

thm_2Eutil__prob_2Elogr__inv
(TMGhqa66BKc7R6EBGBs7fnhKXPLYp6jE3CU)

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Definition 1 We define $c_2Emin_2E_3D$ to be $\lambda A.\lambda x \in A.\lambda y \in A.inj_o (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 2 We define $c_2Ebool_2E_ET$ to be $(ap (ap (c_2Emin_2E_3D (2^2)) (\lambda V0x \in 2.V0x)) (\lambda V1x \in 2.V1x))$

Definition 3 We define $c_2Ebool_2E_21$ to be $\lambda A.\lambda a : \iota.(\lambda V0P \in (2^{A-27a}).(ap (ap (c_2Emin_2E_3D (2^{A-27a}))$

Definition 4 We define $c_2Ebool_2E_EF$ to be $(ap (c_2Ebool_2E_21 2) (\lambda V0t \in 2.V0t))$.

Definition 5 We define $c_2Emin_2E_3D_3D_3E$ to be $\lambda P \in 2.\lambda Q \in 2.inj_o (p P \Rightarrow p Q)$ of type ι .

Definition 6 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap (ap c_2Emin_2E_3D_3D_3E V0t) c_2Ebool_2E_EF$

Definition 7 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap (c_2Ebool_2E_21 2) (\lambda V2t \in 2.V2t))$

Let $ty_2Ehreal_2Ehreal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Ehreal_2Ehreal \tag{1}$$

Let $ty_2Epair_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty_2Epair_2Eprod\ A0\ A1) \tag{2}$$

Let $ty_2Erealax_2Ereal : \iota$ be given. Assume the following.

$$nonempty\ ty_2Erealax_2Ereal \tag{3}$$

Let $c_2Erealax_2Ereal_REP_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_REP_CLASS \in ((2^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)})\ ty_2Erealax_2Ereal) \tag{4}$$

Definition 8 We define $c_2Emin_2E_40$ to be $\lambda A.\lambda P \in 2^A.if (\exists x \in A.p (ap P x)) \mathbf{then} (the (\lambda x.x \in A \wedge p$ of type $\iota \Rightarrow \iota$.

Definition 9 We define $c_2Erealax_2Ereal_REP$ to be $\lambda V0a \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E40 (ty_2Erealax_2Ereal_mul) (c_2Erealax_2Ereal_mul)))$. Assume the following.

$$c_2Erealax_2Ereal_mul \in (((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal) (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \quad (5)$$

Let $c_2Erealax_2Ereal_eq : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_eq \in ((2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)}) (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \quad (6)$$

Let $c_2Erealax_2Ereal_ABS_CLASS : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_ABS_CLASS \in (ty_2Erealax_2Ereal)^{(2^{(ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)})} \quad (7)$$

Definition 10 We define $c_2Erealax_2Ereal_ABS$ to be $\lambda V0r \in (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)$.

Definition 11 We define $c_2Erealax_2Ereal_mul$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$.

Let $c_2Erealax_2Ereal_neg : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_neg \in ((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal) (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \quad (8)$$

Definition 12 We define $c_2Erealax_2Ereal_inv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap c_2Erealax_2Ereal_neg)$.

Let $c_2Erealax_2Ereal_inv : \iota$ be given. Assume the following.

$$c_2Erealax_2Ereal_inv \in ((ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal) (ty_2Epair_2Eprod ty_2Ehreal_2Ehreal ty_2Ehreal_2Ehreal)) \quad (9)$$

Definition 13 We define $c_2Erealax_2Einv$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.(ap c_2Erealax_2Ereal_ABS)$.

Let $c_2Enum_2EZERO_REP : \iota$ be given. Assume the following.

$$c_2Enum_2EZERO_REP \in \omega \quad (10)$$

Let $ty_2Enum_2Enum : \iota$ be given. Assume the following.

$$nonempty ty_2Enum_2Enum \quad (11)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (12)$$

Definition 14 We define c_2Enum_2E0 to be $(ap c_2Enum_2EABS_num c_2Enum_2EZERO_REP)$.

Let $c_Ereal_Ereal_of_num : \iota$ be given. Assume the following.

$$c_Ereal_Ereal_of_num \in (ty_Erealax_Ereal^{ty_Eenum_Eenum}) \quad (13)$$

Let $c_Erealax_Etreal_lt : \iota$ be given. Assume the following.

$$c_Erealax_Etreal_lt \in ((2^{(ty_Epair_Eprod \ ty_Ehreal_Ehreal \ ty_Ehreal_Ehreal)})^{(ty_Epair_Eprod \ ty_Ehreal_Ehreal)}) \quad (14)$$

Definition 15 We define $c_Erealax_Ereal_lt$ to be $\lambda V0T1 \in ty_Erealax_Ereal. \lambda V1T2 \in ty_Erealax_Ereal.$

Let $c_Ereal_Epow : \iota$ be given. Assume the following.

$$c_Ereal_Epow \in ((ty_Erealax_Ereal^{ty_Eenum_Eenum})^{ty_Erealax_Ereal}) \quad (15)$$

Let $c_Earithmetic_EFACT : \iota$ be given. Assume the following.

$$c_Earithmetic_EFACT \in (ty_Eenum_Eenum^{ty_Eenum_Eenum}) \quad (16)$$

Let $c_Epair_EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty \ A_27a \Rightarrow \forall A_27b.nonempty \ A_27b \Rightarrow c_Epair_EABS_prod \ A_27a \ A_27b \in ((ty_Epair_Eprod \ A_27a \ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (17)$$

Definition 16 We define $c_Epair_E_2C$ to be $\lambda A_27a : \iota. \lambda A_27b : \iota. \lambda V0x \in A_27a. \lambda V1y \in A_27b. (ap \ (c_2Epair_EABS_prod \ x \ y) \ V0V1)$

Let $c_Ereal_Esum : \iota$ be given. Assume the following.

$$c_Ereal_Esum \in ((ty_Erealax_Ereal^{(ty_Erealax_Ereal^{ty_Eenum_Eenum})})^{(ty_Epair_Eprod \ ty_Eenum_Eenum)}) \quad (18)$$

Let $c_Eenum_EERP_num : \iota$ be given. Assume the following.

$$c_Eenum_EERP_num \in (omega^{ty_Eenum_Eenum}) \quad (19)$$

Let $c_Eenum_ESUC_REP : \iota$ be given. Assume the following.

$$c_Eenum_ESUC_REP \in (omega^{omega}) \quad (20)$$

Definition 17 We define c_Eenum_ESUC to be $\lambda V0m \in ty_Eenum_Eenum. (ap \ c_Eenum_EABS_num \ m)$

Definition 18 We define $c_Ebool_E_3F$ to be $\lambda A_27a : \iota. (\lambda V0P \in (2^{A_27a}). (ap \ V0P \ (ap \ (c_2Emin_E_40 \ A_27a) \ P)))$

Definition 19 We define $c_Eprim_rec_E_3C$ to be $\lambda V0m \in ty_Eenum_Eenum. \lambda V1n \in ty_Eenum_Eenum. (c_2Eprim_rec \ m \ n)$

Definition 20 We define $c_Earithmetic_E_3E$ to be $\lambda V0m \in ty_Eenum_Eenum. \lambda V1n \in ty_Eenum_Eenum. (c_2Earithmetic \ m \ n)$

Definition 21 We define $c_Ebool_E_5C_2F$ to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (ap \ (c_2Ebool_E_21 \ 2) \ (\lambda V2t \in 2. (c_2Ebool_E_21 \ t1 \ t2))))$

Definition 22 We define $c_Earithmetic_E_3E_3D$ to be $\lambda V0m \in ty_Eenum_Eenum. \lambda V1n \in ty_Eenum_Eenum. (c_2Earithmetic \ m \ n)$

Let $c_2Erealax_2Etreall_add : \iota$ be given. Assume the following.

$$c_2Erealax_2Etreall_add \in (((ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal\ ty_2Ehreal_2Ehreal)(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal))^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)})^{(ty_2Epair_2Eprod\ ty_2Ehreal_2Ehreal)} \quad (21)$$

Definition 23 We define $c_2Erealax_2Ereal_add$ to be $\lambda V0T1 \in ty_2Erealax_2Ereal.\lambda V1T2 \in ty_2Erealax_2Ereal$

Definition 24 We define $c_2Ereal_2Ereal_sub$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 25 We define $c_2Ereal_2Ereal_lte$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal$

Definition 26 We define c_2Ebool_2ECOND to be $\lambda A_27a : \iota.(\lambda V0t \in 2.(\lambda V1t1 \in A_27a.(\lambda V2t2 \in A_27a.($

Definition 27 We define c_2Ereal_2Eabs to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap (ap (ap (c_2Ebool_2ECOND$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \quad (22)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a)^{(ty_2Epair_2Eprod\ A_27a\ A_27b)} \quad (23)$$

Definition 28 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c)^{A_27a}}$

Let $ty_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Emetric_2Emetric\ A0) \quad (24)$$

Let $c_2Emetric_2Emetric : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Emetric\ A_27a \in ((ty_2Emetric_2Emetric\ A_27a)^{(ty_2Erealax_2Ereal)^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}})) \quad (25)$$

Definition 29 We define $c_2Emetric_2Emr1$ to be $(ap (c_2Emetric_2Emetric\ ty_2Erealax_2Ereal) (ap (c$

Let $c_2Emetric_2Edist : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Emetric_2Edist\ A_27a \in ((ty_2Erealax_2Ereal)^{(ty_2Epair_2Eprod\ A_27a\ A_27a)}) \quad (26)$$

Let $ty_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Etopology_2Etopology\ A0) \quad (27)$$

Let $c_2Etopology_2Etopology : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow c_2Etopology_2Etopology\ A_27a \in ((ty_2Etopology_2Etopology\ A_27a)^{(2^{(2^{A_27a})})}) \quad (28)$$

Definition 30 We define $c_2Emetric_2Emtop$ to be $\lambda A_27a : \iota.\lambda V0m \in (ty_2Emetric_2Emetric A_27a).(ap$
 Let $c_2Enets_2Etends : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow c_2Enets_2Etends \\ & A_27a A_27b \in (((2^{(ty_2Epair_2Eprod (ty_2Etopology_2Etopology A_27a) ((2^{A_27b})^{A_27b}))})_{A_27a})_{(A_27a)^{A_27b}})) \end{aligned} \quad (29)$$

Definition 31 We define $c_2Eseq_2E_2D_2D_3E$ to be $\lambda V0x \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).\lambda V1x$

Definition 32 We define c_2Eseq_2Esums to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).\lambda V1s \in ty_2$

Definition 33 We define $c_2Eseq_2Esuminf$ to be $\lambda V0f \in (ty_2Erealax_2Ereal^{ty_2Enum_2Enum}).(ap (c_2E$

Definition 34 We define $c_2Etransc_2Eexp$ to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap c_2Eseq_2Esuminf (\lambda V1$

Definition 35 We define $c_2Etransc_2Eln$ to be $\lambda V0x \in ty_2Erealax_2Ereal.(ap (c_2Emin_2E_40 ty_2Erea$

Definition 36 We define $c_2Ereal_2E_2F$ to be $\lambda V0x \in ty_2Erealax_2Ereal.\lambda V1y \in ty_2Erealax_2Ereal.($

Definition 37 We define $c_2Eutil_prob_2Elogr$ to be $\lambda V0a \in ty_2Erealax_2Ereal.\lambda V1x \in ty_2Erealax_2E$

Assume the following.

$$True \quad (30)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Rightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Rightarrow True) \Leftrightarrow \\ & True) \wedge (((False \Rightarrow (p V0t)) \Leftrightarrow True) \wedge (((p V0t) \Rightarrow (p V0t)) \Leftrightarrow True) \wedge ((\\ & (p V0t) \Rightarrow False) \Leftrightarrow (\neg (p V0t)))))) \end{aligned} \quad (31)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.((V0x = V0x) \Leftrightarrow True)) \quad (32)$$

Assume the following.

$$\forall A_27a.nonempty A_27a \Rightarrow (\forall V0x \in A_27a.(\forall V1y \in A_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (33)$$

Assume the following.

$$\begin{aligned} & (\forall V0t \in 2.(((True \Leftrightarrow (p V0t)) \Leftrightarrow (p V0t)) \wedge (((p V0t) \Leftrightarrow True) \Leftrightarrow \\ & (p V0t)) \wedge (((False \Leftrightarrow (p V0t)) \Leftrightarrow (\neg (p V0t))) \wedge (((p V0t) \Leftrightarrow False) \Leftrightarrow (\neg \\ & p V0t)))))) \end{aligned} \quad (34)$$

Assume the following.

$$\begin{aligned} & (\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow \\ & ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \end{aligned} \quad (35)$$

Assume the following.

$$2.(((p \ V0x) \Leftrightarrow (p \ V1x_27)) \wedge ((p \ V1x_27) \Rightarrow ((p \ V2y) \Leftrightarrow (p \ V3y_27)))) \Rightarrow \\ (((p \ V0x) \Rightarrow (p \ V2y)) \Leftrightarrow ((p \ V1x_27) \Rightarrow (p \ V3y_27)))) \quad (36)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal. (\forall V1y \in ty_2Erealax_2Ereal. \\ ((ap \ (ap \ c_2Erealax_2Ereal_mul \ (ap \ c_2Erealax_2Ereal_neg \ V0x)) \\ V1y) = (ap \ c_2Erealax_2Ereal_neg \ (ap \ (ap \ c_2Erealax_2Ereal_mul \\ V0x \ V1y)))))) \quad (37)$$

Assume the following.

$$(\forall V0x \in ty_2Erealax_2Ereal. ((p \ (ap \ (ap \ c_2Erealax_2Ereal_lt \\ (ap \ c_2Ereal_2Ereal_of_num \ c_2Enum_2E0)) \ V0x)) \Rightarrow ((ap \ c_2Etransc_2Eln \\ (ap \ c_2Erealax_2Einv \ V0x)) = (ap \ c_2Erealax_2Ereal_neg \ (ap \ c_2Etransc_2Eln \\ V0x)))))) \quad (38)$$

Theorem 1

$$(\forall V0b \in ty_2Erealax_2Ereal. (\forall V1x \in ty_2Erealax_2Ereal. \\ ((p \ (ap \ (ap \ c_2Erealax_2Ereal_lt \ (ap \ c_2Ereal_2Ereal_of_num \\ c_2Enum_2E0)) \ V1x)) \Rightarrow ((ap \ (ap \ c_2Eutil_prob_2Elogr \ V0b) \ (ap \ c_2Erealax_2Einv \\ V1x)) = (ap \ c_2Erealax_2Ereal_neg \ (ap \ (ap \ c_2Eutil_prob_2Elogr \\ V0b \ V1x))))))$$