

# thm\_2Eutil\_\_prob\_2EEQ\_\_T\_\_IMP (TMcJUW- bKxgGVqERAhg2bd3iHnvLPA3BXf25)

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**Definition 1** We define `c_2Emin_2E_3D` to be  $\lambda A.\lambda x \in A.\lambda y \in A.inj\_o (x = y)$  of type  $\iota \Rightarrow \iota$ .

**Definition 2** We define `c_2Ebool_2E_2ET` to be  $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

**Definition 3** We define `c_2Emin_2E_40` to be  $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x))$  then (the  $(\lambda x.x \in A \wedge p)$  of type  $\iota \Rightarrow \iota$ .

**Definition 4** We define `c_2Ebool_2E_3F` to be  $\lambda A_{27a} : \iota.(\lambda V0P \in (2^{A_{27a}}).(ap V0P (ap (c_2Emin_2E_40 A_{27a} P))$

**Definition 5** We define `c_2Ebool_2E_21` to be  $\lambda A_{27a} : \iota.(\lambda V0P \in (2^{A_{27a}}).(ap (ap (c_2Emin_2E_3D (2^{A_{27a}})$

**Definition 6** We define `c_2Ebool_2E_2EF` to be  $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$ .

**Definition 7** We define `c_2Emin_2E_3D_3D_3E` to be  $\lambda P \in 2.\lambda Q \in 2.inj\_o (p P \Rightarrow p Q)$  of type  $\iota$ .

**Definition 8** We define `c_2Ebool_2E_5C_2F` to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

**Definition 9** We define `c_2Ebool_2E_2F_5C` to be  $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

**Definition 10** We define `c_2Ebool_2E_7E` to be  $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_21 2))$

Assume the following.

$$True \tag{1}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p V0t1) \Rightarrow (p V1t2)) \Rightarrow (((p V1t2) \Rightarrow (p V0t1)) \Rightarrow ((p V0t1) \Leftrightarrow (p V1t2)))))) \tag{2}$$

Assume the following.

$$(\forall V0t \in 2.(((p V0t) \Rightarrow False) \Rightarrow (\neg(p V0t)))) \tag{3}$$

Assume the following.

$$(\forall V0t \in 2.((\neg(p V0t)) \Rightarrow ((p V0t) \Rightarrow False))) \tag{4}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((p V0t) \Rightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (5)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge ((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)) \quad (6)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t)))))) \quad (7)$$

Assume the following.

$$\forall A\_27a.nonempty A\_27a \Rightarrow (\forall V0P \in (2^{A-27a}).((\neg(\forall V1x \in A\_27a.(p(ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A\_27a.(\neg(p(ap V0P V2x)))))) \quad (8)$$

Assume the following.

$$(\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (9)$$

Assume the following.

$$(\forall V0A \in 2.((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow False))) \quad (10)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg((p V0A) \vee (p V1B))) \Rightarrow False) \Leftrightarrow (((p V0A) \Rightarrow False) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (11)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg(\neg(p V0A)) \vee (p V1B))) \Rightarrow False) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow False)))) \quad (12)$$

Assume the following.

$$(\forall V0A \in 2.(((\neg(p V0A)) \Rightarrow False) \Rightarrow (((p V0A) \Rightarrow False) \Rightarrow False))) \quad (13)$$

Assume the following.

$$(\forall V0p \in 2.(\forall V1q \in 2.(\forall V2r \in 2.(((p V0p) \Leftrightarrow (p V1q) \Leftrightarrow (p V2r)) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee (\neg(p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee (\neg(p V2r)) \vee (\neg(p V0p))) \wedge ((p V2r) \vee ((\neg(p V1q)) \vee (\neg(p V0p)))))))))) \quad (14)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee 0p) \Leftrightarrow ( \\
& (p \vee 1q) \wedge (p \vee 2r))) \Leftrightarrow (((p \vee 0p) \vee (\neg(p \vee 1q)) \vee \neg(p \vee 2r)))) \wedge (((p \vee 1q) \vee \\
& (\neg(p \vee 0p))) \wedge ((p \vee 2r) \vee \neg(p \vee 0p)))))))))
\end{aligned} \tag{15}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee 0p) \Leftrightarrow ( \\
& (p \vee 1q) \vee (p \vee 2r))) \Leftrightarrow (((p \vee 0p) \vee \neg(p \vee 1q)) \wedge ((p \vee 0p) \vee \neg(p \vee 2r))) \wedge \\
& ((p \vee 1q) \vee ((p \vee 2r) \vee \neg(p \vee 0p)))))))))
\end{aligned} \tag{16}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p \vee 0p) \Leftrightarrow ( \\
& (p \vee 1q) \Rightarrow (p \vee 2r))) \Leftrightarrow (((p \vee 0p) \vee (p \vee 1q)) \wedge ((p \vee 0p) \vee \neg(p \vee 2r))) \wedge ( \\
& \neg(p \vee 1q) \vee ((p \vee 2r) \vee \neg(p \vee 0p)))))))))
\end{aligned} \tag{17}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p \vee 0p) \Leftrightarrow \neg(p \vee 1q)) \Leftrightarrow (((p \vee 0p) \vee \\
& (p \vee 1q)) \wedge (\neg(p \vee 1q) \vee \neg(p \vee 0p))))))
\end{aligned} \tag{18}$$

**Theorem 1**  $(\forall V0x \in 2. ((p \vee 0x) \Leftrightarrow (True \Rightarrow (p \vee 0x))))$ .