

thm_2Eutil__prob_2Elg__1
(TMXi5GGWcJVZwyTudj3dA8LA5qCF9qQgUrn)

October 26, 2020

Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_21$ to be $(ap (ap (c_2Emin_2E_3D (2^2))) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x)$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{3}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{ty_2Erealax}) \tag{4}$$

Definition 3 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap\ P\ x))$ **then** (the $(\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$).

Definition 4 We define $c_2Ebool_2E_21$ to be $\lambda A.27a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a})))$

Definition 5 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 (ty$

Let $c_2Erealax_2Etrealm_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_neg \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal}) \tag{5}$$

Let $c_2Erealax_2Etrealm_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Etrealm_eq \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)) \quad (6)$$

Let $c_2Erealax_2Ereal_2EABS_2ECLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_2EABS_2ECLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})} \quad (7)$$

Definition 6 We define $c_2Erealax_2Ereal_2EABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)$

Definition 7 We define $c_2Erealax_2Ereal_2Eneg$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal_2EABS)$

Definition 8 We define $c_2Emin_2E3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o\ (p\ P \Rightarrow Q)$ of type ι .

Definition 9 We define $c_2Ebool_2E2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E21\ 2)\ (\lambda V2t \in 2)))$

Let $ty_2Eenum_2Eenum : \iota$ be given. Assume the following.

$$nonempty\ ty_2Eenum_2Eenum \quad (8)$$

Let $c_2Eenum_2EZERO_2EREPR : \iota$ be given. Assume the following.

$$c_2Eenum_2EZERO_2EREPR \in omega \quad (9)$$

Let $c_2Eenum_2EABS_2Eenum : \iota$ be given. Assume the following.

$$c_2Eenum_2EABS_2Eenum \in (ty_2Eenum_2Eenum)^{omega} \quad (10)$$

Definition 10 We define c_2Eenum_2E0 to be $(ap\ c_2Eenum_2EABS_2Eenum\ c_2Eenum_2EZERO_2EREPR)$.

Let $c_2Eenum_2EEREP_2Eenum : \iota$ be given. Assume the following.

$$c_2Eenum_2EEREP_2Eenum \in (omega)^{ty_2Eenum_2Eenum} \quad (11)$$

Let $c_2Eenum_2ESUC_2EREPR : \iota$ be given. Assume the following.

$$c_2Eenum_2ESUC_2EREPR \in (omega)^{omega} \quad (12)$$

Definition 11 We define c_2Eenum_2ESUC to be $\lambda V0m \in ty_2Eenum_2Eenum.(ap\ c_2Eenum_2EABS_2Eenum)$

Let $c_2Earithmetic_2E2B : \iota$ be given. Assume the following.

$$c_2Earithmetic_2E2B \in ((ty_2Eenum_2Eenum)^{ty_2Eenum_2Eenum})^{ty_2Eenum_2Eenum} \quad (13)$$

Definition 12 We define $c_2Earithmetic_2EBIT1$ to be $\lambda V0n \in ty_2Eenum_2Eenum.(ap\ (ap\ c_2Earithmetic_2E2B))$

Let $c_2Ereal_2Epow : \iota$ be given. Assume the following.

$$c_2Ereal_2Epow \in ((ty_2Erealax_2Ereal^{ty_2Enum_2Enum})^{ty_2Erealax_2Ereal}) \quad (14)$$

Let $c_2Earithmetic_2EFACT : \iota$ be given. Assume the following.

$$c_2Earithmetic_2EFACT \in (ty_2Enum_2Enum^{ty_2Enum_2Enum}) \quad (15)$$

Let $c_2Ereal_2Ereal_of_num : \iota$ be given. Assume the following.

$$c_2Ereal_2Ereal_of_num \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}) \quad (16)$$

Let $c_2Erealax_2Etreal_inv : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_inv \in ((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (17)$$

Definition 13 We define $c_2Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap\ c_2Erealax_2Ereal_ABS$

Let $c_2Erealax_2Etreal_mul : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreal_mul \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)}) \quad (18)$$

Definition 14 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (19)$$

Definition 15 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2EABS_prod\ x\ y))$

Let $c_2Ereal_2Esum : \iota$ be given. Assume the following.

$$c_2Ereal_2Esum \in ((ty_2Erealax_2Ereal^{(ty_2Erealax_2Ereal^{ty_2Enum_2Enum})})^{(ty_2Epair_2Eprod\ ty_2Enum_2Enum)}) \quad (20)$$

Definition 16 We define c_2Ebool_2EF to be $(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 17 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_21\ 2))$

Definition 18 We define $c_2Ebool_2E_3F$ to be $\lambda A_27a : \iota.(\lambda V0P \in (2^{A_27a}).(ap\ V0P\ (ap\ (c_2Emin_2E_40\ 2)\ V0P)))$

Definition 19 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 20 We define $c_2Earithmetic_2E_3E$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 21 We define $c_Ebool_E_5C_2F$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_Ebool_E_21) 2) (\lambda V2t \in$

Definition 22 We define $c_Earithmic_E_3E_3D$ to be $\lambda V0m \in ty_Eenum_Eenum.\lambda V1n \in ty_Eenum_E2$

Let $c_Erealax_Eereal_add : \iota$ be given. Assume the following.

$$c_Erealax_Eereal_add \in (((ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal) (ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal) (ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)) (ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)) (ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal) \quad (21)$$

Definition 23 We define $c_Erealax_Eereal_add$ to be $\lambda V0T1 \in ty_Erealax_Eereal.\lambda V1T2 \in ty_Erealax_Eereal$

Definition 24 We define $c_Ereal_Eereal_sub$ to be $\lambda V0x \in ty_Erealax_Eereal.\lambda V1y \in ty_Erealax_Eereal$

Let $c_Erealax_Eereal_lt : \iota$ be given. Assume the following.

$$c_Erealax_Eereal_lt \in ((2^{(ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal) (ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal))} (ty_Epair_Eprod ty_Ehreal_Ehreal ty_Ehreal_Ehreal)) \quad (22)$$

Definition 25 We define $c_Erealax_Eereal_lt$ to be $\lambda V0T1 \in ty_Erealax_Eereal.\lambda V1T2 \in ty_Erealax_Eereal$

Definition 26 We define $c_Ereal_Eereal_lte$ to be $\lambda V0x \in ty_Erealax_Eereal.\lambda V1y \in ty_Erealax_Eereal$

Definition 27 We define c_Ebool_ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 28 We define c_Ereal_Eeabs to be $\lambda V0x \in ty_Erealax_Eereal.(ap (ap (ap (c_Ebool_ECOND$

Let $c_Epair_EESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_Epair_EESND A_27a A_27b \in (A_27b^{(ty_Epair_Eprod A_27a A_27b)}) \quad (23)$$

Let $c_Epair_EEST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_Epair_EEST A_27a A_27b \in (A_27a^{(ty_Epair_Eprod A_27a A_27b)}) \quad (24)$$

Definition 29 We define $c_Epair_EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c^{A_27a}$

Let $ty_Emetric_Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty A0 \Rightarrow nonempty (ty_Emetric_Emetric A0) \quad (25)$$

Let $c_Emetric_Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow c_Emetric_Emetric A_27a \in ((ty_Emetric_Emetric A_27a)^{(ty_Erealax_Eereal^{(ty_Epair_Eprod A_27a A_27a)}})) \quad (26)$$

Definition 30 We define $c_Emetric_Emr1$ to be $(ap (c_Emetric_Emetric ty_Erealax_Eereal) (ap (c$

Let $c_2Emetric_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Edist\ A_27a \in ((ty_2Erealax_2Ereal^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})) \quad (27)$$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Etopology_2Etopology\ A0) \quad (28)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Etopology\ A_27a \in ((ty_2Etopology_2Etopology\ A_27a)^{(2^{(2^A-27a)}})) \quad (29)$$

Definition 31 We define $c_2Emetric_2Emtop$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Emetric_2Emetric\ A_27a).(ap$

Let $c_2Enets_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Enets_2Etends\ A_27a\ A_27b \in (((2^{(ty_2Epair_2Eprod\ (ty_2Etopology_2Etopology\ A_27a)\ (2^{A-27b})^{A-27b}}))^{A_27a})^{(A_27a^{A-27b})}) \quad (30)$$

Definition 32 We define $c_2Eseq_2E_2D_2D_3E$ to be $\lambda V0x \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).\lambda V1x$

Definition 33 We define c_2Eseq_2Esums to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).\lambda V1s \in ty_2$

Definition 34 We define $c_2Eseq_2Esuminf$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(ap\ (c_2E$

Definition 35 We define $c_2Etransc_2Eexp$ to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ c_2Eseq_2Esuminf\ (\lambda V1$

Definition 36 We define $c_2Etransc_2Eln$ to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ (c_2Emin_2E_40\ ty_2Ereal$

Definition 37 We define $c_2Ereal_2E_2F$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.($

Definition 38 We define $c_2Earithmetic_2EZERO$ to be c_2Enum_2E0 .

Definition 39 We define $c_2Earithmetic_2EBIT2$ to be $\lambda V0n \in ty_2Enum_2Enum.(ap\ (ap\ c_2Earithmetic$

Definition 40 We define $c_2Earithmetic_2ENUMERAL$ to be $\lambda V0x \in ty_2Enum_2Enum.V0x$.

Definition 41 We define $c_2Eutil_prob_2Elogr$ to be $\lambda V0a \in ty_2Erealax_2Ereal.\lambda V1x \in ty_2Erealax_2E$

Definition 42 We define $c_2Eutil_prob_2Elg$ to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap\ (ap\ c_2Eutil_prob_2Elc$

Assume the following.

$$True \quad (31)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (32)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal.((ap (ap c_2Ereal_2E_2F (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) V0x) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))) \quad (33)$$

Assume the following.

$$(\forall V0n \in ty_2Enum_2Enum.(\forall V1m \in ty_2Enum_2Enum.(((ap c_2Ereal_2Ereal_of_num V0n) = (ap c_2Ereal_2Ereal_of_num V1m)) \Leftrightarrow (V0n = V1m)) \wedge (((ap c_2Erealax_2Ereal_neg (ap c_2Ereal_2Ereal_of_num V0n)) = (ap c_2Ereal_2Ereal_of_num V1m)) \Leftrightarrow ((V0n = c_2Enum_2E0) \wedge (V1m = c_2Enum_2E0))) \wedge (((ap c_2Ereal_2Ereal_of_num V0n) = (ap c_2Erealax_2Ereal_neg (ap c_2Ereal_2Ereal_of_num V1m))) \Leftrightarrow ((V0n = c_2Enum_2E0) \wedge (V1m = c_2Enum_2E0))) \wedge (((ap c_2Erealax_2Ereal_neg (ap c_2Ereal_2Ereal_of_num V0n)) = (ap c_2Erealax_2Ereal_neg (ap c_2Ereal_2Ereal_of_num V1m))) \Leftrightarrow (V0n = V1m)))))) \quad (34)$$

Assume the following.

$$((ap c_2Etransc_2EIn (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0)) \quad (35)$$

Theorem 1

$$((ap c_2Eutil_prob_2Elg (ap c_2Ereal_2Ereal_of_num (ap c_2Earithmetic_2ENUMERAL (ap c_2Earithmetic_2EBIT1 c_2Earithmetic_2EZERO)))) = (ap c_2Ereal_2Ereal_of_num c_2Enum_2E0))$$