thm_2Eutil__prob_2EREAL__LE__LT__MUL (TMZfC8aJK5CHwBgKtYcd5eTaw22ag4esdVX)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 7 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Let $ty\_2Ehreal\_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Ehreal\_2Ehreal \tag{1}$$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{2}$$

Let $ty\_2Erealax\_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Erealax\_2Ereal \tag{3}$$

Let $c\_2Erealax\_2Ereal\_\_REP\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_REP\_\_CLASS \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{ty\_2Erealax} \tag{4}$$

Definition 8 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.\mathbf{if}\ (\exists x \in A.p\ (ap\ P\ x))\ \mathbf{then}\ (the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 9 *We define* c_2Erealax_2Ereal__REP *to be* $\lambda V0a \in ty\_2Erealax\_2Ereal.(ap$ (c_2Emin_2E_40 $(ty$

Let $c\_2Erealax\_2Etreal\_\_mul : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_mul \in (((ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehr} \quad (5)$$

Let $c\_2Erealax\_2Etreal\_\_eq : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_eq \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Eh} \quad (6)$$

Let $c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Ereal\_\_ABS\_\_CLASS \in (ty\_2Erealax\_2Ereal^{(2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehr} \quad (7)$$

Definition 10 *We define* c_2Erealax_2Ereal__ABS *to be* $\lambda V0r \in (ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty$

Definition 11 *We define* c_2Erealax_2Ereal__mul *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Ereala$

Let $c\_2Erealax\_2Etreal\_\_lt : \iota$ be given. Assume the following.

$$c\_2Erealax\_2Etreal\_\_lt \in ((2^{(ty\_2Epair\_2Eprod\ ty\_2Ehreal\_2Ehreal\ ty\_2Ehreal\_2Ehreal)})^{(ty\_2Epair\_2Eprod\ ty\_2Ehr} \quad (8)$$

Definition 12 *We define* c_2Erealax_2Ereal__lt *to be* $\lambda V0T1 \in ty\_2Erealax\_2Ereal.\lambda V1T2 \in ty\_2Erealax\_$

Definition 13 *We define* c_2Ereal_2Ereal__lte *to be* $\lambda V0x \in ty\_2Erealax\_2Ereal.\lambda V1y \in ty\_2Erealax\_2Er$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \quad (9)$$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (10)$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \quad (11)$$

Definition 14 *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Ereal\_2Ereal\_\_of\_\_num : \iota$ be given. Assume the following.

$$c\_2Ereal\_2Ereal\_\_of\_\_num \in (ty\_2Erealax\_2Ereal^{ty\_2Enum\_2Enum}) \quad (12)$$

Assume the following.

$$True \quad (13)$$

Assume the following.

$$(\forall V0t \in 2.(((True{\Rightarrow}(p\ V0t)){\Leftrightarrow}(p\ V0t)){\wedge}((((p\ V0t){\Rightarrow}True){\Leftrightarrow} True){\wedge}(((False{\Rightarrow}(p\ V0t)){\Leftrightarrow}True){\wedge}((((p\ V0t){\Rightarrow}(p\ V0t)){\Leftrightarrow}True){\wedge}(((p\ V0t){\Rightarrow}False){\Leftrightarrow}({\neg}(p\ V0t)))))))) \tag{14}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a{\Rightarrow}(\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y){\Leftrightarrow}(V1y = V0x)))) \tag{15}$$

Assume the following.

$$(\forall V0t \in 2.(((True{\Leftrightarrow}(p\ V0t)){\Leftrightarrow}(p\ V0t)){\wedge}((((p\ V0t){\Leftrightarrow}True){\Leftrightarrow} (p\ V0t)){\wedge}(((False{\Leftrightarrow}(p\ V0t)){\Leftrightarrow}({\neg}(p\ V0t))){\wedge}(((p\ V0t){\Leftrightarrow}False){\Leftrightarrow}({\neg}(p\ V0t))))))) \tag{16}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1){\Rightarrow} ((p\ V1t2){\Rightarrow}(p\ V2t3))){\Leftrightarrow}(((p\ V0t1){\wedge}(p\ V1t2)){\Rightarrow}(p\ V2t3)))))) \tag{17}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.(((((p\ V0x){\Leftrightarrow}(p\ V1x\_27)){\wedge}((p\ V1x\_27){\Rightarrow}((p\ V2y){\Leftrightarrow}(p\ V3y\_27)))){\Rightarrow} (((p\ V0x){\Rightarrow}(p\ V2y)){\Leftrightarrow}((p\ V1x\_27){\Rightarrow}(p\ V3y\_27))))))))) \tag{18}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.((ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V0x) = (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))) \tag{19}$$

Assume the following.

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. (\forall V2z \in ty\_2Erealax\_2Ereal.((p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V2z)){\Rightarrow}((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V0x)\ V2z))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V1y)\ V2z))){\Leftrightarrow}(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ V0x)\ V1y))))))) \tag{20}$$

Theorem 1

$$(\forall V0x \in ty\_2Erealax\_2Ereal.(\forall V1y \in ty\_2Erealax\_2Ereal. (((p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V0x)){\wedge}(p\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_lt}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ V1y))){\Rightarrow}(p\ (ap\ (ap\ \mathsf{c\_2Ereal\_2Ereal\_\_lte}\ (ap\ c\_2Ereal\_2Ereal\_\_of\_\_num\ \mathsf{c\_2Enum\_2E0}))\ (ap\ (ap\ \mathsf{c\_2Erealax\_2Ereal\_\_mul}\ V0x)\ V1y))))))$$