thm_2Eupdate_2EUPDATE__EQ (TM-LzNrm8CDuZtMzEWSSNWihAj5kVohaG9vM)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj__o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\text{c_2Emin_2E_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ (ap\ (\text{c_2Emin_2E_3D}\ (2^{A_27$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\text{c_2Ebool_2E_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj__o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\text{c_2Ebool_2E_21}\ 2)\ (\lambda V2t \in 2$

Definition 7 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A$**.if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 8 *We define* c_2Ebool_2ECOND *to be* $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.(a$

Definition 9 *We define* c_2Ecombin_2EUPDATE *to be* $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0a \in A_27a.\lambda V1b \in A_27b.($

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1a \in A_27a.(\forall V2b \in A_27b.(\forall V3c \in A_27b.((ap\ (ap\ (ap\ (\text{c_2Ecombin_2EUPDATE}\ A_27a\ A_27b)\ V1a)\ V3c)\ (ap\ (ap\ (ap\ (\text{c_2Ecombin_2EUPDATE}\ A_27a\ A_27b)\ V1a)\ V2b)\ V0f)) = (ap\ (ap\ (ap\ (\text{c_2Ecombin_2EUPDATE}\ A_27a\ A_27b)\ V1a)\ V3c)\ V0f)))))) \tag{1}$$

Theorem 1

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\forall V0f \in (A_27b^{A_27a}).(\forall V1a \in A_27a.(\forall V2b \in A_27b.(\forall V3c \in A_27b.((ap\ (ap\ (ap\ (\text{c_2Ecombin_2EUPDATE}\ A_27a\ A_27b)\ V1a)\ V3c)\ (ap\ (ap\ (ap\ (\text{c_2Ecombin_2EUPDATE}\ A_27a\ A_27b)\ V1a)\ V2b)\ V0f)) = (ap\ (ap\ (ap\ (\text{c_2Ecombin_2EUPDATE}\ A_27a\ A_27b)\ V1a)\ V3c)\ V0f))))))$$