

thm_2Eutil__prob_2EMINIMAL__EQ
(TMduFGtphf3cHcyYw17yjU2hL6rvQZAQEHX)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_2T$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ecombin_2E_2K$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.(\lambda V0x \in A_27a.(\lambda V1y \in A_27b.V0x))$

Definition 4 We define $c_2Ecombin_2E_2S$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.(\lambda V0f \in ((A_27c^{A_27b})^{A_27a}))$

Definition 5 We define $c_2Ecombin_2E_2I$ to be $\lambda A_27a : \iota.(ap (ap (c_2Ecombin_2E_2S A_27a (A_27a^{A_27a})) A_27a))$

Definition 6 We define $c_2Ebool_2E_21$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap (ap (c_2Emin_2E_3D (2^{A_27a})) P)))$

Definition 7 We define $c_2Ebool_2E_2F$ to be $(ap (c_2Ebool_2E_21 2)) (\lambda V0t \in 2.V0t)$.

Definition 8 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p \Rightarrow q)$ of type ι .

Definition 9 We define $c_2Ebool_2E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2)) (\lambda V2t \in 2.V2t)))$

Definition 10 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \text{ then } (the (\lambda x.x \in A \wedge P x))$ of type $\iota \Rightarrow \iota$.

Definition 11 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap V0P (ap (c_2Emin_2E_40) P)))$

Definition 12 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_2F))$

Definition 13 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2)) (\lambda V2t \in 2.V2t)))$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Enum_2Enum \tag{1}$$

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \tag{2}$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{omega}) \quad (3)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{omega}) \quad (4)$$

Definition 14 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum. (ap\ c_2Enum_2EABS_num$

Definition 15 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum. \lambda V1n \in ty_2Enum_2Enum$

Definition 16 We define $c_2Eutil_prob_2Eminimal$ to be $\lambda V0p \in (2^{ty_2Enum_2Enum}). (ap\ (c_2Emin_2E.40\ ty$

Assume the following.

$$True \quad (5)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (6)$$

Assume the following.

$$(\forall V0t \in 2. (False \Rightarrow (p\ V0t))) \quad (7)$$

Assume the following.

$$\forall A_27a. nonempty\ A_27a \Rightarrow (\forall V0t \in 2. ((\forall V1x \in A_27a. (p\ V0t) \Leftrightarrow (p\ V0t)))) \quad (8)$$

Assume the following.

$$(\forall V0t \in 2. (((p\ V0t) \Rightarrow False) \Rightarrow (\neg(p\ V0t)))) \quad (9)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(p\ V0t)) \Rightarrow ((p\ V0t) \Rightarrow False))) \quad (10)$$

Assume the following.

$$(\forall V0t \in 2. (((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (11)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg(p\ V0t)))))) \quad (12)$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True))) \quad (13)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (14)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (15)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg(p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg(p V0t))))) \quad (16)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).((\neg(\forall V1x \in A_27a.(p (ap V0P V1x)))) \Leftrightarrow (\exists V2x \in A_27a.(\neg(p (ap V0P V2x))))) \quad (17)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).((\neg(\exists V1x \in A_27a.(p (ap V0P V1x)))) \Leftrightarrow (\forall V2x \in A_27a.(\neg(p (ap V0P V2x))))) \quad (18)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in (2^{A_27a}).(\forall V1Q \in 2.(((\exists V2x \in A_27a.(p (ap V0P V2x))) \vee (p V1Q)) \Leftrightarrow (\exists V3x \in A_27a.((p (ap V0P V3x)) \vee (p V1Q))))) \quad (19)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0P \in 2.(\forall V1Q \in (2^{A_27a}).(((p V0P) \vee (\exists V2x \in A_27a.(p (ap V1Q V2x)))) \Leftrightarrow (\exists V3x \in A_27a.((p V0P) \vee (p (ap V1Q V3x))))) \quad (20)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(\forall V2C \in 2.(((p V0A) \vee ((p V1B) \vee (p V2C))) \Leftrightarrow (((p V0A) \vee (p V1B)) \vee (p V2C))))) \quad (21)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \vee (p V1B)) \Leftrightarrow ((p V1B) \vee (p V0A)))) \quad (22)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \wedge (p V1B))) \Leftrightarrow ((\neg(p V0A) \vee \neg(p V1B)))) \wedge ((\neg((p V0A) \vee (p V1B))) \Leftrightarrow ((\neg(p V0A) \wedge \neg(p V1B))))))) \quad (23)$$

Assume the following.

$$(\forall V0t1 \in 2. (\forall V1t2 \in 2. (\forall V2t3 \in 2. (((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (24)$$

Assume the following.

$$(\forall V0x \in 2. (\forall V1x_{27} \in 2. (\forall V2y \in 2. (\forall V3y_{27} \in 2. (((p V0x) \Leftrightarrow (p V1x_{27})) \wedge ((p V1x_{27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{27})))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{27}) \Rightarrow (p V3y_{27})))))) \quad (25)$$

Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow \forall A_{27b}. \text{nonempty } A_{27b} \Rightarrow (\forall V0P \in ((2^{A_{27b}})^{A_{27a}}). ((\forall V1x \in A_{27a}. (\exists V2y \in A_{27b}. (p (ap (ap V0P V1x) V2y)))) \Leftrightarrow (\exists V3f \in (A_{27b}^{A_{27a}}). (\forall V4x \in A_{27a}. (p (ap (ap V0P V4x) (ap V3f V4x))))))) \quad (26)$$

Assume the following.

$$\forall A_{27a}. \text{nonempty } A_{27a} \Rightarrow (\forall V0x \in A_{27a}. ((ap (c_{2Ecombin_2El} A_{27a}) V0x) = V0x)) \quad (27)$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p V0t))) \Leftrightarrow (p V0t))) \quad (28)$$

Assume the following.

$$(\forall V0A \in 2. ((p V0A) \Rightarrow ((\neg(p V0A)) \Rightarrow \text{False}))) \quad (29)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg(\neg((p V0A) \vee (p V1B))) \Rightarrow \text{False}) \Leftrightarrow ((p V0A) \Rightarrow ((\neg(p V1B)) \Rightarrow \text{False})))))) \quad (31)$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p V0A)) \Rightarrow \text{False}) \Rightarrow (((p V0A) \Rightarrow \text{False}) \Rightarrow \text{False}))) \quad (32)$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Leftrightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee ((p V1q) \vee (p V2r))) \wedge (((p V0p) \vee ((\neg(\\
& p V2r)) \vee (\neg(p V1q)))) \wedge (((p V1q) \vee ((\neg(p V2r)) \vee (\neg(p V0p)))) \wedge ((p V2r) \vee \\
& ((\neg(p V1q)) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{33}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{34}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{35}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge ((\\
& \neg(p V1q)) \vee ((p V2r) \vee (\neg(p V0p))))))
\end{aligned} \tag{36}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{37}$$

Assume the following.

$$\begin{aligned}
& (\forall V0P \in (2^{ty_2Enum_2Enum}). ((\exists V1n \in ty_2Enum_2Enum. \\
& (p (ap V0P V1n))) \Leftrightarrow ((p (ap V0P (ap c_2Util_prob_2Eminimal V0P))) \wedge \\
& (\forall V2n \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C \\
& V2n) (ap c_2Util_prob_2Eminimal V0P))) \Rightarrow (\neg(p (ap V0P V2n))))))
\end{aligned} \tag{38}$$

Assume the following.

$$\begin{aligned}
& (\forall V0m \in ty_2Enum_2Enum. (\forall V1p \in (2^{ty_2Enum_2Enum}). \\
& (((p (ap V1p V0m)) \wedge (\forall V2n \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C \\
& V2n) V0m))) \Rightarrow (\neg(p (ap V1p V2n)))))) \Rightarrow (V0m = (ap c_2Util_prob_2Eminimal \\
& V1p))))
\end{aligned} \tag{39}$$

Theorem 1

$$\begin{aligned}
& (\forall V0p \in (2^{ty_2Enum_2Enum}). (\forall V1m \in ty_2Enum_2Enum. \\
& (((p (ap V0p V1m)) \wedge (V1m = (ap c_2Util_prob_2Eminimal V0p))) \Leftrightarrow \\
& ((p (ap V0p V1m)) \wedge (\forall V2n \in ty_2Enum_2Enum. ((p (ap (ap c_2Eprim_rec_2E_3C \\
& V2n) V1m))) \Rightarrow (\neg(p (ap V0p V2n))))))
\end{aligned}$$