# thm_2Eveblen_2Eincreasing (TMXPfsUEVKKq3Mz7YQtuXigKXDQhrQLwGvQ)

October 26, 2020

**Definition 1** *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

**Definition 2** *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V1$

**Definition 3** *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27a}$

**Definition 4** *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

**Definition 5** *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* $\iota$.

**Definition 6** *We define* c_2Ebool_2E_5C_2F *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

**Definition 7** *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

**Definition 8** *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}\ A$

**Definition 9** *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

**Definition 10** *We define* c_2Ebool_2EIN *to be* $\lambda A\_27a : \iota.(\lambda V0x \in A\_27a.(\lambda V1f \in (2^{A\_27a}).(ap\ V1f\ V0x))$

Let $ty\_2Enum\_2Enum : \iota$ be given. Assume the following.

$$nonempty\ ty\_2Enum\_2Enum \quad (1)$$

Let $ty\_2Esum\_2Esum : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Esum\_2Esum\ A0\ A1) \quad (2)$$

Let $ty\_2Ewellorder\_2Ewellorder : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Ewellorder\_2Ewellorder\ A0) \quad (3)$$

Let $ty\_2Eordinal\_2Eordinal : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow nonempty\ (ty\_2Eordinal\_2Eordinal\ A0) \tag{4}$$

Let $c\_2Eordinal\_2Eordinal\_\_REP\_\_CLASS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eordinal\_2Eordinal\_\_REP\_\_CLASS\ A\_27a \in ((2^{(ty\_2Ewellorder\_2Ewellorder\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ A\_27a))})^{(ty\_2Eordinal\_2Eordinal\ A\_27a)} \tag{5}$$

**Definition 11** *We define* c_2Eordinal_2Eordinal__REP *to be* $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Eordinal\_2Eordinal\ A$

Let $ty\_2Epair\_2Eprod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A0.nonempty\ A0 \Rightarrow \forall A1.nonempty\ A1 \Rightarrow nonempty\ (ty\_2Epair\_2Eprod\ A0\ A1) \tag{6}$$

Let $c\_2Ewellorder\_2Ewellorder\_\_REP : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewellorder\_2Ewellorder\_\_REP\ A\_27a \in ((2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})^{(ty\_2Ewellorder\_2Ewellorder\ A\_27a)}) \tag{7}$$

**Definition 12** *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap$ (c_2Ebool_2E_21 2) $(\lambda V2t \in$

Let $c\_2Epair\_2EABS\_\_prod : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EABS\_\_prod\ A\_27a\ A\_27b \in ((ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)^{((2^{A\_27b})^{A\_27a})}) \tag{8}$$

**Definition 13** *We define* c_2Epair_2E_2C *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0x \in A\_27a.\lambda V1y \in A\_27b.(ap\ (c\_2$

Let $c\_2Epair\_2ESND : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2ESND\ A\_27a\ A\_27b \in (A\_27b^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{9}$$

Let $c\_2Epair\_2EFST : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epair\_2EFST\ A\_27a\ A\_27b \in (A\_27a^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27b)}) \tag{10}$$

**Definition 14** *We define* c_2Epair_2EUNCURRY *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda A\_27c : \iota.\lambda V0f \in ((A\_27c^{A\_27}$

Let $c\_2Epred\_\_set\_2EGSPEC : \iota \Rightarrow \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow c\_2Epred\_\_set\_2EGSPEC\ A\_27a\ A\_27b \in ((2^{A\_27a})^{((ty\_2Epair\_2Eprod\ A\_27a\ 2)^{A\_27b})}) \tag{11}$$

**Definition 15** *We define* c_2Eset__relation_2Estrict *to be* $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)}$

**Definition 16** *We define* c_2Ewellorder_2Eiseg *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Ewellorder\_2Ewellorder\ A\_2$

**Definition 17** *We define* c_2Eset__relation_2Errestrict *to be* $\lambda A\_27a : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_2}$

Let $c\_2Ewellorder\_2Ewellorder\_\_ABS : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Ewellorder\_2Ewellorder\_\_ABS\ A\_27a \in ((ty\_2Ewellorder\_2Ewellorder\ A\_27a)^{(2^{(ty\_2Epair\_2Eprod\ A\_27a\ A\_27a)})}) \tag{12}$$

**Definition 18** *We define* c_2Ewellorder_2Ewobound *to be* $\lambda A\_27a : \iota.\lambda V0x \in A\_27a.\lambda V1w \in (ty\_2Ewellord$

**Definition 19** *We define* c_2Eset__relation_2Erange *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod\ A}$

**Definition 20** *We define* c_2Eset__relation_2Edomain *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0r \in (2^{(ty\_2Epair\_2Eprod}$

**Definition 21** *We define* c_2Epred__set_2EUNION *to be* $\lambda A\_27a : \iota.\lambda V0s \in (2^{A\_27a}).\lambda V1t \in (2^{A\_27a}).(ap\ (c$

**Definition 22** *We define* c_2Ewellorder_2EelsOf *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Ewellorder\_2Ewellorder\ A$

**Definition 23** *We define* c_2Ewellorder_2Eorderiso *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0w1 \in (ty\_2Ewellorder\_2$

**Definition 24** *We define* c_2Ewellorder_2Eorderlt *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0w1 \in (ty\_2Ewellorder\_2E$

**Definition 25** *We define* c_2Eordinal_2Eordlt *to be* $\lambda A\_27a : \iota.\lambda V0T1 \in (ty\_2Eordinal\_2Eordinal\ A\_27a).\lambda$

**Definition 26** *We define* c_2Eordinal_2Epreds *to be* $\lambda A\_27a : \iota.\lambda V0w \in (ty\_2Eordinal\_2Eordinal\ A\_27a).($

**Definition 27** *We define* c_2Eordinal_2Eoleast *to be* $\lambda A\_27a : \iota.\lambda V0P \in (2^{(ty\_2Eordinal\_2Eordinal\ A\_27a)}).(ap$

**Definition 28** *We define* c_2Eordinal_2EordSUC *to be* $\lambda A\_27a : \iota.\lambda V0a \in (ty\_2Eordinal\_2Eordinal\ A\_27a)$

Let $c\_2Enum\_2EZERO\_\_REP : \iota$ be given. Assume the following.

$$c\_2Enum\_2EZERO\_\_REP \in omega \tag{13}$$

Let $c\_2Enum\_2EABS\_\_num : \iota$ be given. Assume the following.

$$c\_2Enum\_2EABS\_\_num \in (ty\_2Enum\_2Enum^{omega}) \tag{14}$$

**Definition 29** *We define* c_2Enum_2E0 *to be* $(ap\ c\_2Enum\_2EABS\_\_num\ c\_2Enum\_2EZERO\_\_REP)$.

Let $c\_2Eordinal\_2EfromNat : \iota \Rightarrow \iota$ be given. Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow c\_2Eordinal\_2EfromNat\ A\_27a \in ((ty\_2Eordinal\_2Eordinal\ A\_27a)^{ty\_2Enum\_2Enum}) \tag{15}$$

**Definition 30** *We define* c_2Epred__set_2EIMAGE *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in$

**Definition 31** *We define* c_2Epred__set_2EBIGUNION *to be* $\lambda A\_27a : \iota.\lambda V0P \in (2^{(2^{A\_27a})}).(ap\ (c\_2Epred\_\_s$

**Definition 32** *We define* c_2Eordinal_2Esup *to be* $\lambda A\_27a : \iota.\lambda V0ordset \in (2^{(ty\_2Eordinal\_2Eordinal\ A\_27a)}).($

**Definition 33** *We define* c_2Epred__set_2EUNIV *to be* $\lambda A_27a : \iota.(\lambda V0x \in A_27a.$c_2Ebool_2ET$)$.

**Definition 34** *We define* c_2Epred__set_2EINJ *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in (A\_27b^{A\_27a}).\lambda V1s \in (2^{A}$

**Definition 35** *We define* c_2Ecardinal_2Ecardleq *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0s1 \in (2^{A\_27a}).\lambda V1s2 \in (2^{A}$

**Definition 36** *We define* c_2Eveblen_2Econtinuous *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in ((ty\_2Eordinal\_2Eo$

**Definition 37** *We define* c_2Eveblen_2Estrict__mono *to be* $\lambda A\_27a : \iota.\lambda A\_27b : \iota.\lambda V0f \in ((ty\_2Eordinal\_2E$

Assume the following.

$$True \tag{16}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(((p\ V0t1) \Rightarrow (p\ V1t2)) \Rightarrow (((p\ V1t2) \Rightarrow (p\ V0t1)) \Rightarrow ((p\ V0t1) \Leftrightarrow (p\ V1t2)))))) \tag{17}$$

Assume the following.

$$(\forall V0t \in 2.(False \Rightarrow (p\ V0t))) \tag{18}$$

Assume the following.

$$(\forall V0t \in 2.((p\ V0t) \lor (\neg(p\ V0t)))) \tag{19}$$

Assume the following.

$$(\forall V0t \in 2.(((True \land (p\ V0t)) \Leftrightarrow (p\ V0t)) \land ((((p\ V0t) \land True) \Leftrightarrow (p\ V0t)) \land (((False \land (p\ V0t)) \Leftrightarrow False) \land ((((p\ V0t) \land False) \Leftrightarrow False) \land (((p\ V0t) \land (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{20}$$

Assume the following.

$$(\forall V0t \in 2.(((True \lor (p\ V0t)) \Leftrightarrow True) \land ((((p\ V0t) \lor True) \Leftrightarrow True) \land (((False \lor (p\ V0t)) \Leftrightarrow (p\ V0t)) \land ((((p\ V0t) \lor False) \Leftrightarrow (p\ V0t)) \land (((p\ V0t) \lor (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{21}$$

Assume the following.

$$((\forall V0t \in 2.((\neg(\neg(p\ V0t))) \Leftrightarrow (p\ V0t))) \land (((\neg True) \Leftrightarrow False) \land ((\neg False) \Leftrightarrow True))) \tag{22}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in A\_27a.(\forall V1y \in A\_27a.((V0x = V1y) \Leftrightarrow (V1y = V0x)))) \tag{23}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{24}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{A\_27a}).((\neg (\exists V1x \in A\_27a.(p\ (ap\ V0P\ V1x)))) \Leftrightarrow (\forall V2x \in A\_27a.(\neg (p\ (ap\ V0P\ V2x)))))) \tag{25}$$

Assume the following.

$$(\forall V0A \in 2.(\forall V1B \in 2.(((\neg ((p\ V0A) \wedge (p\ V1B))) \Leftrightarrow ((\neg (p\ V0A)) \vee (\neg (p\ V1B)))) \wedge ((\neg ((p\ V0A) \vee (p\ V1B))) \Leftrightarrow ((\neg (p\ V0A)) \wedge (\neg (p\ V1B))))))) \tag{26}$$

Assume the following.

$$(\forall V0t1 \in 2.(\forall V1t2 \in 2.(\forall V2t3 \in 2.(((p\ V0t1) \Rightarrow ((p\ V1t2) \Rightarrow (p\ V2t3))) \Leftrightarrow (((p\ V0t1) \wedge (p\ V1t2)) \Rightarrow (p\ V2t3)))))) \tag{27}$$

Assume the following.

$$(\forall V0x \in 2.(\forall V1x\_27 \in 2.(\forall V2y \in 2.(\forall V3y\_27 \in 2.((((p\ V0x) \Leftrightarrow (p\ V1x\_27)) \wedge ((p\ V1x\_27) \Rightarrow ((p\ V2y) \Leftrightarrow (p\ V3y\_27)))) \Rightarrow (((p\ V0x) \Rightarrow (p\ V2y)) \Leftrightarrow ((p\ V1x\_27) \Rightarrow (p\ V3y\_27)))))))) \tag{28}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow \forall A\_27c.nonempty\ A\_27c \Rightarrow (\forall V0f \in (A\_27c^{A\_27a}).(\forall V1s \in (2^{A\_27a}).(\forall V2t \in (2^{A\_27b}).((p\ (ap\ (ap\ (\mathsf{c\_2Ecardinal\_2Ecardleq}\ A\_27a\ A\_27b)\ V1s)\ V2t)) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Ecardinal\_2Ecardleq}\ A\_27c\ A\_27b)\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27a\ A\_27c)\ V0f)\ V1s))\ V2t)))))) \tag{29}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0w \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(\neg (p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V0w)\ V0w)))) \tag{30}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(\forall V1w \in (ty\_2Eordinal\_2Eordinal\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Eordinal\_2Eordinal\ A\_27a))\ V0x)\ (ap\ (\mathsf{c\_2Eordinal\_2Epreds}\ A\_27a)\ V1w))) \Leftrightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V0x)\ V1w))))) \tag{31}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0ord \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(p\ (ap\ (ap\ (\mathsf{c\_2Ecardinal\_2Ecardleq}\ (ty\_2Eordinal\_2Eordinal\ A\_27a)\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ A\_27a))\ (ap\ (\mathsf{c\_2Eordinal\_2Epreds}\ A\_27a)\ V0ord))\ (\mathsf{c\_2Epred\_\_set\_2EUNIV}\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ A\_27a)))))) \tag{32}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0b \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(\forall V1a \in (ty\_2Eordinal\_2Eordinal\ A\_27a).((\neg(p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V0b)\ V1a))) \Leftrightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V1a)\ V0b)) \vee (V1a = V0b))))) \tag{33}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(\forall V1y \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(\forall V2z \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(((\neg(p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V1y)\ V0x))) \wedge (\neg(p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V2z)\ V1y)))) \Rightarrow (\neg(p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V2z)\ V0x))))))) \tag{34}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0x \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(\forall V1y \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(\forall V2z \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(((\neg(p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V1y)\ V0x))) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V1y)\ V2z))) \Rightarrow (p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V0x)\ V2z)))))) \tag{35}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(\neg(p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V0a)\ (ap\ (c\_2Eordinal\_2EfromNat\ A\_27a)\ \mathsf{c\_2Enum\_2E0}))))) \tag{36}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V0a)\ (ap\ (\mathsf{c\_2Eordinal\_2EordSUC}\ A\_27a)\ V0a)))) \tag{37}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0a \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(\forall V1b \in (ty\_2Eordinal\_2Eordinal\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V0a)\ (ap\ (\mathsf{c\_2Eordinal\_2EordSUC}\ A\_27a)\ V1b))) \Leftrightarrow ((p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V0a)\ V1b)) \vee (V0a = V1b))))) \tag{38}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0s \in (2^{(ty\_2Eordinal\_2Eordinal\ A\_27a)}). ((p\ (ap\ (ap\ (\mathsf{c\_2Ecardinal\_2Ecardleq}\ (ty\_2Eordinal\_2Eordinal\ A\_27a)\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ A\_27a))\ V0s)\ (\mathsf{c\_2Epred\_\_set\_2EUNIV}\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ A\_27a))))) \Rightarrow (\forall V1a \in (ty\_2Eordinal\_2Eordinal\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V1a)\ (ap\ (\mathsf{c\_2Eordinal\_2Esup}\ A\_27a)\ V0s))) \Leftrightarrow (\exists V2b \in (ty\_2Eordinal\_2Eordinal\ A\_27a). ((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Eordinal\_2Eordinal\ A\_27a))\ V2b)\ V0s)) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V1a)\ V2b))))))))) \tag{39}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0b \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(\forall V1s \in (2^{(ty\_2Eordinal\_2Eordinal\ A\_27a)}).(( (p\ (ap\ (ap\ (\mathsf{c\_2Ecardinal\_2Ecardleq}\ (ty\_2Eordinal\_2Eordinal\ A\_27a)\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ A\_27a))\ V1s)\ (\mathsf{c\_2Epred\_\_set\_2EUNIV}\ (ty\_2Esum\_2Esum\ ty\_2Enum\_2Enum\ A\_27a)))) \wedge (p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ (ty\_2Eordinal\_2Eordinal\ A\_27a))\ V0b)\ V1s))) \Rightarrow (\neg (p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ (ap\ (\mathsf{c\_2Eordinal\_2Esup}\ A\_27a)\ V1s))\ V0b)))))) \tag{40}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow ( \forall V0x \in A\_27a.(\forall V1s \in (2^{A\_27a}).((p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27a)\ V0x)\ V1s)) \Rightarrow (\forall V2f \in (A\_27b^{A\_27a}).(p\ (ap\ (ap\ (\mathsf{c\_2Ebool\_2EIN}\ A\_27b)\ (ap\ V2f\ V0x))\ (ap\ (ap\ (\mathsf{c\_2Epred\_\_set\_2EIMAGE}\ A\_27a\ A\_27b)\ V2f)\ V1s))))))) \tag{41}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0P \in (2^{(ty\_2Eordinal\_2Eordinal\ A\_27a)}). (((p\ (ap\ V0P\ (ap\ (c\_2Eordinal\_2EfromNat\ A\_27a)\ \mathsf{c\_2Enum\_2E0}))) \wedge ((\forall V1a \in (ty\_2Eordinal\_2Eordinal\ A\_27a).((p\ (ap\ V0P\ V1a)) \Rightarrow (p\ (ap\ V0P\ (ap\ (\mathsf{c\_2Eordinal\_2EordSUC}\ A\_27a)\ V1a)))))) \wedge (\forall V2a \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(((p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ (ap\ (c\_2Eordinal\_2EfromNat\ A\_27a)\ \mathsf{c\_2Enum\_2E0}))\ V2a)) \wedge (\forall V3b \in (ty\_2Eordinal\_2Eordinal\ A\_27a).((p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ V3b)\ V2a)) \Rightarrow (p\ (ap\ V0P\ V3b))))) \Rightarrow (p\ (ap\ V0P\ (ap\ (\mathsf{c\_2Eordinal\_2Esup}\ A\_27a)\ (ap\ (\mathsf{c\_2Eordinal\_2Epreds}\ A\_27a)\ V2a)))))))) \Rightarrow (\forall V4a \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(p\ (ap\ V0P\ V4a))))) \tag{42}$$

**Theorem 1**

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0f \in ((ty\_2Eordinal\_2Eordinal\ A\_27a)^{(ty\_2Eordinal\_2Eordinal\ A\_27a)}).(\forall V1x \in (ty\_2Eordinal\_2Eordinal\ A\_27a).(((p\ (ap\ (\mathsf{c\_2Eveblen\_2Estrict\_\_mono}\ A\_27a\ A\_27a)\ V0f)) \land (p\ (ap\ (\mathsf{c\_2Eveblen\_2Econtinuous}\ A\_27a\ A\_27a)\ V0f))) \Rightarrow (\neg(p\ (ap\ (ap\ (\mathsf{c\_2Eordinal\_2Eordlt}\ A\_27a)\ (ap\ V0f\ V1x))\ V1x))))))$$