a\ (A\_27a^{A\_27a})\ A$

Let $c\_2Elist\_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Elist\_2ENIL\ A\_27a \in (ty\_2Elist\_2Elist\ A\_27a) \tag{9}$$

Let $c\_2Eupdate\_2ELIST\_\_UPDATE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Eupdate\_2ELIST\_\_UPDATE\ A\_27a\ A\_27b \in (((A\_27b^{A\_27a})^{(A\_27b^{A\_27a})})^{(ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))}) \tag{10}$$

Assume the following.

$$True \tag{11}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((V0x = V0x) \Leftrightarrow True)) \tag{12}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{13}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.nonempty\ A\_27c \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27a^{A\_27c}).(\forall V2x \in A\_27c.((ap\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27c\ A\_27b\ A\_27a)\ V0f)\ V1g)\ V2x) = (ap\ V0f\ (ap\ V1g\ V2x)))))) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.nonempty\ A\_27c \Rightarrow \forall A\_27d.nonempty\ A\_27d \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1g \in (A\_27a^{A\_27c}).(\forall V2h \in (A\_27c^{A\_27d}).((ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27d\ A\_27b\ A\_27a)\ V0f)\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27d\ A\_27a\ A\_27c)\ V1g)\ V2h)) = (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27d\ A\_27b\ A\_27c)\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27c\ A\_27b\ A\_27a)\ V0f)\ V1g))\ V2h))))) \tag{15}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.((ap\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27a)\ V0x) = V0x)) \tag{16}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(((ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27a\ A\_27b\ A\_27b)\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27b))\ V0f) = V0f) \land ((ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ A\_27a\ A\_27b\ A\_27a)\ V0f)\ (\mathsf{c\_2Ecombin\_2EI}\ A\_27a)) = V0f))) \tag{17}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0l \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ (c\_2Elist\_2ENIL\ A\_27a))\ V0l) = V0l)) \land (\forall V1l1 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V2l2 \in (ty\_2Elist\_2Elist\ A\_27a).(\forall V3h \in A\_27a.((ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V3h)\ V1l1))\ V2l2) = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V3h)\ (ap\ (ap\ (c\_2Elist\_2EAPPEND\ A\_27a)\ V1l1)\ V2l2))))))) \tag{18}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Elist\_2Elist\ A\_27a)}). (((p\ (ap\ V0P\ (c\_2Elist\_2ENIL\ A\_27a))) \wedge (\forall V1t \in (ty\_2Elist\_2Elist\ A\_27a).((p\ (ap\ V0P\ V1t)) \Rightarrow (\forall V2h \in A\_27a.(p\ (ap\ V0P\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2h)\ V1t))))))) \Rightarrow (\forall V3l \in (ty\_2Elist\_2Elist\ A\_27a).(p\ (ap\ V0P\ V3l)))))) \tag{19}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow ((\forall V0x \in A\_27a.((ap\ (ap\ (c\_2Elist\_2ESNOC\ A\_27a)\ V0x)\ (c\_2Elist\_2ENIL\ A\_27a)) = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V0x)\ (c\_2Elist\_2ENIL\ A\_27a)))) \wedge (\forall V1x \in A\_27a.(\forall V2x\_27 \in A\_27a.(\forall V3l \in (ty\_2Elist\_2Elist\ A\_27a).((ap\ (ap\ (c\_2Elist\_2ESNOC\ A\_27a)\ V1x)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2x\_27)\ V3l)) = (ap\ (ap\ (c\_2Elist\_2ECONS\ A\_27a)\ V2x\_27)\ (ap\ (ap\ (c\_2Elist\_2ESNOC\ A\_27a)\ V1x)\ V3l))))))) \tag{20}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27b.((ap\ (c\_2Epair\_2EFST\ A\_27a\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y)) = V0x))) \tag{21}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27b.((ap\ (c\_2Epair\_2ESND\ A\_27a\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epair\_2E\_2C}\ A\_27a\ A\_27b)\ V0x)\ V1y)) = V1y))) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (((ap\ (c\_2Eupdate\_2ELIST\_\_UPDATE\ A\_27a\ A\_27b)\ (c\_2Elist\_2ENIL\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))) = (\mathsf{c\_2Ecombin\_2EI}\ (A\_27b^{A\_27a}))) \wedge (\forall V0h \in (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b).(\forall V1t \in (ty\_2Elist\_2Elist\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)).((ap\ (c\_2Eupdate\_2ELIST\_\_UPDATE\ A\_27a\ A\_27b)\ (ap\ (ap\ (c\_2Elist\_2ECONS\ (ty\_2Epair\_2Eprod\ A\_27a\ A\_27b))\ V0h)\ V1t)) = (ap\ (ap\ (\mathsf{c\_2Ecombin\_2Eo}\ (A\_27b^{A\_27a})\ (A\_27b^{A\_27a})\ (A\_27b^{A\_27a}))\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2EUPDATE}\ A\_27a\ A\_27b)\ (ap\ (c\_2Epair\_2EFST\ A\_27a\ A\_27b)\ V0h))\ (ap\ (c\_2Epair\_2ESND\ A\_27a\ A\_27b)\ V0h)))\ (ap\ (c\_2Eupdate\_2ELIST\_\_UPDATE\ A\_27a\ A\_27b)\ V1t)))))) \tag{23}$$

Theorem 1

$$\begin{gathered}
\forall A_27a.\mathit{nonempty}\ A_27a \Rightarrow \forall A_27b.\mathit{nonempty}\ A_27b \Rightarrow \forall A_27c. \\
\mathit{nonempty}\ A_27c \Rightarrow \forall A_27d.\mathit{nonempty}\ A_27d \Rightarrow \forall A_27e.\mathit{nonempty} \\
A_27e \Rightarrow \forall A_27f.\mathit{nonempty}\ A_27f \Rightarrow \forall A_27g.\mathit{nonempty}\ A_27g \Rightarrow \\
\forall A_27h.\mathit{nonempty}\ A_27h \Rightarrow (((\forall V0l1 \in A_27a.(\forall V1l2 \in \\
A_27a.(\forall V2r1 \in A_27b.(\forall V3r2 \in A_27b.((ap\ (ap\ (\mathsf{c_2Ecombin_2Eo} \\
(A_27b^{A_27a})\ (A_27b^{A_27a})\ (A_27b^{A_27a}))\ (ap\ (ap\ (\mathsf{c_2Ecombin_2EUPDATE} \\
A_27a\ A_27b)\ V0l1)\ V2r1))\ (ap\ (ap\ (\mathsf{c_2Ecombin_2EUPDATE}\ A_27a\ A_27b) \\
V1l2)\ V3r2)) = (ap\ (\mathit{c_2Eupdate_2ELIST__UPDATE}\ A_27a\ A_27b)\ (ap \\
(ap\ (\mathit{c_2Elist_2ECONS}\ (\mathit{ty_2Epair_2Eprod}\ A_27a\ A_27b))\ (ap\ (ap\ (\\
\mathsf{c_2Epair_2E_2C}\ A_27a\ A_27b)\ V0l1)\ V2r1))\ (ap\ (ap\ (\mathit{c_2Elist_2ECONS} \\
(\mathit{ty_2Epair_2Eprod}\ A_27a\ A_27b))\ (ap\ (ap\ (\mathsf{c_2Epair_2E_2C}\ A_27a \\
A_27b)\ V1l2)\ V3r2))\ (\mathit{c_2Elist_2ENIL}\ (\mathit{ty_2Epair_2Eprod}\ A_27a\ A_27b))))))))))\wedge \\
((\forall V4l \in A_27c.(\forall V5r \in A_27d.(\forall V6t \in (\mathit{ty_2Elist_2Elist} \\
(\mathit{ty_2Epair_2Eprod}\ A_27c\ A_27d)).((ap\ (ap\ (\mathsf{c_2Ecombin_2Eo}\ (A_27d^{A_27c}) \\
(A_27d^{A_27c})\ (A_27d^{A_27c}))\ (ap\ (ap\ (\mathsf{c_2Ecombin_2EUPDATE}\ A_27c \\
A_27d)\ V4l)\ V5r))\ (ap\ (\mathit{c_2Eupdate_2ELIST__UPDATE}\ A_27c\ A_27d) \\
V6t)) = (ap\ (\mathit{c_2Eupdate_2ELIST__UPDATE}\ A_27c\ A_27d)\ (ap\ (ap\ (\mathit{c_2Elist_2ECONS} \\
(\mathit{ty_2Epair_2Eprod}\ A_27c\ A_27d))\ (ap\ (ap\ (\mathsf{c_2Epair_2E_2C}\ A_27c \\
A_27d)\ V4l)\ V5r))\ V6t))))))\wedge((\forall V7l1 \in A_27e.(\forall V8l2 \in \\
A_27e.(\forall V9r1 \in A_27f.(\forall V10r2 \in A_27f.(\forall V11f \in \\
(A_27f^{A_27e}).((ap\ (ap\ (ap\ (\mathsf{c_2Ecombin_2EUPDATE}\ A_27e\ A_27f)\ V7l1) \\
V9r1)\ (ap\ (ap\ (ap\ (\mathsf{c_2Ecombin_2EUPDATE}\ A_27e\ A_27f)\ V8l2)\ V10r2) \\
V11f)) = (ap\ (ap\ (\mathit{c_2Eupdate_2ELIST__UPDATE}\ A_27e\ A_27f)\ (ap\ (ap \\
(\mathit{c_2Elist_2ECONS}\ (\mathit{ty_2Epair_2Eprod}\ A_27e\ A_27f))\ (ap\ (ap\ (\mathsf{c_2Epair_2E_2C} \\
A_27e\ A_27f)\ V7l1)\ V9r1))\ (ap\ (ap\ (\mathit{c_2Elist_2ECONS}\ (\mathit{ty_2Epair_2Eprod} \\
A_27e\ A_27f))\ (ap\ (ap\ (\mathsf{c_2Epair_2E_2C}\ A_27e\ A_27f)\ V8l2)\ V10r2)) \\
(\mathit{c_2Elist_2ENIL}\ (\mathit{ty_2Epair_2Eprod}\ A_27e\ A_27f)))))\ V11f)))))))\wedge \\
(\forall V12l \in A_27g.(\forall V13r \in A_27h.(\forall V14t \in (\mathit{ty_2Elist_2Elist} \\
(\mathit{ty_2Epair_2Eprod}\ A_27g\ A_27h)).(\forall V15f \in (A_27h^{A_27g}). \\
((ap\ (ap\ (ap\ (\mathsf{c_2Ecombin_2EUPDATE}\ A_27g\ A_27h)\ V12l)\ V13r)\ (ap\ (\\
ap\ (\mathit{c_2Eupdate_2ELIST__UPDATE}\ A_27g\ A_27h)\ V14t)\ V15f)) = (ap\ (\\
ap\ (\mathit{c_2Eupdate_2ELIST__UPDATE}\ A_27g\ A_27h)\ (ap\ (ap\ (\mathit{c_2Elist_2ECONS} \\
(\mathit{ty_2Epair_2Eprod}\ A_27g\ A_27h))\ (ap\ (ap\ (\mathsf{c_2Epair_2E_2C}\ A_27g \\
A_27h)\ V12l)\ V13r))\ V14t))\ V15f))))))))\wedge((\forall V16l1 \in (\mathit{ty_2Elist_2Elist} \\
(\mathit{ty_2Epair_2Eprod}\ A_27a\ A_27b)).(\forall V17l2 \in (\mathit{ty_2Elist_2Elist} \\
(\mathit{ty_2Epair_2Eprod}\ A_27a\ A_27b)).((ap\ (ap\ (\mathsf{c_2Ecombin_2Eo}\ (A_27b^{A_27a}) \\
(A_27b^{A_27a})\ (A_27b^{A_27a}))\ (ap\ (\mathit{c_2Eupdate_2ELIST__UPDATE}\ A_27a \\
A_27b)\ V16l1))\ (ap\ (\mathit{c_2Eupdate_2ELIST__UPDATE}\ A_27a\ A_27b)\ V17l2)) = \\
(ap\ (\mathit{c_2Eupdate_2ELIST__UPDATE}\ A_27a\ A_27b)\ (ap\ (ap\ (\mathit{c_2Elist_2EAPPEND} \\
(\mathit{ty_2Epair_2Eprod}\ A_27a\ A_27b))\ V16l1)\ V17l2)))))\wedge((\forall V18l1 \in \\
(\mathit{ty_2Elist_2Elist}\ (\mathit{ty_2Epair_2Eprod}\ A_27c\ A_27d)).(\forall V19l2 \in \\
A_27c.(\forall V20r \in A_27d.((ap\ (ap\ (\mathsf{c_2Ecombin_2Eo}\ (A_27d^{A_27c}) \\
(A_27d^{A_27c})\ (A_27d^{A_27c}))\ (ap\ (\mathit{c_2Eupdate_2ELIST__UPDATE}\ A_27c \\
A_27d)\ V18l1))\ (ap\ (ap\ (\mathsf{c_2Ecombin_2EUPDATE}\ A_27c\ A_27d)\ V19l2) \\
V20r)) = (ap\ (\mathit{c_2Eupdate_2ELIST__UPDATE}\ A_27c\ A_27d)\ (ap\ (ap\ (\mathit{c_2Elist_2ESNOC} \\
(\mathit{ty_2Epair_2Eprod}\ A_27c\ A_27d))\ (ap\ (ap\ (\mathsf{c_2Epair_2E_2C}\ A_27c \\
A_27d)\ V19l2)\ V20r))\ V18l1))))))\wedge((\forall V21l1 \in (\mathit{ty_2Elist_2Elist} \\
(\mathit{ty_2Epair_2Eprod}\ A_27e\ A_27f)).(\forall V22l2 \in (\mathit{ty_2Elist_2Elist} \\
(\mathit{ty_2Epair_2Eprod}\ A_27e\ A_27f)).(\forall V23f \in (A_27f^{A_27e}). \\
((ap\ (ap\ (\mathit{c_2Eupdate_2ELIST__UPDATE}\ A_27e\ A_27f)\ V21l1)\ (ap\ (ap \\
(\mathit{c_2Eupdate_2ELIST__UPDATE}\ A_27e\ A_27f)\ V22l2)\ V23f)) = (ap\ (ap \\
(\mathit{c_2Eupdate_2ELIST__UPDATE}\ A_27e\ A_27f)\ (ap\ (ap\ (\mathit{c_2Elist_2EAPPEND} \\
(\mathit{ty_2Epair_2Eprod}\ A_27e\ A_27f))\ V21l1)\ V22l2))\ V23f)))))\wedge(\forall V24l1 \in \\
(\mathit{ty_2Elist_2Elist}\ (\mathit{ty_2Epair_2Eprod}\ A_27g\ A_27h)).(\forall V25l2 \in \\
A_27g.(\forall V26r \in A_27h.(\forall V27f \in (A_27h^{A_27g}).((ap \\
(ap\ (\mathit{c_2Eupdate_2ELIST__UPDATE}\ A_27g\ A_27h)\ V24l1)\ (ap\ (ap\ (ap \\
(\mathsf{c_2Ecombin_2EUPDATE}\ A_27g\ A_27h)\ V25l2)\ V26r)\ V27f)) = (ap\ (ap \\
(\mathit{c_2Eupdate_2ELIST__UPDATE}\ A_27g\ A_27h)\ (ap\ (ap\ (\mathit{c_2Elist_2ESNOC} \\
(\mathit{ty_2Epair_2Eprod}\ A_27g\ A_27h))\ (ap\ (ap\ (\mathsf{c_2Epair_2E_2C}\ A_27g
\end{gathered}$$