thm_2Ewellorder_2EWF__REC (TMWt85FAj83w1LZbZcRGAn6px3uV8xWviyh)

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Definition 1 *We define* c_2Emin_2E_3D *to be* $\lambda A.\lambda x \in A.\lambda y \in A.inj\_\_o\ (x = y)$ *of type* $\iota \Rightarrow \iota$.

Definition 2 *We define* c_2Ebool_2ET *to be* $(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^2))\ (\lambda V0x \in 2.V0x))\ (\lambda V1x \in 2.V$

Definition 3 *We define* c_2Ebool_2E_21 *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ (ap\ (\mathsf{c\_2Emin\_2E\_3D}\ (2^{A\_27}$

Definition 4 *We define* c_2Ebool_2EF *to be* $(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V0t \in 2.V0t))$.

Definition 5 *We define* c_2Emin_2E_3D_3D_3E *to be* $\lambda P \in 2.\lambda Q \in 2.inj\_\_o\ (p\ P \Rightarrow p\ Q)$ *of type* ι.

Definition 6 *We define* c_2Ebool_2E_7E *to be* $(\lambda V0t \in 2.(ap\ (ap\ \mathsf{c\_2Emin\_2E\_3D\_3D\_3E}\ V0t)\ \mathsf{c\_2Ebool\_2EF}$

Definition 7 *We define* c_2Emin_2E_40 *to be* $\lambda A.\lambda P \in 2^A.$**if** $(\exists x \in A.p\ (ap\ P\ x))$ **then** $(the\ (\lambda x.x \in A \wedge p$ *of type* $\iota \Rightarrow \iota$.

Definition 8 *We define* c_2Ebool_2E_3F *to be* $\lambda A\_27a : \iota.(\lambda V0P \in (2^{A\_27a}).(ap\ V0P\ (ap\ (\mathsf{c\_2Emin\_2E\_40}\ A$

Definition 9 *We define* c_2Ebool_2E_2F_5C *to be* $(\lambda V0t1 \in 2.(\lambda V1t2 \in 2.(ap\ (\mathsf{c\_2Ebool\_2E\_21}\ 2)\ (\lambda V2t \in 2$

Definition 10 *We define* c_2Erelation_2EWF *to be* $\lambda A\_27a : \iota.\lambda V0R \in ((2^{A\_27a})^{A\_27a}).(ap\ (\mathsf{c\_2Ebool\_2E\_21}$

Assume the following.

$$True \tag{1}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0t1 \in A\_27a.(\forall V1t2 \in A\_27b.((ap\ (\lambda V2x \in A\_27b. V0t1)\ V1t2) = V0t1))) \tag{2}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow (\forall V0t \in 2.((\forall V1x \in A\_27a.(p\ V0t)) \Leftrightarrow (p\ V0t))) \tag{3}$$

Assume the following.

$$(\forall V0t \in 2.(((True \wedge (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \wedge True) \Leftrightarrow (p\ V0t)) \wedge (((False \wedge (p\ V0t)) \Leftrightarrow False) \wedge ((((p\ V0t) \wedge False) \Leftrightarrow False) \wedge (((p\ V0t) \wedge (p\ V0t)) \Leftrightarrow (p\ V0t))))))) \tag{4}$$

Assume the following.

$$(\forall V0t \in 2.(((True \Leftrightarrow (p\ V0t)) \Leftrightarrow (p\ V0t)) \wedge ((((p\ V0t) \Leftrightarrow True) \Leftrightarrow (p\ V0t)) \wedge (((False \Leftrightarrow (p\ V0t)) \Leftrightarrow (\neg (p\ V0t))) \wedge (((p\ V0t) \Leftrightarrow False) \Leftrightarrow (\neg (p\ V0t))))))) \tag{5}$$

Assume the following.

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0\_3C\_3C \in ((2^{A\_27a})^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Erelation\_2EWF}\ A\_27a)\ V0\_3C\_3C)) \Rightarrow (\forall V1H \in ((A\_27b^{A\_27a})^{(A\_27b^{A\_27a})}).(\forall V2S \in ((2^{A\_27b})^{A\_27a}).((\forall V3f \in (A\_27b^{A\_27a}).(\forall V4g \in (A\_27b^{A\_27a}).(\forall V5x \in A\_27a.((\forall V6z \in A\_27a.((p\ (ap\ (ap\ V0\_3C\_3C\ V6z)\ V5x)) \Rightarrow (((ap\ V3f\ V6z) = (ap\ V4g\ V6z)) \wedge (p\ (ap\ (ap\ V2S\ V6z)\ (ap\ V3f\ V6z)))))) \Rightarrow (((ap\ (ap\ V1H\ V3f)\ V5x) = (ap\ (ap\ V1H\ V4g)\ V5x)) \wedge (p\ (ap\ (ap\ V2S\ V5x)\ (ap\ (ap\ V1H\ V3f)\ V5x)))))))) \Rightarrow (\exists V7f \in (A\_27b^{A\_27a}).(\forall V8x \in A\_27a.((ap\ V7f\ V8x) = (ap\ (ap\ V1H\ V7f)\ V8x))))))))) \tag{6}$$

Theorem 1

$$\forall A\_27a.nonempty\ A\_27a \Rightarrow \forall A\_27b.nonempty\ A\_27b \Rightarrow (\forall V0\_3C\_3C \in ((2^{A\_27a})^{A\_27a}).((p\ (ap\ (\mathsf{c\_2Erelation\_2EWF}\ A\_27a)\ V0\_3C\_3C)) \Rightarrow (\forall V1H \in ((A\_27b^{A\_27a})^{(A\_27b^{A\_27a})}).((\forall V2f \in (A\_27b^{A\_27a}).(\forall V3g \in (A\_27b^{A\_27a}).(\forall V4x \in A\_27a.((\forall V5z \in A\_27a.((p\ (ap\ (ap\ V0\_3C\_3C\ V5z)\ V4x)) \Rightarrow ((ap\ V2f\ V5z) = (ap\ V3g\ V5z)))) \Rightarrow ((ap\ (ap\ V1H\ V2f)\ V4x) = (ap\ (ap\ V1H\ V3g)\ V4x)))))) \Rightarrow (\exists V6f \in (A\_27b^{A\_27a}).(\forall V7x \in A\_27a.((ap\ V6f\ V7x) = (ap\ (ap\ V1H\ V6f)\ V7x)))))))))$$