thm_2Eupdate_2EUPDATE__COMMUTES (TMXdJb-BLYN2wP1oNqy4N5MRXaLe4TZ4mKL7)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ ($c_2Emin_2E_3D $(2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ ($c_2Emin_2E_3D $(2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ ($c_2Ebool_2E_21 $2)\ (\lambda V2t \in 2$

Definition 7 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 8 *We define* c_2Ebool_2ECOND *to be* $\lambda A\_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A\_27a.(\lambda V2t2 \in A\_27a.(a$

Definition 9 *We define* c_2Ecombin_2EUPDATE *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0a \in A\_27a.\lambda V1b \in A\_27b.($

Definition 10 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap$ c_2Emin_2E_3D_3D_3E $V0t)$ c_2Ebool_2E

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1a \in A\_27a.(\forall V2b \in A\_27a. (\forall V3c \in A\_27b.(\forall V4d \in A\_27b.((\neg(V1a = V2b)) \Rightarrow ((ap\ (ap\ (ap\ (\text{c\_2Ecombin\_2EUPDATE}\ A\_27a\ A\_27b)\ V1a)\ V3c)\ (ap\ (ap\ (ap\ (\text{c\_2Ecombin\_2EUPDATE}\ A\_27a\ A\_27b)\ V2b)\ V4d)\ V0f)) = (ap\ (ap\ (ap\ (\text{c\_2Ecombin\_2EUPDATE}\ A\_27a\ A\_27b)\ V2b)\ V4d)\ (ap\ (ap\ (ap\ (\text{c\_2Ecombin\_2EUPDATE}\ A\_27a\ A\_27b)\ V1a)\ V3c)\ V0f)))))))))) \tag{1}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0f \in (A\_27b^{A\_27a}).(\forall V1a \in A\_27a.(\forall V2b \in A\_27a. (\forall V3c \in A\_27b.(\forall V4d \in A\_27b.((\neg(V1a = V2b)) \Rightarrow ((ap\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2EUPDATE}\ A\_27a\ A\_27b)\ V1a)\ V3c)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2EUPDATE}\ A\_27a\ A\_27b)\ V2b)\ V4d)\ V0f)) = (ap\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2EUPDATE}\ A\_27a\ A\_27b)\ V2b)\ V4d)\ (ap\ (ap\ (ap\ (\mathsf{c\_2Ecombin\_2EUPDATE}\ A\_27a\ A\_27b)\ V1a)\ V3c)\ V0f)))))))))$$