

# thm\_2Eupdate\_2EUPDATE\_\_APPLY\_\_IMP\_\_ID (TMdZrDFkqPyd3adsz2ez19461TViTm2hGs7)

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**Definition 1** We define  $c\_2Emin\_2E\_3D$  to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define  $c\_2Ebool\_2E\_ET$  to be  $(ap (ap (c\_2Emin\_2E\_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define  $c\_2Ebool\_2E\_21$  to be  $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap (ap (c\_2Emin\_2E\_3D (2^{A\_27a}))$

**Definition 4** We define  $c\_2Ebool\_2E\_EF$  to be  $(ap (c\_2Ebool\_2E\_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 5** We define  $c\_2Emin\_2E\_3D\_3D\_3E$  to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 6** We define  $c\_2Ebool\_2E\_2F\_5C$  to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c\_2Ebool\_2E\_21 2) (\lambda V2t \in 2.V2t))$

**Definition 7** We define  $c\_2Emin\_2E\_40$  to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge p x))$  of type  $\iota \Rightarrow \iota$ .

**Definition 8** We define  $c\_2Ebool\_2E\_COND$  to be  $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(ap$

**Definition 9** We define  $c\_2Ecombin\_2E\_UPDATE$  to be  $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0a \in A\_27a.\lambda V1b \in A\_27b.(ap$

Assume the following.

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow ( \\ & \quad \forall V0f \in (A\_27b^{A\_27a}).(\forall V1b \in A\_27b.(\forall V2a \in A\_27a. \\ & \quad ((ap V0f V2a) = V1b) \Rightarrow ((ap (ap (ap (c\_2Ecombin\_2E\_UPDATE A\_27a A\_27b) \\ & \quad \quad V2a) V1b) V0f) = V0f)))))) \end{aligned} \tag{1}$$

**Theorem 1**

$$\begin{aligned} & \forall A\_27a.nonempty A\_27a \Rightarrow \forall A\_27b.nonempty A\_27b \Rightarrow ( \\ & \quad \forall V0f \in (A\_27b^{A\_27a}).(\forall V1b \in A\_27b.(\forall V2a \in A\_27a. \\ & \quad ((ap V0f V2a) = V1b) \Rightarrow ((ap (ap (ap (c\_2Ecombin\_2E\_UPDATE A\_27a A\_27b) \\ & \quad \quad V2a) V1b) V0f) = V0f)))))) \end{aligned}$$