

thm_2Eupdate_2EOVERRIDE_ind (TMZ1Xu72kb8CAHgqfUshBp9CCLozjkYX5v9)

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Definition 1 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \ x)) \text{ of type } \iota \Rightarrow \iota.$

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota.$

Definition 3 We define `c_2Ebool_2E_3F` to be $\lambda A_{27a} : \iota. (\lambda V0P \in (2^{A_{27a}}). (\text{ap } V0P \text{ (ap (c_2Emin_2E_40 } A_{27a} \text{))}))$
Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$\text{nonempty ty_2Enum_2Enum} \tag{1}$$

Let `c_2Earithmetic_2EEVEN` : ι be given. Assume the following.

$$\text{c_2Earithmetic_2EEVEN} \in (2^{\text{ty_2Enum_2Enum}}) \tag{2}$$

Let `c_2Earithmetic_2EODD` : ι be given. Assume the following.

$$\text{c_2Earithmetic_2EODD} \in (2^{\text{ty_2Enum_2Enum}}) \tag{3}$$

Definition 4 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) \text{ (}\lambda V0x \in 2. V0x)) \text{ (}\lambda V1x \in 2. V1x))$

Definition 5 We define `c_2Ebool_2E_21` to be $\lambda A_{27a} : \iota. (\lambda V0P \in (2^{A_{27a}}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A_{27a}})) \text{ (}\lambda V1t \in 2. V1t)) \text{ (}\lambda V2t \in 2. V2t))$

Definition 6 We define `c_2Ebool_2EF` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) \text{ (}\lambda V0t \in 2. V0t))$.

Definition 7 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow p \ Q)$ of type $\iota.$

Definition 8 We define `c_2Ebool_2E_7E` to be $(\lambda V0t \in 2. (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D_3D_3E } V0t) \text{ c_2Ebool_2EF } 2) \text{ (}\lambda V1t \in 2. V1t))$

Definition 9 We define `c_2Ebool_2E_2F_5C` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) \text{ (}\lambda V2t \in 2. V2t)) \text{ (}\lambda V3t \in 2. V3t))$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (4)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (5)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (6)$$

Definition 10 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num$

Definition 11 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 12 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 13 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in$

Definition 14 We define $c_2Earithmetic_2E_3E_3D$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (7)$$

Definition 15 We define c_2Enum_2E0 to be $(ap\ c_2Enum_2EABS_num\ c_2Enum_2EZERO_REP)$.

Definition 16 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota. (\lambda V0t \in 2. (\lambda V1t1 \in A_27a. (\lambda V2t2 \in A_27a. ($

Definition 17 We define $c_2Eprim_rec_2EPRE$ to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ (ap\ (ap\ (c_2Ebool_2E$

Let $c_2Earithmetic_2EEXP : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EEXP \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (8)$$

Let $c_2Earithmetic_2E_2D : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2D \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (9)$$

Let $c_2Earithmetic_2E_2A : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2A \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (10)$$

Definition 18 We define $c_2Enumeral_2EiZ$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Definition 19 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum. V0x$.

Let $c_2Earithmetic_2E_2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E_2B \in ((ty_2Enum_2Enum^{ty_2Enum_2Enum})^{ty_2Enum_2Enum}) \quad (11)$$

Definition 20 We define $c_2\text{Earithmic_2EBIT2}$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2\text{Earithmic_2EBIT2})$

Definition 21 We define $c_2\text{Earithmic_2EBIT1}$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap (ap c_2\text{Earithmic_2EBIT1})$

Definition 22 We define $c_2\text{Earithmic_2EZERO}$ to be c_2Enum_2E0 .

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \quad (12)$$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (13)$$

Definition 23 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap (c_2Epair_2E_2C)$

Definition 24 We define $c_2Erelation_2Einv_image$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0R \in ((2^{A_27b})^{A_27a}).\lambda V1$

Definition 25 We define $c_2Eprim_rec_2Emeasure$ to be $\lambda A_27a : \iota.(ap (c_2Erelation_2Einv_image\ A_27a\ ty_2Eprim_rec_2Emeasure)$

Let $c_2Ebool_2EARB : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Ebool_2EARB\ A_27a \in A_27a \quad (14)$$

Definition 26 We define $c_2Erelation_2ERESTRICT$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1$

Let $ty_2Elist_2Elist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Elist_2Elist\ A0) \quad (15)$$

Let $c_2Elist_2ELENGTH : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ELENGTH\ A_27a \in (ty_2Enum_2Enum^{(ty_2Elist_2Elist\ A_27a)}) \quad (16)$$

Definition 27 We define $c_2\text{Earithmic_2E_3C_3D}$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum.$

Let $c_2Elist_2ENIL : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ENIL\ A_27a \in (ty_2Elist_2Elist\ A_27a) \quad (17)$$

Definition 28 We define $c_2Ecombin_2EK$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 29 We define $c_2Ecombin_2ES$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}).\lambda V1$

Definition 30 We define $c_2Ecombin_2EI$ to be $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2ES\ A_27a\ (A_27a^{A_27a})\ A_27a)$

Let $c_2Elist_2Elist_CASE : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Elist_2Elist_CASE \\ & A_27a\ A_27b \in (((A_27b^{(A_27b^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}})^{A_27b})^{(ty_2Elist_2Elist\ A_27a)}) \end{aligned} \quad (18)$$

Let $c_2Elist_2ECONS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2ECONS\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}) \quad (19)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST \\ & A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \end{aligned} \quad (20)$$

Let $c_2Elist_2EFILTER : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Elist_2EFILTER\ A_27a \in (((ty_2Elist_2Elist\ A_27a)^{(ty_2Elist_2Elist\ A_27a)})^{(2^{A_27a})}) \quad (21)$$

Definition 31 We define $c_2Erelation_2EWF$ to be $\lambda A_27a : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). (ap\ (c_2Ebool_2E_21$

Definition 32 We define $c_2Erelation_2ETC$ to be $\lambda A_27a : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1a \in A_27a. \lambda V2b$

Definition 33 We define $c_2Erelation_2Eapprox$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1M$

Definition 34 We define $c_2Erelation_2Ethe_fun$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1M$

Definition 35 We define $c_2Erelation_2EWFREC$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0R \in ((2^{A_27a})^{A_27a}). \lambda V1M$

Definition 36 We define $c_2Eupdate_2EOVERRIDE$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. (ap\ (ap\ (c_2Erelation_2EWF$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & ((ap\ (ap\ c_2Earithmetic_2E_2B\ c_2Enum_2E0)\ V0m) = V0m) \wedge (((ap\ (\\ & ap\ c_2Earithmetic_2E_2B\ V0m)\ c_2Enum_2E0) = V0m) \wedge (((ap\ (ap\ c_2Earithmetic_2E_2B \\ & (ap\ c_2Enum_2ESUC\ V0m))\ V1n) = (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B \\ & V0m)\ V1n))) \wedge ((ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ (ap\ c_2Enum_2ESUC \\ & V1n)) = (ap\ c_2Enum_2ESUC\ (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n)))))) \end{aligned} \quad (22)$$

Assume the following.

$$\begin{aligned} & (\forall V0m \in ty_2Enum_2Enum. (\forall V1n \in ty_2Enum_2Enum. (\\ & (ap\ (ap\ c_2Earithmetic_2E_2B\ V0m)\ V1n) = (ap\ (ap\ c_2Earithmetic_2E_2B \\ & V1n)\ V0m)))) \end{aligned} \quad (23)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(p (ap (ap c_2Earithmetic_2E_3C_3D c_2Enum_2E0) V0n))) \quad (24)$$

Assume the following.

$$(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\neg(p (ap (ap c_2Eprim_rec_2E_3C V0m) V1n))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V0m))))) \quad (25)$$

Assume the following.

$$\begin{aligned} &(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ &((ap (ap c_2Earithmetic_2E_2A c_2Enum_2E0) V0m) = c_2Enum_2E0) \wedge \\ &(((ap (ap c_2Earithmetic_2E_2A V0m) c_2Enum_2E0) = c_2Enum_2E0) \wedge \\ &(((ap (ap c_2Earithmetic_2E_2A (ap c_2Earithmetic_2ENUMERAL \\ &(ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0m) = V0m) \wedge \\ &(((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Earithmetic_2ENUMERAL \\ &(ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) = V0m) \wedge (\\ &((ap (ap c_2Earithmetic_2E_2A (ap c_2Enum_2ESUC V0m)) V1n) = (ap \\ &(ap c_2Earithmetic_2E_2B (ap (ap c_2Earithmetic_2E_2A V0m) V1n)) \\ &V1n)) \wedge ((ap (ap c_2Earithmetic_2E_2A V0m) (ap c_2Enum_2ESUC V1n)) = \\ &(ap (ap c_2Earithmetic_2E_2B V0m) (ap (ap c_2Earithmetic_2E_2A \\ &V0m) V1n)))))))))) \quad (26) \end{aligned}$$

Assume the following.

$$\begin{aligned} &(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ &\forall V2p \in ty_2Enum_2Enum.(((p (ap (ap c_2Earithmetic_2E_3C_3D \\ &V0m) V1n)) \wedge (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p))) \Rightarrow (p (\\ &ap (ap c_2Earithmetic_2E_3C_3D V0m) V2p)))))) \quad (27) \end{aligned}$$

Assume the following.

$$\begin{aligned} &(\forall V0m \in ty_2Enum_2Enum.(\forall V1n \in ty_2Enum_2Enum.(\\ &\forall V2p \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmetic_2E_3C_3D \\ &(ap (ap c_2Earithmetic_2E_2B V0m) V1n)) (ap (ap c_2Earithmetic_2E_2B \\ &V0m) V2p))) \Leftrightarrow (p (ap (ap c_2Earithmetic_2E_3C_3D V1n) V2p)))))) \quad (28) \end{aligned}$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.((ap c_2Enum_2ESUC V0n) = (ap (ap c_2Earithmetic_2E_2B (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO))) V0n))) \quad (29)$$

Assume the following.

$$True \quad (30)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p \ V0t))) \quad (31)$$

Assume the following.

$$\forall A_27a. nonempty \ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p \ V0t)) \Leftrightarrow (p \ V0t))) \quad (32)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge (((p \ V0t) \Rightarrow (p \ V0t)) \Leftrightarrow True) \wedge ((p \ V0t) \Rightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (33)$$

Assume the following.

$$((\forall V0t \in 2. ((\neg(\neg(p \ V0t))) \Leftrightarrow (p \ V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (34)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p \ V0t)) \Leftrightarrow (p \ V0t)) \wedge (((p \ V0t) \Leftrightarrow True) \Leftrightarrow (p \ V0t)) \wedge (((False \Leftrightarrow (p \ V0t)) \Leftrightarrow (\neg(p \ V0t))) \wedge (((p \ V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p \ V0t)))))) \quad (35)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p \ V0A) \wedge (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A)) \vee (\neg(p \ V1B)))) \wedge ((\neg((p \ V0A) \vee (p \ V1B))) \Leftrightarrow ((\neg(p \ V0A)) \wedge (\neg(p \ V1B))))))) \quad (36)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((p \ V0A) \Rightarrow (p \ V1B)) \Leftrightarrow ((\neg(p \ V0A)) \vee (p \ V1B)))) \quad (37)$$

Assume the following.

$$(\forall V0t \in 2. (((p \ V0t) \Rightarrow False) \Leftrightarrow ((p \ V0t) \Leftrightarrow False))) \quad (38)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p \ V0t1) \Rightarrow ((p \ V1t2) \Rightarrow (p \ V2t3))) \Leftrightarrow (((p \ V0t1) \wedge (p \ V1t2)) \Rightarrow (p \ V2t3)))))) \quad (39)$$

Assume the following.

$$\forall A_27a. nonempty \ A_27a \Rightarrow (\forall V0x \in A_27a. ((ap \ (c_2Ecombin_2El \ A_27a) \ V0x) = V0x)) \quad (40)$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\
& (\forall V0v \in A_27b. (\forall V1f \in ((A_27b^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}). \\
& ((ap\ (ap\ (ap\ (c_2Elist_2Elist_CASE\ A_27a\ A_27b)\ (c_2Elist_2ENIL \\
& A_27a))\ V0v)\ V1f) = V0v))) \wedge (\forall V2a0 \in A_27a. (\forall V3a1 \in \\
& (ty_2Elist_2Elist\ A_27a). (\forall V4v \in A_27b. (\forall V5f \in (\\
& (A_27b^{(ty_2Elist_2Elist\ A_27a)})^{A_27a}). ((ap\ (ap\ (ap\ (c_2Elist_2Elist_CASE \\
& A_27a\ A_27b)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V2a0)\ V3a1))\ V4v)\ V5f) = \\
& (ap\ (ap\ V5f\ V2a0)\ V3a1))))))
\end{aligned} \tag{41}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (((ap\ (c_2Elist_2ELENGTH\ A_27a) \\
& (c_2Elist_2ENIL\ A_27a)) = c_2Enum_2E0) \wedge (\forall V0h \in A_27a. (\\
& \forall V1t \in (ty_2Elist_2Elist\ A_27a). ((ap\ (c_2Elist_2ELENGTH \\
& A_27a)\ (ap\ (ap\ (c_2Elist_2ECONS\ A_27a)\ V0h)\ V1t)) = (ap\ c_2Enum_2ESUC \\
& (ap\ (c_2Elist_2ELENGTH\ A_27a)\ V1t))))))
\end{aligned} \tag{42}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0l \in (ty_2Elist_2Elist \\
& A_27a). ((V0l = (c_2Elist_2ENIL\ A_27a)) \vee (\exists V1h \in A_27a. (\\
& \exists V2t \in (ty_2Elist_2Elist\ A_27a). (V0l = (ap\ (ap\ (c_2Elist_2ECONS \\
& A_27a)\ V1h)\ V2t))))))
\end{aligned} \tag{43}$$

$$\begin{aligned}
& ((\forall V0n \in ty_2Enum_2Enum.((ap (ap c_2Earithmic_2E_2B \\
& c_2Enum_2E0) V0n) = V0n)) \wedge ((\forall V1n \in ty_2Enum_2Enum.((ap \\
& (ap c_2Earithmic_2E_2B V1n) c_2Enum_2E0) = V1n)) \wedge ((\forall V2n \in \\
& ty_2Enum_2Enum.(\forall V3m \in ty_2Enum_2Enum.((ap (ap c_2Earithmic_2E_2B \\
& (ap c_2Earithmic_2ENUMERAL V2n)) (ap c_2Earithmic_2ENUMERAL \\
& V3m))) = (ap c_2Earithmic_2ENUMERAL (ap c_2Enumeral_2EiZ (ap \\
& (ap c_2Earithmic_2E_2B V2n) V3m)))))) \wedge ((\forall V4n \in ty_2Enum_2Enum. \\
& ((ap (ap c_2Earithmic_2E_2A c_2Enum_2E0) V4n) = c_2Enum_2E0)) \wedge \\
& ((\forall V5n \in ty_2Enum_2Enum.((ap (ap c_2Earithmic_2E_2A \\
& V5n) c_2Enum_2E0) = c_2Enum_2E0)) \wedge ((\forall V6n \in ty_2Enum_2Enum. \\
& (\forall V7m \in ty_2Enum_2Enum.((ap (ap c_2Earithmic_2E_2A (\\
& ap c_2Earithmic_2ENUMERAL V6n)) (ap c_2Earithmic_2ENUMERAL \\
& V7m))) = (ap c_2Earithmic_2ENUMERAL (ap (ap c_2Earithmic_2E_2A \\
& V6n) V7m)))))) \wedge ((\forall V8n \in ty_2Enum_2Enum.((ap (ap c_2Earithmic_2E_2D \\
& c_2Enum_2E0) V8n) = c_2Enum_2E0)) \wedge ((\forall V9n \in ty_2Enum_2Enum. \\
& ((ap (ap c_2Earithmic_2E_2D V9n) c_2Enum_2E0) = V9n)) \wedge ((\forall V10n \in \\
& ty_2Enum_2Enum.(\forall V11m \in ty_2Enum_2Enum.((ap (ap c_2Earithmic_2E_2D \\
& (ap c_2Earithmic_2ENUMERAL V10n)) (ap c_2Earithmic_2ENUMERAL \\
& V11m))) = (ap c_2Earithmic_2ENUMERAL (ap (ap c_2Earithmic_2E_2D \\
& V10n) V11m)))))) \wedge ((\forall V12n \in ty_2Enum_2Enum.((ap (ap c_2Earithmic_2EEXP \\
& c_2Enum_2E0) (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 \\
& V12n))) = c_2Enum_2E0)) \wedge ((\forall V13n \in ty_2Enum_2Enum.((ap \\
& (ap c_2Earithmic_2EEXP c_2Enum_2E0) (ap c_2Earithmic_2ENUMERAL \\
& (ap c_2Earithmic_2EBIT2 V13n))) = c_2Enum_2E0)) \wedge ((\forall V14n \in \\
& ty_2Enum_2Enum.((ap (ap c_2Earithmic_2EEXP V14n) c_2Enum_2E0) = \\
& (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 c_2Earithmic_2EZERO)))))) \wedge \\
& ((\forall V15n \in ty_2Enum_2Enum.(\forall V16m \in ty_2Enum_2Enum. \\
& ((ap (ap c_2Earithmic_2EEXP (ap c_2Earithmic_2ENUMERAL V15n)) \\
& (ap c_2Earithmic_2ENUMERAL V16m))) = (ap c_2Earithmic_2ENUMERAL \\
& (ap (ap c_2Earithmic_2EEXP V15n) V16m)))))) \wedge (((ap c_2Enum_2ESUC \\
& c_2Enum_2E0) = (ap c_2Earithmic_2ENUMERAL (ap c_2Earithmic_2EBIT1 \\
& c_2Earithmic_2EZERO))) \wedge ((\forall V17n \in ty_2Enum_2Enum.(\\
& (ap c_2Enum_2ESUC (ap c_2Earithmic_2ENUMERAL V17n)) = (ap c_2Earithmic_2ENUMERAL \\
& (ap c_2Enum_2ESUC V17n)))))) \wedge (((ap c_2Eprim_rec_2EPRE c_2Enum_2E0) = \\
& c_2Enum_2E0) \wedge ((\forall V18n \in ty_2Enum_2Enum.((ap c_2Eprim_rec_2EPRE \\
& (ap c_2Earithmic_2ENUMERAL V18n)) = (ap c_2Earithmic_2ENUMERAL \\
& (ap c_2Eprim_rec_2EPRE V18n)))))) \wedge ((\forall V19n \in ty_2Enum_2Enum. \\
& (((ap c_2Earithmic_2ENUMERAL V19n) = c_2Enum_2E0) \Leftrightarrow (V19n = c_2Earithmic_2EZERO)))))) \wedge \\
& ((\forall V20n \in ty_2Enum_2Enum.((c_2Enum_2E0 = (ap c_2Earithmic_2ENUMERAL \\
& V20n)) \Leftrightarrow (V20n = c_2Earithmic_2EZERO))) \wedge ((\forall V21n \in ty_2Enum_2Enum. \\
& (\forall V22m \in ty_2Enum_2Enum.(((ap c_2Earithmic_2ENUMERAL \\
& V21n) = (ap c_2Earithmic_2ENUMERAL V22m)) \Leftrightarrow (V21n = V22m)))))) \wedge \\
& ((\forall V23n \in ty_2Enum_2Enum.((p (ap (ap c_2Eprim_rec_2E_3C \\
& V23n) c_2Enum_2E0)) \Leftrightarrow False)) \wedge ((\forall V24n \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Eprim_rec_2E_3C c_2Enum_2E0) (ap c_2Earithmic_2ENUMERAL \\
& V24n))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmic_2EZERO) \\
& V24n)))))) \wedge ((\forall V25n \in ty_2Enum_2Enum.(\forall V26m \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Eprim_rec_2E_3C (ap c_2Earithmic_2ENUMERAL \\
& V25n)) (ap c_2Earithmic_2ENUMERAL V26m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& V25n) V26m)))))) \wedge ((\forall V27n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmic_2E_3E \\
& c_2Enum_2E0) V27n)) \Leftrightarrow False)) \wedge ((\forall V28n \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Earithmic_2E_3E (ap c_2Earithmic_2ENUMERAL \\
& V28n)) c_2Enum_2E0)) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C c_2Earithmic_2EZERO) \\
& V28n)))))) \wedge ((\forall V29n \in ty_2Enum_2Enum.(\forall V30m \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Earithmic_2E_3E (ap c_2Earithmic_2ENUMERAL \\
& V29n)) (ap c_2Earithmic_2ENUMERAL V30m))) \Leftrightarrow (p (ap (ap c_2Eprim_rec_2E_3C \\
& V30m) V29n)))))) \wedge ((\forall V31n \in ty_2Enum_2Enum.((p (ap (ap c_2Earithmic_2E_3C_3D \\
& c_2Enum_2E0) V31n)) \Leftrightarrow True)) \wedge ((\forall V32n \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2ENUMERAL
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0n \in ty_2Enum_2Enum. (\forall V1m \in ty_2Enum_2Enum. (\\
& ((p (ap (ap c_2Earithmic_2E_3C_3D c_2Earithmic_2EZERO) V0n)) \Leftrightarrow \\
& True) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT1 \\
& V0n)) c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D \\
& (ap c_2Earithmic_2EBIT2 V0n)) c_2Earithmic_2EZERO)) \Leftrightarrow False) \wedge \\
& (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT1 \\
& V0n)) (ap c_2Earithmic_2EBIT1 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT1 \\
& V0n)) (ap c_2Earithmic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V0n) V1m))) \wedge (((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT2 \\
& V0n)) (ap c_2Earithmic_2EBIT1 V1m))) \Leftrightarrow (\neg (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V1m) V0n)))) \wedge ((p (ap (ap c_2Earithmic_2E_3C_3D (ap c_2Earithmic_2EBIT2 \\
& V0n)) (ap c_2Earithmic_2EBIT2 V1m))) \Leftrightarrow (p (ap (ap c_2Earithmic_2E_3C_3D \\
& V0n) V1m)))))))))) \\
\end{aligned} \tag{45}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow (\\
& \forall V0x \in A_27a. (\forall V1y \in A_27b. ((ap (c_2Epair_2EFS T A_27a \\
& A_27b) (ap (ap (c_2Epair_2E_2C A_27a A_27b) V0x) V1y)) = V0x))) \\
\end{aligned} \tag{46}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a. nonempty A_27a \Rightarrow \forall A_27b. nonempty A_27b \Rightarrow (\\
& \forall V0P \in (2^{(ty_2Epair_2Eprod A_27a A_27b)}). ((\forall V1p \in \\
& (ty_2Epair_2Eprod A_27a A_27b). (p (ap V0P V1p))) \Leftrightarrow (\forall V2p_1 \in \\
& A_27a. (\forall V3p_2 \in A_27b. (p (ap V0P (ap (ap (c_2Epair_2E_2C \\
& A_27a A_27b) V2p_1) V3p_2)))))) \\
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a. nonempty A_27a \Rightarrow (\forall V0m \in (ty_2Enum_2Enum^{A_27a}). \\
& (p (ap (c_2Erelation_2EWF A_27a) (ap (c_2Eprim_rec_2Emeasure \\
& A_27a) V0m)))) \\
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a. nonempty A_27a \Rightarrow (\forall V0R \in ((2^{A_27a})^{A_27a}). \\
& ((p (ap (c_2Erelation_2EWF A_27a) V0R)) \Rightarrow (\forall V1P \in (2^{A_27a}). \\
& ((\forall V2x \in A_27a. ((\forall V3y \in A_27a. ((p (ap (ap V0R V3y) V2x)) \Rightarrow \\
& (p (ap V1P V3y)))) \Rightarrow (p (ap V1P V2x)))) \Rightarrow (\forall V4x \in A_27a. (p (ap \\
& V1P V4x)))))) \\
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0f \in (A_27b^{A_27a}).(\forall V1R \in ((2^{A_27a})^{A_27a}).(\forall V2y \in \\ & \quad A_27a.(\forall V3z \in A_27a.((p\ (ap\ (ap\ V1R\ V2y)\ V3z)) \Rightarrow ((ap\ (ap\ (ap \\ & \quad (ap\ (c_2Erelation_2ERESTRICT\ A_27a\ A_27b)\ V0f)\ V1R)\ V3z)\ V2y) = \\ & \quad (ap\ V0f\ V2y))))))))) \end{aligned} \quad (50)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0M \in ((A_27b^{A_27a})^{(A_27b^{A_27a})}).(\forall V1R \in ((2^{A_27a})^{A_27a}). \\ & \quad (\forall V2f \in (A_27b^{A_27a}).((V2f = (ap\ (ap\ (c_2Erelation_2EWFREC \\ & \quad A_27a\ A_27b)\ V1R)\ V0M)) \Rightarrow ((p\ (ap\ (c_2Erelation_2EWF\ A_27a)\ V1R)) \Rightarrow \\ & \quad (\forall V3x \in A_27a.((ap\ V2f\ V3x) = (ap\ (ap\ V0M\ (ap\ (ap\ (ap\ (c_2Erelation_2ERESTRICT \\ & \quad A_27a\ A_27b)\ V2f)\ V1R)\ V3x))\ V3x))))))))) \end{aligned} \quad (51)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1l \in \\ & \quad (ty_2Elist_2Elist\ A_27a).(p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D \\ & \quad (ap\ (c_2Elist_2ELENGTH\ A_27a)\ (ap\ (ap\ (c_2Elist_2EFILTER\ A_27a) \\ & \quad V0P)\ V1l)))\ (ap\ (c_2Elist_2ELENGTH\ A_27a)\ V1l)))))) \end{aligned} \quad (52)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (53)$$

Assume the following.

$$(\forall V0A \in 2.((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (54)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & \quad (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (55)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2.(\forall V1B \in 2.(((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & \quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \end{aligned} \quad (56)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p\ V0A)) \Rightarrow False) \Rightarrow ((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (57)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p\ V0p) \Leftrightarrow (\\ & \quad (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\ & \quad p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ & \quad ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \end{aligned} \quad (58)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(((p \ V0p) \Leftrightarrow (\neg(p \ V1q))) \Leftrightarrow (((p \ V0p) \vee (p \ V1q)) \wedge ((\neg(p \ V1q)) \vee (\neg(p \ V0p))))))) \quad (59)$$

Theorem 1

$$\begin{aligned} & \forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow (\\ & \quad \forall V0P \in (2^{(ty_2Elist_2Elist \ (ty_2Epair_2Eprod \ A_27a \ A_27b))}). \\ & \quad (((p \ (ap \ V0P \ (c_2Elist_2ENIL \ (ty_2Epair_2Eprod \ A_27a \ A_27b)))) \wedge \\ & \quad (\forall V1x \in (ty_2Epair_2Eprod \ A_27a \ A_27b).(\forall V2t \in (ty_2Elist_2Elist \\ & \quad (ty_2Epair_2Eprod \ A_27a \ A_27b)).((p \ (ap \ V0P \ (ap \ (ap \ (c_2Elist_2EFILTER \\ & \quad (ty_2Epair_2Eprod \ A_27a \ A_27b)) \ (\lambda V3y \in (ty_2Epair_2Eprod \\ & \quad A_27a \ A_27b).(ap \ c_2Ebool_2E7E \ (ap \ (ap \ (c_2Emin_2E3D \ A_27a) \\ & \quad (ap \ (c_2Epair_2EFST \ A_27a \ A_27b) \ V1x)) \ (ap \ (c_2Epair_2EFST \ A_27a \\ & \quad A_27b) \ V3y)))))) \ V2t))) \Rightarrow (p \ (ap \ V0P \ (ap \ (ap \ (c_2Elist_2ECONS \ (ty_2Epair_2Eprod \\ & \quad A_27a \ A_27b)) \ V1x) \ V2t)))))) \Rightarrow (\forall V4v \in (ty_2Elist_2Elist \\ & \quad (ty_2Epair_2Eprod \ A_27a \ A_27b)).(p \ (ap \ V0P \ V4v)))) \end{aligned}$$