

thm_2Eveblen_2Eclub__INTER (TMZB- waSw8ahP3E1vm5FbrRsnMq9wZCDxEv2)

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Definition 1 We define `c_2Emin_2E_40` to be $\lambda A. \lambda P \in 2^A. \text{if } (\exists x \in A. p \text{ (ap } P \ x)) \text{ then (the } (\lambda x. x \in A \wedge p \text{ of type } \iota \Rightarrow \iota).$

Let `ty_2Epair_2Eprod` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty (ty_2Epair_2Eprod } A0 \ A1) \tag{1}$$

Definition 2 We define `c_2Emin_2E_3D` to be $\lambda A. \lambda x \in A. \lambda y \in A. \text{inj_o } (x = y)$ of type $\iota \Rightarrow \iota$.

Definition 3 We define `c_2Ebool_2ET` to be $(\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^2)) (\lambda V0x \in 2. V0x)) (\lambda V1x \in 2. V1x))$

Definition 4 We define `c_2Ebool_2E_21` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } (\text{ap } (\text{c_2Emin_2E_3D } (2^{A-27a}))))$

Definition 5 We define `c_2Ebool_2EF` to be $(\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V0t \in 2. V0t))$.

Definition 6 We define `c_2Emin_2E_3D_3D_3E` to be $\lambda P \in 2. \lambda Q \in 2. \text{inj_o } (p \Rightarrow q)$ of type ι .

Definition 7 We define `c_2Ebool_2E_5C_2F` to be $(\lambda V0t1 \in 2. (\lambda V1t2 \in 2. (\text{ap } (\text{c_2Ebool_2E_21 } 2) (\lambda V2t \in 2. V2t))))$

Definition 8 We define `c_2Ebool_2E_3F` to be $\lambda A. 27a : \iota. (\lambda V0P \in (2^{A-27a}). (\text{ap } V0P \ (\text{ap } (\text{c_2Emin_2E_40 } A))))$

Let `ty_2Enum_2Enum` : ι be given. Assume the following.

$$\text{nonempty ty_2Enum_2Enum} \tag{2}$$

Let `ty_2Esum_2Esum` : $\iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \forall A1. \text{nonempty } A1 \Rightarrow \text{nonempty (ty_2Esum_2Esum } A0 \ A1) \tag{3}$$

Let `ty_2Ewellorder_2Ewellorder` : $\iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0. \text{nonempty } A0 \Rightarrow \text{nonempty (ty_2Ewellorder_2Ewellorder } A0) \tag{4}$$

Let $ty_2Eordinal_2Eordinal : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty_2Eordinal_2Eordinal\ A0) \quad (5)$$

Let $c_2Eordinal_2Eordinal_REP_CLASS : \iota \Rightarrow \iota$ be given. Assume the following.

$$A_27a \in ((2^{(ty_2Ewellorder_2Ewellorder\ (ty_2Esum_2Esum\ ty_2Enum_2Enum\ A_27a))})^{(ty_2Eordinal_2Eordinal\ A_27a)}) \quad (6)$$

Definition 9 We define $c_2Eordinal_2Eordinal_REP$ to be $\lambda A_27a : \iota.\lambda V0a \in (ty_2Eordinal_2Eordinal\ A_27a)$

Let $c_2Ewellorder_2Ewellorder_REP : \iota \Rightarrow \iota$ be given. Assume the following.

$$A_27a \in ((2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})^{(ty_2Ewellorder_2Ewellorder\ A_27a)}) \quad (7)$$

Definition 10 We define $c_2Ebool_2E_7E$ to be $(\lambda V0t \in 2.(ap\ (ap\ c_2Emin_2E_3D_3D_3E\ V0t)\ c_2Ebool_2E_7E))$

Definition 11 We define $c_2Ebool_2E_2F_5C$ to be $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (c_2Ebool_2E_21\ 2)\ (\lambda V2t \in 2)))$

Let $c_2Epair_2EABS_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EABS_prod\ A_27a\ A_27b \in ((ty_2Epair_2Eprod\ A_27a\ A_27b)^{(2^{A_27b})^{A_27a}}) \quad (8)$$

Definition 12 We define $c_2Epair_2E_2C$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0x \in A_27a.\lambda V1y \in A_27b.(ap\ (c_2Epair_2E_2C\ V0x\ V1y))$

Definition 13 We define c_2Ebool_2EIN to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.(\lambda V1f \in (2^{A_27a}).(ap\ V1f\ V0x)))$

Let $c_2Epair_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2ESND\ A_27a\ A_27b \in (A_27b^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (9)$$

Let $c_2Epair_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epair_2EFST\ A_27a\ A_27b \in (A_27a^{(ty_2Epair_2Eprod\ A_27a\ A_27b)}) \quad (10)$$

Definition 14 We define $c_2Epair_2EUNCURRY$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda A_27c : \iota.\lambda V0f \in ((A_27c)^{A_27b})$

Let $c_2Epred_set_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow c_2Epred_set_2EGSPEC\ A_27a\ A_27b \in ((2^{A_27a})^{(ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}}) \quad (11)$$

Definition 15 We define $c_Eset_relation_Estrict$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$,

Definition 16 We define $c_Ewellorder_Eiseg$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Ewellorder_2Ewellorder\ A_27a)$,

Definition 17 We define $c_Eset_relation_Erestrict$ to be $\lambda A_27a : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})$,

Let $c_Ewellorder_2Ewellorder_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow c_Ewellorder_2Ewellorder_ABS \\ & A_27a \in ((ty_2Ewellorder_2Ewellorder\ A_27a)^{(2^{(ty_2Epair_2Eprod\ A_27a\ A_27a)})}) \end{aligned} \quad (12)$$

Definition 18 We define $c_Ewellorder_Ewobound$ to be $\lambda A_27a : \iota.\lambda V0x \in A_27a.\lambda V1w \in (ty_2Ewellorder\ A_27a)$,

Definition 19 We define $c_Eset_relation_Erange$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})$,

Definition 20 We define $c_Eset_relation_Edomain$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0r \in (2^{(ty_2Epair_2Eprod\ A_27a\ A_27b)})$,

Definition 21 We define $c_Epred_set_EUNION$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap\ (c_2Epred_set_EUNION\ s)\ t)$,

Definition 22 We define $c_Ewellorder_EelsOf$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Ewellorder_2Ewellorder\ A_27a)$,

Definition 23 We define $c_Ewellorder_Eorderiso$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0w1 \in (ty_2Ewellorder_2Ewellorder\ A_27a)$,

Definition 24 We define $c_Ewellorder_Eorderlt$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0w1 \in (ty_2Ewellorder_2Ewellorder\ A_27a)$,

Definition 25 We define $c_Eordinal_Eordlt$ to be $\lambda A_27a : \iota.\lambda V0T1 \in (ty_2Eordinal_2Eordinal\ A_27a).$

Definition 26 We define $c_Eveblen_Eunbounded$ to be $\lambda A_27a : \iota.\lambda V0A \in (2^{(ty_2Eordinal_2Eordinal\ A_27a)})$,

Definition 27 We define $c_Eordinal_Epreds$ to be $\lambda A_27a : \iota.\lambda V0w \in (ty_2Eordinal_2Eordinal\ A_27a).$

Definition 28 We define $c_Epred_set_EIMAGE$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^{A_27a}).(ap\ (c_2Epred_set_EIMAGE\ f)\ s)$,

Definition 29 We define $c_Epred_set_EBIGUNION$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(ap\ (c_2Epred_set_EBIGUNION\ P)\ s)$,

Definition 30 We define $c_Eordinal_Eoleast$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(ty_2Eordinal_2Eordinal\ A_27a)}).$

Definition 31 We define $c_Eordinal_Esup$ to be $\lambda A_27a : \iota.\lambda V0ordset \in (2^{(ty_2Eordinal_2Eordinal\ A_27a)}).$

Definition 32 We define $c_Eveblen_Eclosed$ to be $\lambda A_27a : \iota.\lambda V0A \in (2^{(ty_2Eordinal_2Eordinal\ A_27a)}).$

Definition 33 We define $c_Epred_set_EUNIV$ to be $\lambda A_27a : \iota.(\lambda V0x \in A_27a.c_2Ebool_2ET)$.

Definition 34 We define $c_Epred_set_EINJ$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0f \in (A_27b^{A_27a}).\lambda V1s \in (2^{A_27a}).(ap\ (c_2Epred_set_EINJ\ f)\ s)$,

Definition 35 We define $c_Ecardinal_Ecardleq$ to be $\lambda A_27a : \iota.\lambda A_27b : \iota.\lambda V0s1 \in (2^{A_27a}).\lambda V1s2 \in (2^{A_27a}).(ap\ (c_2Ecardinal_Ecardleq\ s1)\ s2)$,

Definition 36 We define $c_Epred_set_EBIGINTER$ to be $\lambda A_27a : \iota.\lambda V0P \in (2^{(2^{A_27a})}).(ap\ (c_2Epred_set_EBIGINTER\ P)\ s)$,

Let $c_2Enum_2EREP_num : \iota$ be given. Assume the following.

$$c_2Enum_2EREP_num \in (\omega^{ty_2Enum_2Enum}) \quad (13)$$

Let $c_2Enum_2ESUC_REP : \iota$ be given. Assume the following.

$$c_2Enum_2ESUC_REP \in (\omega^{\omega}) \quad (14)$$

Let $c_2Enum_2EABS_num : \iota$ be given. Assume the following.

$$c_2Enum_2EABS_num \in (ty_2Enum_2Enum^{\omega}) \quad (15)$$

Definition 37 We define c_2Enum_2ESUC to be $\lambda V0m \in ty_2Enum_2Enum.(ap\ c_2Enum_2EABS_num$

Definition 38 We define $c_2Eprim_rec_2E_3C$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 39 We define $c_2Earithmetic_2E_3C_3D$ to be $\lambda V0m \in ty_2Enum_2Enum.\lambda V1n \in ty_2Enum_2Enum$

Definition 40 We define $c_2Epred_set_2ESUBSET$ to be $\lambda A_27a : \iota.\lambda V0s \in (2^{A_27a}).\lambda V1t \in (2^{A_27a}).(ap$

Definition 41 We define $c_2Eveblen_2Eclub$ to be $\lambda A_27a : \iota.\lambda V0A \in (2^{(ty_2Eordinal_2Eordinal\ A_27a)}).(ap$

Assume the following.

$$True \quad (16)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \quad (17)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \quad (18)$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge (((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)))))) \quad (19)$$

Assume the following.

$$(\forall V0t \in 2.(((True \Rightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Rightarrow True) \Leftrightarrow True) \wedge (((False \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow (p\ V0t)) \Leftrightarrow True) \wedge (((p\ V0t) \Rightarrow False) \Leftrightarrow (\neg (p\ V0t)))))) \quad (20)$$

Assume the following.

$$((\forall V0t \in 2.((\neg (\neg (p\ V0t))) \Leftrightarrow (p\ V0t)) \wedge (((\neg True) \Leftrightarrow False) \wedge ((\neg False) \Leftrightarrow True)))) \quad (21)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1y \in A_27a. ((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \quad (22)$$

Assume the following.

$$(\forall V0t \in 2. (((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow \neg(p\ V0t)) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow \neg(p\ V0t)))))) \quad (23)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). ((p\ V0P) \wedge (\forall V2x \in A_27a. (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow (\forall V3x \in A_27a. ((p\ V0P) \wedge (p\ (ap\ V1Q\ V3x)))))) \quad (24)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in 2. ((\exists V2x \in A_27a. ((p\ (ap\ V0P\ V2x)) \wedge (p\ V1Q))) \Leftrightarrow ((\exists V3x \in A_27a. (p\ (ap\ V0P\ V3x)) \wedge (p\ V1Q)))))) \quad (25)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ V0P) \vee (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \vee (\forall V3x \in A_27a. (p\ (ap\ V1Q\ V3x)))))) \quad (26)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in (2^{A_27a}). (\forall V1Q \in 2. ((\forall V2x \in A_27a. ((p\ (ap\ V0P\ V2x)) \Rightarrow (p\ V1Q))) \Leftrightarrow ((\exists V3x \in A_27a. (p\ (ap\ V0P\ V3x)) \Rightarrow (p\ V1Q)))))) \quad (27)$$

Assume the following.

$$\forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0P \in 2. (\forall V1Q \in (2^{A_27a}). ((\forall V2x \in A_27a. ((p\ V0P) \Rightarrow (p\ (ap\ V1Q\ V2x)))) \Leftrightarrow ((p\ V0P) \Rightarrow (\forall V3x \in A_27a. (p\ (ap\ V1Q\ V3x)))))) \quad (28)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \vee \neg(p\ V1B)))) \wedge ((\neg((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg(p\ V0A) \wedge \neg(p\ V1B)))))) \quad (29)$$

Assume the following.

$$(\forall V0A \in 2. (\forall V1B \in 2. (\forall V2C \in 2. (((p\ V0A) \vee ((p\ V1B) \wedge (p\ V2C))) \Leftrightarrow (((p\ V0A) \vee (p\ V1B)) \wedge ((p\ V0A) \vee (p\ V2C)))))) \quad (30)$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((p V0A) \Rightarrow (p V1B)) \Leftrightarrow ((\neg(p V0A)) \vee (p V1B)))))) \quad (31)$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p V0t1) \Rightarrow ((p V1t2) \Rightarrow (p V2t3))) \Leftrightarrow (((p V0t1) \wedge (p V1t2)) \Rightarrow (p V2t3)))))) \quad (32)$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x_{.27} \in 2.(\forall V2y \in 2.(\forall V3y_{.27} \in 2.(((p V0x) \Leftrightarrow (p V1x_{.27})) \wedge ((p V1x_{.27}) \Rightarrow ((p V2y) \Leftrightarrow (p V3y_{.27})))))) \Rightarrow ((p V0x) \Rightarrow (p V2y)) \Leftrightarrow ((p V1x_{.27}) \Rightarrow (p V3y_{.27})))))) \quad (33)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0P \in (2^{A_{.27a}}).(\forall V1a \in A_{.27a}.((\exists V2x \in A_{.27a}.((V2x = V1a) \wedge (p (ap V0P V2x)))) \Leftrightarrow (p (ap V0P V1a)))))) \quad (34)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0f \in (2^{A_{.27a}}).(\forall V1v \in A_{.27a}.((\forall V2x \in A_{.27a}.((V2x = V1v) \Rightarrow (p (ap V0f V2x)))) \Leftrightarrow (p (ap V0f V1v)))))) \quad (35)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0Q \in (2^{(ty_{.2Eordinal_{.2Eordinal}} A_{.27a})}).(\forall V1P \in (2^{(ty_{.2Eordinal_{.2Eordinal}} A_{.27a})}).(((\exists V2a \in (ty_{.2Eordinal_{.2Eordinal}} A_{.27a}).(p (ap V1P V2a))) \wedge (\forall V3a \in (ty_{.2Eordinal_{.2Eordinal}} A_{.27a}).(((\forall V4b \in (ty_{.2Eordinal_{.2Eordinal}} A_{.27a}).(p (ap (ap (c_{.2Eordinal_{.2Eordlt}} A_{.27a}) V4b) V3a))) \Rightarrow (\neg (p (ap V1P V4b)))))) \wedge (p (ap V1P V3a)))) \Rightarrow (p (ap V0Q V3a)))))) \Rightarrow (p (ap V0Q (ap (c_{.2Eordinal_{.2Eoleast}} A_{.27a}) V1P)))))) \quad (36)$$

Assume the following.

$$\forall A_{.27a}.nonempty A_{.27a} \Rightarrow (\forall V0s \in (2^{(ty_{.2Eordinal_{.2Eordinal}} A_{.27a})}).((p (ap (ap (c_{.2Ecardinal_{.2Ecardleq}} (ty_{.2Eordinal_{.2Eordinal}} A_{.27a}) (ty_{.2Esum_{.2Esum}} ty_{.2Enum_{.2Enum}} A_{.27a})) V0s) (c_{.2Epred_{.set_{.2EUNIV}} (ty_{.2Esum_{.2Esum}} ty_{.2Enum_{.2Enum}} A_{.27a})))) \Rightarrow (\forall V1a \in (ty_{.2Eordinal_{.2Eordinal}} A_{.27a}).((p (ap (ap (c_{.2Eordinal_{.2Eordlt}} A_{.27a}) V1a) (ap (c_{.2Eordinal_{.2Esup}} A_{.27a}) V0s))) \Leftrightarrow (\exists V2b \in (ty_{.2Eordinal_{.2Eordinal}} A_{.27a}).((p (ap (ap (c_{.2Ebool_{.2EIN}} (ty_{.2Eordinal_{.2Eordinal}} A_{.27a}) V2b) V0s)) \wedge (p (ap (ap (c_{.2Eordinal_{.2Eordlt}} A_{.27a}) V1a) V2b)))))))))) \quad (37)$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0x \in A_27a. (\forall V1y \in A_27b. (\forall V2a \in A_27a. (\forall V3b \in \\ & \quad A_27b. (((ap\ (ap\ (c_2Epair_2E_2C\ A_27a\ A_27b)\ V0x)\ V1y) = (ap\ (ap \\ & \quad (c_2Epair_2E_2C\ A_27a\ A_27b)\ V2a)\ V3b)) \Leftrightarrow ((V0x = V2a) \wedge (V1y = V3b)))))) \\ & \hspace{15em} (38) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow \forall A_27b.nonempty\ A_27b \Rightarrow (\\ & \quad \forall V0f \in ((ty_2Epair_2Eprod\ A_27a\ 2)^{A_27b}). (\forall V1v \in \\ & \quad A_27a. ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V1v)\ (ap\ (c_2Epred_set_2EGSPEC \\ & \quad A_27a\ A_27b)\ V0f))) \Leftrightarrow (\exists V2x \in A_27b. ((ap\ (ap\ (c_2Epair_2E_2C \\ & \quad A_27a\ 2)\ V1v)\ c_2Ebool_2ET) = (ap\ V0f\ V2x)))))) \\ & \hspace{15em} (39) \end{aligned}$$

Assume the following.

$$\begin{aligned} & \forall A_27a.nonempty\ A_27a \Rightarrow (\forall V0x \in A_27a. (\forall V1B \in \\ & \quad (2^{(2^{A_27a})}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ (ap\ (c_2Epred_set_2EBIGINTER \\ & \quad A_27a)\ V1B))) \Leftrightarrow (\forall V2P \in (2^{A_27a}). ((p\ (ap\ (ap\ (c_2Ebool_2EIN \\ & \quad (2^{A_27a})\ V2P)\ V1B)) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN\ A_27a)\ V0x)\ V2P)))))) \\ & \hspace{15em} (40) \end{aligned}$$

Assume the following.

$$(\forall V0t \in 2. ((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \quad (41)$$

Assume the following.

$$(\forall V0A \in 2. ((p\ V0A) \Rightarrow ((\neg(p\ V0A)) \Rightarrow False))) \quad (42)$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((p\ V0A) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & \quad (((p\ V0A) \Rightarrow False) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \\ & \hspace{15em} (43) \end{aligned}$$

Assume the following.

$$\begin{aligned} & (\forall V0A \in 2. (\forall V1B \in 2. (((\neg((\neg(p\ V0A)) \vee (p\ V1B))) \Rightarrow False) \Leftrightarrow \\ & \quad ((p\ V0A) \Rightarrow ((\neg(p\ V1B)) \Rightarrow False)))))) \\ & \hspace{15em} (44) \end{aligned}$$

Assume the following.

$$(\forall V0A \in 2. (((\neg(p\ V0A)) \Rightarrow False) \Rightarrow (((p\ V0A) \Rightarrow False) \Rightarrow False))) \quad (45)$$

Assume the following.

$$\begin{aligned} & (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p\ V0p) \Leftrightarrow (\\ & \quad (p\ V1q) \Leftrightarrow (p\ V2r))) \Leftrightarrow (((p\ V0p) \vee ((p\ V1q) \vee (p\ V2r))) \wedge (((p\ V0p) \vee ((\neg \\ & \quad p\ V2r)) \vee (\neg(p\ V1q)))) \wedge (((p\ V1q) \vee ((\neg(p\ V2r)) \vee (\neg(p\ V0p)))) \wedge ((p\ V2r) \vee \\ & \quad ((\neg(p\ V1q)) \vee (\neg(p\ V0p)))))))))) \\ & \hspace{15em} (46) \end{aligned}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \wedge (p V2r))) \Leftrightarrow (((p V0p) \vee ((\neg(p V1q)) \vee (\neg(p V2r)))) \wedge (((p V1q) \vee \\
& (\neg(p V0p))) \wedge ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{47}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \vee (p V2r))) \Leftrightarrow (((p V0p) \vee (\neg(p V1q))) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge \\
& ((p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{48}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (\forall V2r \in 2. (((p V0p) \Leftrightarrow (\\
& (p V1q) \Rightarrow (p V2r))) \Leftrightarrow (((p V0p) \vee (p V1q)) \wedge (((p V0p) \vee (\neg(p V2r))) \wedge (\\
& \neg(p V1q) \vee ((p V2r) \vee (\neg(p V0p))))))))))
\end{aligned} \tag{49}$$

Assume the following.

$$\begin{aligned}
& (\forall V0p \in 2. (\forall V1q \in 2. (((p V0p) \Leftrightarrow (\neg(p V1q))) \Leftrightarrow (((p V0p) \vee \\
& (p V1q)) \wedge ((\neg(p V1q)) \vee (\neg(p V0p))))))
\end{aligned} \tag{50}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (p V0p))) \tag{51}$$

Assume the following.

$$(\forall V0p \in 2. (\forall V1q \in 2. ((\neg((p V0p) \Rightarrow (p V1q))) \Rightarrow (\neg(p V1q)))) \tag{52}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow \forall A_27b.nonempty A_27b \Rightarrow (\\
& \forall V0f \in (A_27b^{ty_2Enum_2Enum}). (p (ap (ap (c_2Ecardinal_2Ecardleq \\
& A_27b (ty_2Esum_2Esum ty_2Enum_2Enum A_27a)) (ap (c_2Epred_set_2EGSPEC \\
& A_27b ty_2Enum_2Enum) (\lambda V1n \in ty_2Enum_2Enum. (ap (ap (c_2Epair_2E_2C \\
& A_27b 2) (ap V0f V1n)) c_2Ebool_2ET)))) (c_2Epred_set_2EUNIV \\
& (ty_2Esum_2Esum ty_2Enum_2Enum A_27a))))))
\end{aligned} \tag{53}$$

Assume the following.

$$\begin{aligned}
& \forall A_27a.nonempty A_27a \Rightarrow (\forall V0f \in ((ty_2Eordinal_2Eordinal \\
& A_27a)^{ty_2Enum_2Enum}). ((\forall V1n \in ty_2Enum_2Enum. (\neg(p (\\
& ap (ap (c_2Eordinal_2Eordlt A_27a) (ap V0f (ap c_2Enum_2ESUC V1n))) \\
& (ap V0f V1n)))))) \Rightarrow (\forall V2m \in ty_2Enum_2Enum. (\forall V3n \in ty_2Enum_2Enum. \\
& ((p (ap (ap c_2Earithmetic_2E_3C_3D V2m) V3n)) \Rightarrow (\neg(p (ap (ap (c_2Eordinal_2Eordlt \\
& A_27a) (ap V0f V3n)) (ap V0f V2m))))))))))
\end{aligned} \tag{54}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0A \in ((2^{(ty_2Eordinal_2Eordinal\ A.27a)})_{ty_2Enum_2Enum}). \\
& \quad (\forall V1f \in ((ty_2Eordinal_2Eordinal\ A.27a)_{ty_2Enum_2Enum}). \\
& \quad ((\forall V2n \in ty_2Enum_2Enum.(p\ (ap\ (c_2Eveblen_2Eclub\ A.27a) \\
& \quad (ap\ V0A\ V2n)))) \wedge (\forall V3n \in ty_2Enum_2Enum.(p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET \\
& \quad (ty_2Eordinal_2Eordinal\ A.27a))\ (ap\ V0A\ (ap\ c_2Enum_2ESUC\ V3n))) \\
& \quad (ap\ V0A\ V3n)))) \wedge (\forall V4n \in ty_2Enum_2Enum.(p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad (ty_2Eordinal_2Eordinal\ A.27a))\ (ap\ V1f\ V4n))\ (ap\ V0A\ V4n)))) \wedge \\
& \quad (\forall V5m \in ty_2Enum_2Enum.(\forall V6n \in ty_2Enum_2Enum.(\\
& \quad (p\ (ap\ (ap\ c_2Earithmetic_2E_3C_3D\ V5m)\ V6n)) \Rightarrow (\neg(p\ (ap\ (ap\ (c_2Eordinal_2Eordlt \\
& \quad A.27a)\ (ap\ V1f\ V6n))\ (ap\ V1f\ V5m)))))) \Rightarrow (p\ (ap\ (ap\ (c_2Ebool_2EIN \\
& \quad (ty_2Eordinal_2Eordinal\ A.27a))\ (ap\ (c_2Eordinal_2Esup\ A.27a) \\
& \quad (ap\ (c_2Epred_set_2EGSPEC\ (ty_2Eordinal_2Eordinal\ A.27a)\ ty_2Enum_2Enum) \\
& \quad (\lambda V7n \in ty_2Enum_2Enum.(ap\ (ap\ (c_2Epair_2E_2C\ (ty_2Eordinal_2Eordinal \\
& \quad A.27a)\ 2)\ (ap\ V1f\ V7n))\ c_2Ebool_2ET))))))\ (ap\ (c_2Epred_set_2EBIGINTER \\
& \quad (ty_2Eordinal_2Eordinal\ A.27a))\ (ap\ (c_2Epred_set_2EGSPEC \\
& \quad (2^{(ty_2Eordinal_2Eordinal\ A.27a)})\ ty_2Enum_2Enum)\ (\lambda V8n \in \\
& \quad ty_2Enum_2Enum.(ap\ (ap\ (c_2Epair_2E_2C\ (2^{(ty_2Eordinal_2Eordinal\ A.27a)}) \\
& \quad 2)\ (ap\ V0A\ V8n))\ c_2Ebool_2ET))))))))) \\
& \hspace{15em} (55)
\end{aligned}$$

Assume the following.

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0P \in (2^{(ty_2Eordinal_2Eordinal\ A.27a)}). \\
& \quad (\forall V1a \in (ty_2Eordinal_2Eordinal\ A.27a).((p\ (ap\ V0P\ V1a)) \Rightarrow \\
& \quad (\neg(p\ (ap\ (ap\ (c_2Eordinal_2Eordlt\ A.27a)\ V1a)\ (ap\ (c_2Eordinal_2Eoleast \\
& \quad A.27a)\ V0P)))))) \\
& \hspace{15em} (56)
\end{aligned}$$

Theorem 1

$$\begin{aligned}
& \forall A.27a.nonempty\ A.27a \Rightarrow (\forall V0A \in ((2^{(ty_2Eordinal_2Eordinal\ A.27a)})_{ty_2Enum_2Enum}). \\
& \quad ((\forall V1n \in ty_2Enum_2Enum.(p\ (ap\ (c_2Eveblen_2Eclub\ A.27a) \\
& \quad (ap\ V0A\ V1n)))) \wedge (\forall V2n \in ty_2Enum_2Enum.(p\ (ap\ (ap\ (c_2Epred_set_2ESUBSET \\
& \quad (ty_2Eordinal_2Eordinal\ A.27a))\ (ap\ V0A\ (ap\ c_2Enum_2ESUC\ V2n))) \\
& \quad (ap\ V0A\ V2n)))) \Rightarrow (p\ (ap\ (c_2Eveblen_2Eclub\ A.27a)\ (ap\ (c_2Epred_set_2EBIGINTER \\
& \quad (ty_2Eordinal_2Eordinal\ A.27a))\ (ap\ (c_2Epred_set_2EGSPEC \\
& \quad (2^{(ty_2Eordinal_2Eordinal\ A.27a)})\ ty_2Enum_2Enum)\ (\lambda V3n \in \\
& \quad ty_2Enum_2Enum.(ap\ (ap\ (c_2Epair_2E_2C\ (2^{(ty_2Eordinal_2Eordinal\ A.27a)}) \\
& \quad 2)\ (ap\ V0A\ V3n))\ c_2Ebool_2ET)))))))))
\end{aligned}$$